8–1. A spherical gas tank has an inner radius of $r = 1.5$ m. If it is subjected to an internal pressure of $p = 300$ kPa, determine its required thickness if the maximum normal stress is not to exceed 12 MPa.

$$\sigma_{\text{allow}} = \frac{p\,r}{2\,t}; \qquad 12(10^6) = \frac{300(10^3)(1.5)}{2\,t}$$

$t = 0.0188 \text{ m} = 18.8 \text{ mm}$ **Ans.**

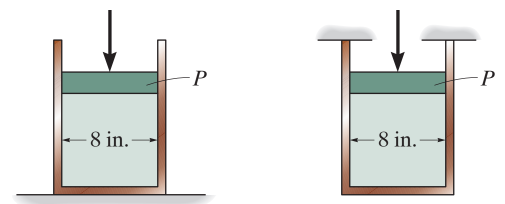
8–2. A pressurized spherical tank is to be made of 0.5-in.-thick steel. If it is subjected to an internal pressure of $p = 200$ psi, determine its outer radius if the maximum normal stress is not to exceed 15 ksi.

$$\sigma_{\text{allow}} = \frac{p\,r}{2\,t}; \qquad 15(10^3) = \frac{200\,r_i}{2(0.5)}$$

$r_i = 75$ in.

$r_o = 75 \text{ in.} + 0.5 \text{ in.} = 75.5 \text{ in.}$ **Ans.**

8–3. The thin-walled cylinder can be supported in one of two ways as shown. Determine the state of stress in the wall of the cylinder for both cases if the piston P causes the internal pressure to be 65 psi. The wall has a thickness of 0.25 in. and the inner diameter of the cylinder is 8 in.



(a) (b)

Case (a):

$$\sigma_1 = \frac{pr}{t}; \qquad \sigma_1 = \frac{65(4)}{0.25} = 1.04 \text{ ksi}$$ **Ans.**

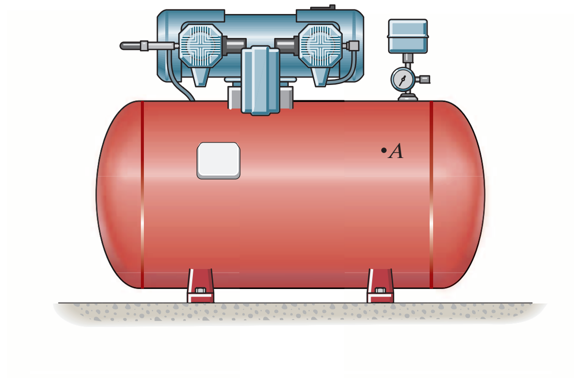
$$\sigma_2 = 0$$ **Ans.**

Case (b):

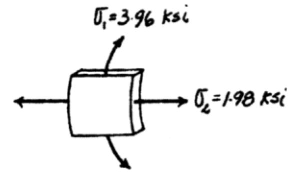
$$\sigma_1 = \frac{pr}{t}; \qquad \sigma_1 = \frac{65(4)}{0.25} = 1.04 \text{ ksi}$$ **Ans.**

$$\sigma_2 = \frac{pr}{2t}; \qquad \sigma_2 = \frac{65(4)}{2(0.25)} = 520 \text{ psi}$$ **Ans.**

***8–4.** The tank of the air compressor is subjected to an internal pressure of 90 psi. If the internal diameter of the tank is 22 in., and the wall thickness is 0.25 in., determine the stress components acting at point A. Draw a volume element of the material at this point, and show the results on the element.



Hoop Stress for Cylindrical Vessels: Since $\frac{r}{t} = \frac{11}{0.25} = 44 > 10$, then *thin wall* analysis can be used. Applying Eq. 8–1

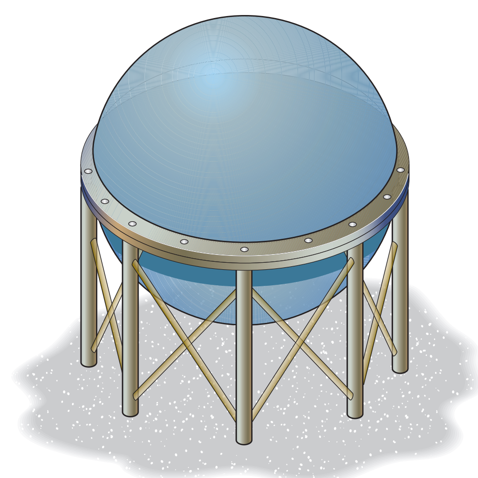


$$\sigma_1 = \frac{pr}{t} = \frac{90(11)}{0.25} = 3960 \text{ psi} = 3.96 \text{ ksi} \qquad \textbf{Ans.}$$

Longitudinal Stress for Cylindrical Vessels: Applying Eq. 8–2

$$\sigma_2 = \frac{pr}{2t} = \frac{90(11)}{2(0.25)} = 1980 \text{ psi} = 1.98 \text{ ksi} \qquad \textbf{Ans.}$$

•8–5. The spherical gas tank is fabricated by bolting together two hemispherical thin shells of thickness 30 mm. If the gas contained in the tank is under a gauge pressure of 2 MPa, determine the normal stress developed in the wall of the tank and in each of the bolts. The tank has an inner diameter of 8 m and is sealed with 900 bolts each 25 mm in diameter.



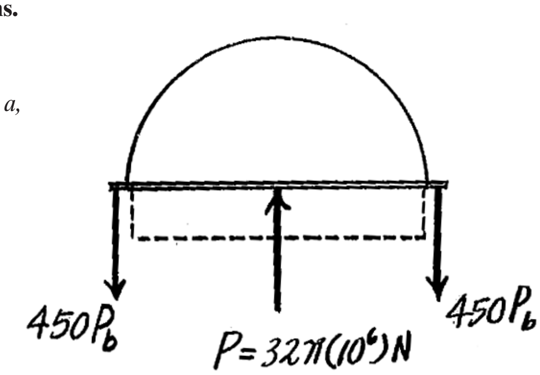
Normal Stress: Since $\frac{r}{t} = \frac{4}{0.03} = 133.33 > 10$, thin-wall analysis is valid. For the spherical tank's wall,

$$\sigma = \frac{pr}{2t} = \frac{2(4)}{2(0.03)} = 133 \text{ MPa} \qquad \textbf{Ans.}$$

Referring to the free-body diagram shown in Fig. *a*, $P = pA = 2\left(10^6\right)\left[\frac{\pi}{4}\left(8^2\right)\right] = 32\pi\left(10^6\right)$ N. Thus,

$$+\uparrow \Sigma F_y = 0; \qquad 32\pi\left(10^6\right) - 450P_b - 450P_b = 0$$

$$P_b = 35.56\left(10^3\right)\pi \text{ N}$$

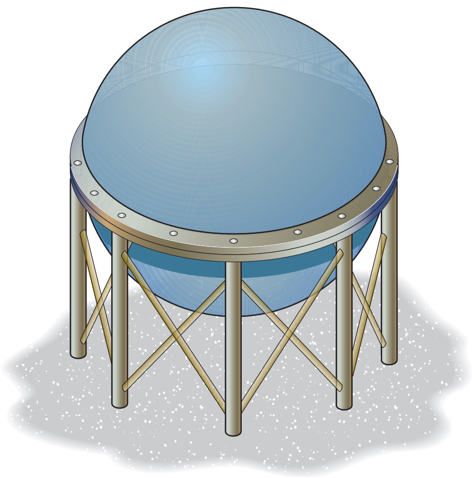


(a)

The normal stress developed in each bolt is then

$$\sigma_b = \frac{P_b}{A_b} = \frac{35.56\left(10^3\right)\pi}{\frac{\pi}{4}\left(0.025^2\right)} = 228 \text{ MPa} \qquad \textbf{Ans.}$$

8–6. The spherical gas tank is fabricated by bolting together two hemispherical thin shells. If the 8-m inner diameter tank is to be designed to withstand a gauge pressure of 2 MPa, determine the minimum wall thickness of the tank and the minimum number of 25-mm diameter bolts that must be used to seal it. The tank and the bolts are made from material having an allowable normal stress of 150 MPa and 250 MPa, respectively.



Normal Stress: For the spherical tank's wall,

$$\sigma_{\text{allow}} = \frac{pr}{2t}$$

$$150\left(10^6\right) = \frac{2\left(10^6\right)(4)}{2t}$$

$$t = 0.02667 \text{ m} = 26.7 \text{ mm} \qquad \textbf{Ans.}$$

Since $\frac{r}{t} = \frac{4}{0.02667} = 150 > 10$, thin-wall analysis is valid.

Referring to the free-body diagram shown in Fig. *a*, $P = pA = 2\left(10^6\right)\left[\frac{\pi}{4}\left(8^2\right)\right] = 32\pi\left(10^6\right)$ N. Thus,

$$+\uparrow \Sigma F_y = 0; \qquad 32\pi\left(10^6\right) - \frac{n}{2}(P_b)_{\text{allow}} - \frac{n}{2}(P_b)_{\text{allow}} = 0$$

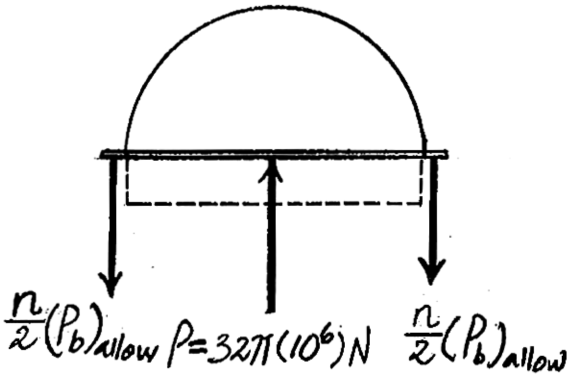
$$n = \frac{32\pi\left(10^6\right)}{(P_b)_{\text{allow}}} \qquad \textbf{(1)}$$

The allowable tensile force for each bolt is

$$(P_b)_{\text{allow}} = \sigma_{\text{allow}} A_b = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.025^2\right)\right] = 39.0625\left(10^3\right)\pi \text{N}$$

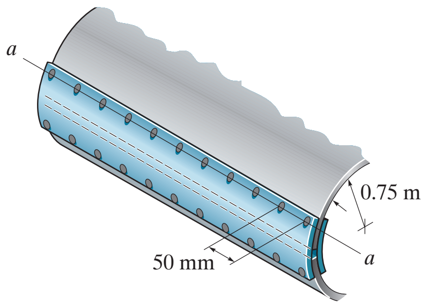
Substituting this result into Eq. (1),

$$n = \frac{32\pi\left(10^6\right)}{39.0625\pi\left(10^3\right)} = 819.2 = 820 \qquad \textbf{Ans.}$$



(a)

8–7. A boiler is constructed of 8-mm thick steel plates that are fastened together at their ends using a butt joint consisting of two 8-mm cover plates and rivets having a diameter of 10 mm and spaced 50 mm apart as shown. If the steam pressure in the boiler is 1.35 MPa, determine (a) the circumferential stress in the boiler's plate apart from the seam, (b) the circumferential stress in the outer cover plate along the rivet line *a–a*, and (c) the shear stress in the rivets.



a) $$\sigma_1 = \frac{pr}{t} = \frac{1.35(10^6)(0.75)}{0.008} = 126.56(10^6) = 127 \text{ MPa}$$ **Ans.**

b) $$126.56\,(10^6)(0.05)(0.008) = \sigma_1'(2)(0.04)(0.008)$$

$$\sigma_1' = 79.1\,\text{MPa}$$ **Ans.**

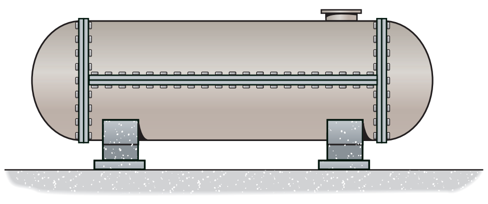
c) From FBD(a)

$$+\uparrow \Sigma F_y = 0; \qquad F_b - 79.1(10^6)[(0.008)(0.04)] = 0$$

$$F_b = 25.3 \text{ kN}$$

$$(\tau_{avg})_b = \frac{F_b}{A} - \frac{25312.5}{\frac{\pi}{4}(0.01)^2} = 322\ MPa$$ **Ans.**

***8–8.** The gas storage tank is fabricated by bolting together two half cylindrical thin shells and two hemispherical shells as shown. If the tank is designed to withstand a pressure of 3 MPa, determine the required minimum thickness of the cylindrical and hemispherical shells and the minimum required number of longitudinal bolts per meter length at each side of the cylindrical shell. The tank and the 25 mm diameter bolts are made from material having an allowable normal stress of 150 MPa and 250 MPa, respectively. The tank has an inner diameter of 4 m.



Normal Stress: For the cylindrical portion of the tank, the hoop stress is twice as large as the longitudinal stress.

$$\sigma_{\text{allow}} = \frac{pr}{t}; \qquad 150\left(10^6\right) = \frac{3\left(10^6\right)(2)}{t_c}$$

$$t_c = 0.04\,\text{m} = 40\,\text{mm} \qquad \textbf{Ans.}$$

For the hemispherical cap,

$$\sigma_{\text{allow}} = \frac{pr}{t}; \qquad 150\left(10^6\right) = \frac{3\left(10^6\right)(2)}{2t_s}$$

$$t_s = 0.02\,\text{m} = 20\text{ mm} \qquad \textbf{Ans.}$$

Since $\frac{r}{t} < 10$, thin-wall analysis is valid.

Referring to the free-body diagram of the per meter length of the cylindrical portion, Fig. *a*, where $P = pA = 3\left(10^6\right)[4(1)] = 12\left(10^6\right)\text{N}$, we have

$$+\uparrow \Sigma F_y = 0; \qquad 12\left(10^6\right) - n_c(P_b)_{\text{allow}} - n_c(P_b)_{\text{allow}} = 0$$

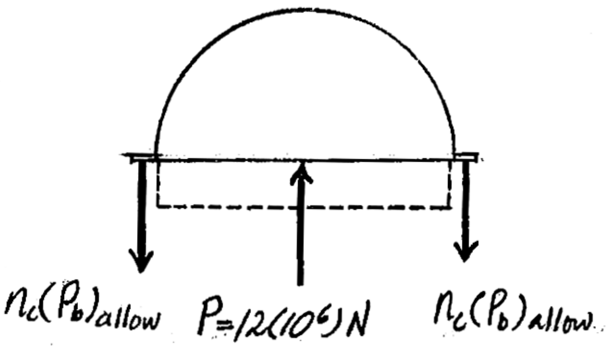
$$n_c = \frac{6\left(10^6\right)}{(P_b)_{\text{allow}}} \qquad \textbf{(1)}$$

The allowable tensile force for each bolt is

$$(P_b)_{\text{allow}} = \sigma_{\text{allow}} A_b = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.025^2\right)\right] = 122.72\left(10^3\right)\text{N}$$

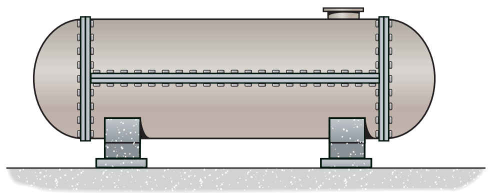
Substituting this result into Eq. (1),

$$n_c = 48.89 = 49 \text{ bolts/meter} \qquad \textbf{Ans.}$$

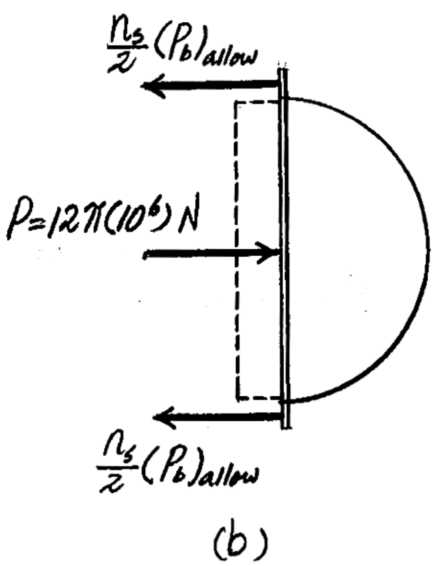


(a)

•**8–9.** The gas storage tank is fabricated by bolting together two half cylindrical thin shells and two hemispherical shells as shown. If the tank is designed to withstand a pressure of 3 MPa, determine the required minimum thickness of the cylindrical and hemispherical shells and the minimum required number of bolts for each hemispherical cap. The tank and the 25 mm diameter bolts are made from material having an allowable normal stress of 150 MPa and 250 MPa, respectively. The tank has an inner diameter of 4 m.



Normal Stress: For the cylindrical portion of the tank, the hoop stress is twice as large as the longitudinal stress.



$$\sigma_{\text{allow}} = \frac{pr}{t}; \qquad 150\left(10^6\right) = \frac{3\left(10^6\right)(2)}{t_c}$$

$$t_c = 0.04 \text{ m} = 40 \text{ mm} \qquad \textbf{Ans.}$$

For the hemispherical cap,

$$\sigma_{\text{allow}} = \frac{pr}{t}; \qquad 150\left(10^6\right) = \frac{3\left(10^6\right)(2)}{2t_s}$$

$$t_s = 0.02 \text{ m} = 20 \text{ mm} \qquad \textbf{Ans.}$$

Since $\frac{r}{t} < 10$, thin-wall analysis is valid.

The allowable tensile force for each bolt is

$$(P_b)_{\text{allow}} = \sigma_{\text{allow}} A_b = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.025^2\right)\right] = 122.72\left(10^3\right)\text{N}$$

Referring to the free-body diagram of the hemispherical cap, Fig. *b*, where $P = pA = 3\left(10^6\right)\left[\frac{\pi}{4}\left(4^2\right)\right] = 12\pi\left(10^6\right)\text{N}$,

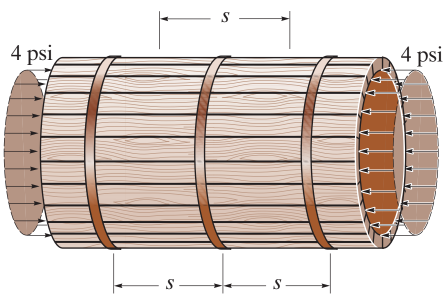
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 12\pi\left(10^6\right) - \frac{n_s}{2}(P_b)_{\text{allow}} - \frac{n_s}{2}(P_b)_{\text{allow}} = 0$$

$$n_s = \frac{12\pi\left(10^6\right)}{(P_b)_{\text{allow}}} \qquad \textbf{(1)}$$

Substituting this result into Eq. (1),

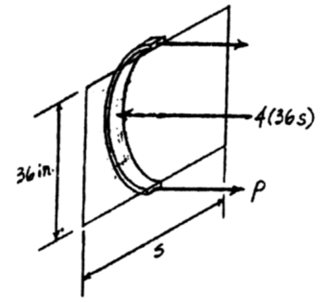
$$n_s = 307.2 = 308 \text{ bolts} \qquad \textbf{Ans.}$$

8–10. A wood pipe having an inner diameter of 3 ft is bound together using steel hoops each having a cross-sectional area of 0.2 in^2. If the allowable stress for the hoops is $\sigma_{\text{allow}} = 12$ ksi, determine their maximum spacing s along the section of pipe so that the pipe can resist an internal gauge pressure of 4 psi. Assume each hoop supports the pressure loading acting along the length s of the pipe.



Equilibrium for the steel Hoop: From the FBD

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 2P - 4(36s) = 0 \qquad P = 72.0s$



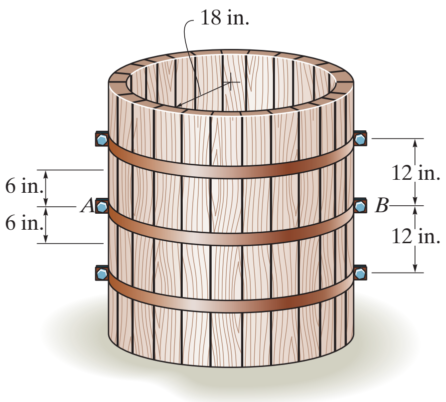
Hoop Stress for the Steel Hoop:

$$\sigma_1 = \sigma_{\text{allow}} = \frac{P}{A}$$

$$12(10^3) = \frac{72.0s}{0.2}$$

$$s = 33.3 \text{ in.}$$ **Ans.**

8–11. The staves or vertical members of the wooden tank are held together using semicircular hoops having a thickness of 0.5 in. and a width of 2 in. Determine the normal stress in hoop AB if the tank is subjected to an internal gauge pressure of 2 psi and this loading is transmitted directly to the hoops. Also, if 0.25-in.-diameter bolts are used to connect each hoop together, determine the tensile stress in each bolt at A and B. Assume hoop AB supports the pressure loading within a 12-in. length of the tank as shown.

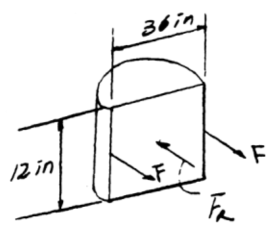


$F_R = 2(36)(12) = 864$ lb

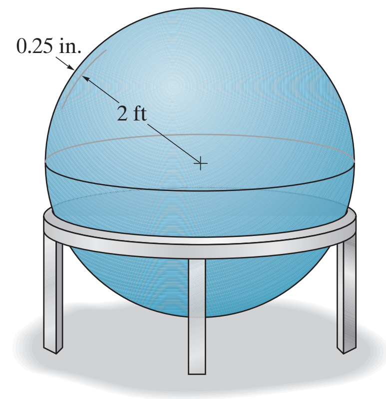
$\Sigma F = 0; \quad 864 - 2F = 0; \quad F = 432$ lb

$$\sigma_h = \frac{F}{A_h} = \frac{432}{0.5(2)} = 432 \text{ psi}$$ **Ans.**

$$\sigma_b = \frac{F}{A_b} = \frac{432}{\frac{\pi}{4}(0.25)^2} = 8801 \text{ psi} = 8.80 \text{ ksi}$$ **Ans.**

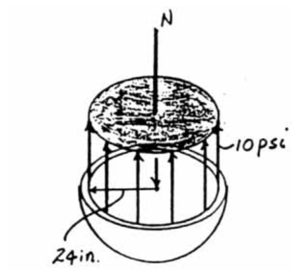


***8–12.** Two hemispheres having an inner radius of 2 ft and wall thickness of 0.25 in. are fitted together, and the inside gauge pressure is reduced to −10 psi. If the coefficient of static friction is $\mu_s = 0.5$ between the hemispheres, determine (a) the torque T needed to initiate the rotation of the top hemisphere relative to the bottom one, (b) the vertical force needed to pull the top hemisphere off the bottom one, and (c) the horizontal force needed to slide the top hemisphere off the bottom one.



Normal Pressure: Vertical force equilibrium for FBD(a).

$$+\uparrow \Sigma F_y = 0; \qquad 10\left[\pi(24^2)\right] - N = 0 \qquad N = 5760\pi \text{ lb}$$



The Friction Force: Applying friction formula

$$F_f = \mu_s N = 0.5(5760\pi) = 2880\pi \text{ lb}$$

a) ***The Required Torque:*** In order to initiate rotation of the two hemispheres relative to each other, the torque must overcome the moment produced by the friction force about the center of the sphere.

$$T = F_f r = 2880\pi(2 + 0.125/12) = 18190 \text{ lb}\cdot\text{ft} = 18.2 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans.}$$

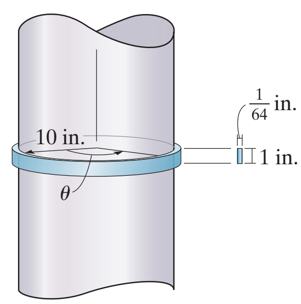
b) ***The Required Vertical Force:*** In order to just pull the two hemispheres apart, the vertical force P must overcome the normal force.

$$P = N = 5760\pi = 18096 \text{ lb} = 18.1 \text{ kip} \qquad \textbf{Ans.}$$

c) ***The Required Horizontal Force:*** In order to just cause the two hemispheres to slide relative to each other, the horizontal force F must overcome the friction force.

$$F = F_f = 2880\pi = 9048 \text{ lb} = 9.05 \text{ kip} \qquad \textbf{Ans.}$$

•**8–13.** The 304 stainless steel band initially fits snugly around the smooth rigid cylinder. If the band is then subjected to a nonlinear temperature drop of $\Delta T = 20 \sin^2\theta$ °F, where θ is in radians, determine the circumferential stress in the band.



Compatibility: Since the band is fixed to a rigid cylinder (it does not deform under load), then

$$\delta_F - \delta_T = 0$$

$$\frac{P(2\pi r)}{AE} - \int_0^{2\pi} \alpha \Delta T r\, d\theta = 0$$

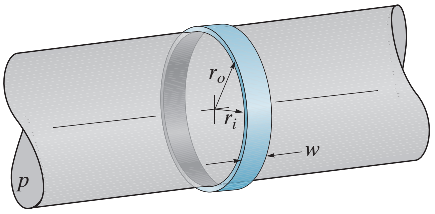
$$\frac{2\pi r}{E}\left(\frac{P}{A}\right) = 20\alpha r \int_0^{2\pi} \sin^2\theta\, d\theta \qquad \text{however, } \frac{P}{A} = \sigma_c$$

$$\frac{2\pi}{E}\sigma_c = 10\alpha \int_0^{2\pi} (1 - \cos 2\theta)\, d\theta$$

$$\sigma_c = 10\alpha E$$

$$= 10(9.60)\left(10^{-6}\right) 28.0\left(10^3\right) = 2.69 \text{ ksi} \qquad \textbf{Ans.}$$

8–14. The ring, having the dimensions shown, is placed over a flexible membrane which is pumped up with a pressure p. Determine the change in the internal radius of the ring after this pressure is applied. The modulus of elasticity for the ring is E.



Equilibrium for the Ring: Form the FBD

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2P - 2pr_iw = 0 \qquad P = pr_iw$$

Hoop Stress and Strain for the Ring:

$$\sigma_1 = \frac{P}{A} = \frac{pr_iw}{(r_s - r_i)w} = \frac{pr_i}{r_s - r_i}$$

Using Hooke's Law

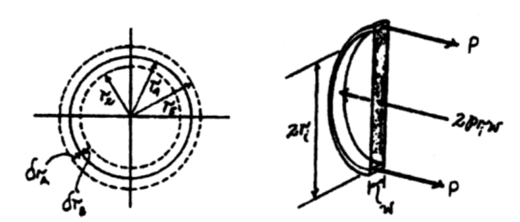
$$\varepsilon_1 = \frac{\sigma_1}{E} = \frac{pr_i}{E(r_s - r_i)} \qquad \textbf{[1]}$$

However, $\qquad \varepsilon_1 = \dfrac{2\pi(r_i)_1 - 2\pi r_i}{2\pi r} = \dfrac{(r_i)_1 - r_i}{r_i} = \dfrac{\delta r_i}{r_i}$.

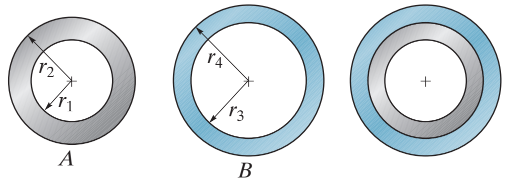
Then, from Eq. [1]

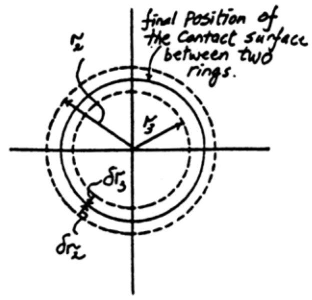
$$\frac{\delta r_i}{r_i} = \frac{pr_i}{E(r_s - r_i)}$$

$$\delta r_i = \frac{pr_i^2}{E(r_s - r_i)} \qquad \textbf{Ans.}$$



8–15. The inner ring A has an inner radius r_1 and outer radius r_2. Before heating, the outer ring B has an inner radius r_3 and an outer radius r_4, and $r_2 > r_3$. If the outer ring is heated and then fitted over the inner ring, determine the pressure between the two rings when ring B reaches the temperature of the inner ring. The material has a modulus of elasticity of E and a coefficient of thermal expansion of α.





Equilibrium for the Ring: From the FBD

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2P - 2pr_iw = 0 \qquad P = pr_iw$$

Hoop Stress and Strain for the Ring:

$$\sigma_1 = \frac{P}{A} = \frac{pr_iw}{(r_o - r_i)w} = \frac{pr_i}{r_o - r_i}$$

Using Hooke's law

$$\varepsilon_1 = \frac{\sigma_1}{E} = \frac{pr_i}{E(r_o - r_i)} \qquad \textbf{[1]}$$

However, $\varepsilon_1 = \dfrac{2\pi(r_i)_1 - 2\pi r_i}{2\pi r} = \dfrac{(r_i)_1 - r_i}{r_i} = \dfrac{\delta r_i}{r_i}$.

Then, from Eq. [1]

$$\frac{\delta r_i}{r_i} = \frac{pr_i}{E(r_o - r_i)}$$

$$\delta r_i = \frac{pr_i^2}{E(r_o - r_i)}$$

Compatibility: The pressure between the rings requires

$$\delta r_2 + \delta r_3 = r_2 - r_3 \qquad \textbf{[2]}$$

From the result obtained above

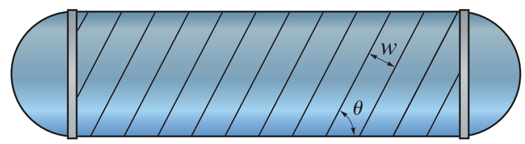
$$\delta r_2 = \frac{pr_2^2}{E(r_2 - r_1)} \qquad \delta r_3 = \frac{pr_3^2}{E(r_4 - r_3)}$$

Substitute into Eq. [2]

$$\frac{pr_2^2}{E(r_2 - r_1)} + \frac{pr_3^2}{E(r_4 - r_3)} = r_2 - r_3$$

$$p = \frac{E(r_2 - r_3)}{\dfrac{r_2^2}{r_2 - r_1} + \dfrac{r_3^2}{r_4 - r_3}} \qquad \textbf{Ans.}$$

***8–16.** The cylindrical tank is fabricated by welding a strip of thin plate helically, making an angle θ with the longitudinal axis of the tank. If the strip has a width w and thickness t, and the gas within the tank of diameter d is pressured to p, show that the normal stress developed along the strip is given by $\sigma_\theta = (pd/8t)(3 - \cos 2\theta)$.



Normal Stress:

$$\sigma_h = \sigma_1 = \frac{pr}{t} = \frac{p(d/2)}{t} = \frac{pd}{2t}$$

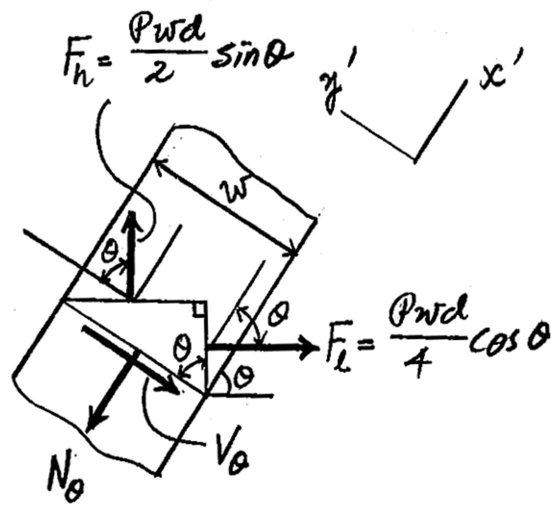
$$\sigma_l = \sigma_2 = \frac{pr}{2t} = \frac{p(d/2)}{2t} = \frac{pd}{4t}$$

Equilibrium: We will consider the triangular element cut from the strip shown in Fig. *a*. Here,

$A_h = (w \sin\theta)t$ and $A_l = (w \cos\theta)t$. Thus,

$F_h = \sigma_h A_h = \dfrac{pd}{2t}(w \sin\theta)t = \dfrac{pwd}{2}\sin\theta$ and

$$F_l = \sigma_l A_l = \frac{pd}{4t}(w \cos\theta)t = \frac{pwd}{4}\cos\theta.$$



(a)

Writing the force equation of equilibrium along the x' axis,

$$\Sigma F_{x'} = 0; \quad \left[\frac{pwd}{2}\sin\theta\right]\sin\theta + \left[\frac{pwd}{4}\cos\theta\right]\cos\theta - N_\theta = \theta$$

$$N_\theta = \frac{pwd}{4}\left(2\sin^2\theta + \cos^2\theta\right)$$

However, $\sin^2\theta + \cos^2\theta = 1$. This equation becomes

$$N_\theta = \frac{pwd}{4}\left(\sin^2\theta + 1\right)$$

Also, $\sin^2\theta = \dfrac{1}{2}(1 - \cos 2\theta)$, so that

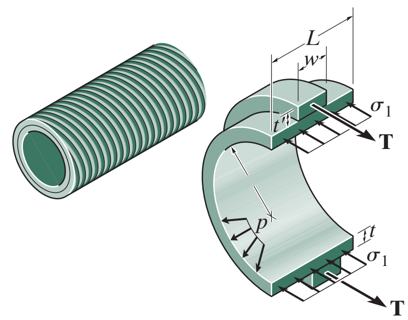
$$N_\theta = \frac{pwd}{8}(3 - \cos 2\theta)$$

Since $A_\theta = wt$, then

$$\sigma_\theta = \frac{N_\theta}{A_\theta} = \frac{\frac{pwd}{8}(3 - \cos 2\theta)}{wt}$$

$$\sigma_\theta = \frac{pd}{8t}(3 - \cos 2\theta) \qquad \textbf{(Q.E.D.)}$$

8–17. In order to increase the strength of the pressure vessel, filament winding of the same material is wrapped around the circumference of the vessel as shown. If the pretension in the filament is T and the vessel is subjected to an internal pressure p, determine the hoop stresses in the filament and in the wall of the vessel. Use the free-body diagram shown, and assume the filament winding has a thickness t' and width w for a corresponding length of the vessel.



Normal Stress in the Wall and Filament Before the Internal Pressure is Applied: The entire length w of wall is subjected to pretension filament force T. Hence, from equilibrium, the normal stress in the wall at this state is

$$2T - (\sigma_l')_w (2wt) = 0 \qquad (\sigma_l')_w = \frac{T}{wt}$$

and for the filament the normal stress is

$$(\sigma_l')_{fil} = \frac{T}{wt'}$$

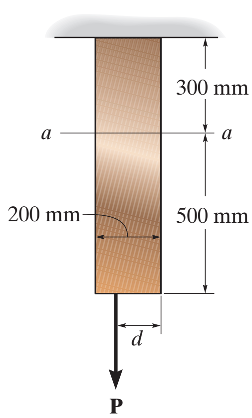
Normal Stress in the Wall and Filament After the Internal Pressure is Applied: The stress in the filament becomes

$$\sigma_{fil} = \sigma_l + (\sigma_l')_{fil} = \frac{pr}{(t + t')} + \frac{T}{wt'} \qquad \textbf{Ans.}$$

And for the wall,

$$\sigma_w = \sigma_l - (\sigma_l')_w = \frac{p\,r}{(t + t')} - \frac{T}{wt} \qquad \textbf{Ans.}$$

8–18. The vertical force **P** acts on the bottom of the plate having a negligible weight. Determine the shortest distance d to the edge of the plate at which it can be applied so that it produces no compressive stresses on the plate at section a–a. The plate has a thickness of 10 mm and **P** acts along the center line of this thickness.



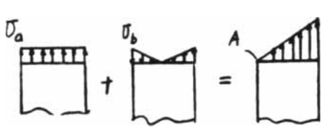
$$\sigma_A = 0 = \sigma_a - \sigma_b$$

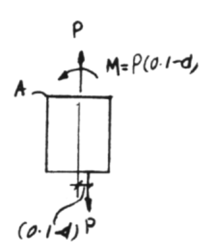
$$0 = \frac{P}{A} - \frac{M\,c}{I}$$

$$0 = \frac{P}{(0.2)(0.01)} - \frac{P(0.1 - d)(0.1)}{\frac{1}{12}(0.01)(0.2^3)}$$

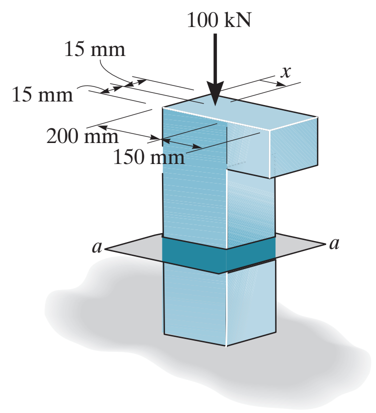
$$P(-1000 + 15000\,d) = 0$$

$$d = 0.0667 \text{ m} = 66.7 \text{ mm} \qquad \textbf{Ans.}$$





8–19. Determine the maximum and minimum normal stress in the bracket at section *a–a* when the load is applied at $x = 0$.



Consider the equilibrium of the FBD of the top cut segment in Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad N - 100 = 0 \qquad N = 100 \text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \qquad 100(0.1) - M = 0 \qquad M = 10 \text{ kN} \cdot \text{m}$

$A = 0.2(0.03) = 0.006 \text{ m}^2 \qquad I = \frac{1}{12}(0.03)(0.2^3) = 20.0(10^{-6}) \text{ m}^4$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

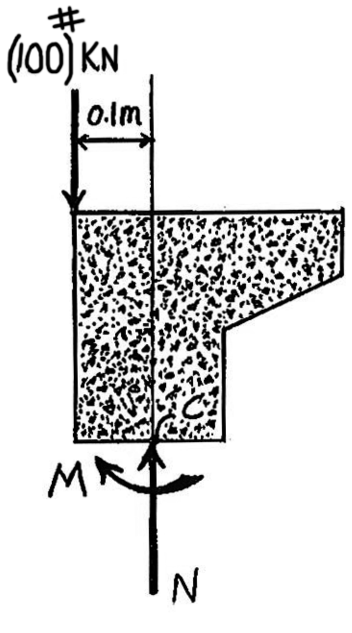
For the left edge fiber, $y = C = 0.1$ m. Then

$$\sigma_L = -\frac{100(10^3)}{0.006} - \frac{10(10^3)(0.1)}{20.0(10^{-6})}$$

$$= -66.67(10^6) \text{ Pa} = 66.7 \text{ MPa (C) (Max)}$$ **Ans.**

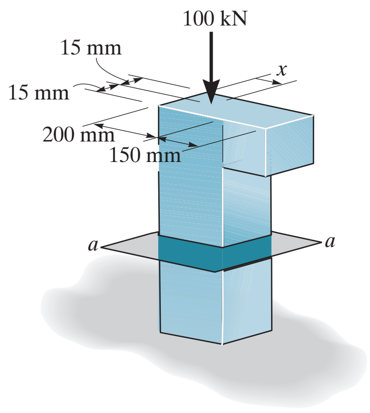
For the right edge fiber, $y = 0.1$ m. Then

$$\sigma_R = -\frac{100\,(10^3)}{0.006} + \frac{10(10^3)(0.1)}{20.0(10^{-6})} = 33.3 \text{ MPa (T)}$$ **Ans.**



(a)

***8–20.** Determine the maximum and minimum normal stress in the bracket at section *a–a* when the load is applied at $x = 300$ mm.



Consider the equilibrium of the FBD of the top cut segment in Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad N - 100 = 0 \qquad N = 100 \text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \qquad M - 100(0.2) = 0 \qquad M = 20 \text{ kN}\cdot\text{m}$

$A = 0.2\,(0.03) = 0.006 \text{ m}^2 \qquad I = \frac{1}{12}(0.03)(0.2^3) = 20.0(10^{-6}) \text{ m}^4$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{M_y}{I}$$

For the left edge fiber, $y = C = 0.1$ m. Then

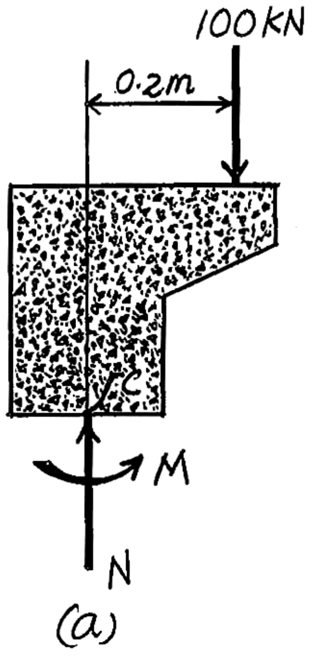
$$\sigma_C = -\frac{100(10^3)}{0.006} + \frac{20.0(10^3)(0.1)}{20.0(10^{-6})}$$

$$= 83.33(10^6) \text{ Pa} = 83.3 \text{ MPa (T)(Min)}$$ **Ans.**

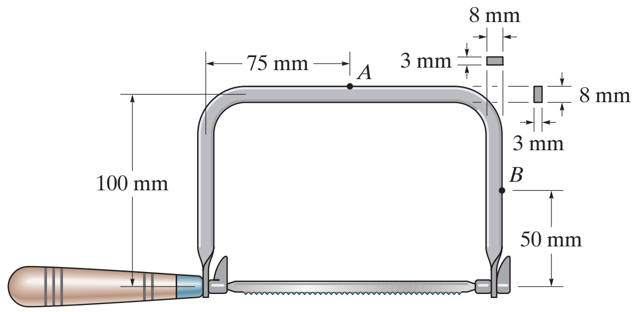
For the right edge fiber, $y = C = 0.1$ m. Thus

$$\sigma_R = -\frac{100(10^3)}{0.006} - \frac{20.0(10^3)(0.1)}{20.0(10^{-6})}$$

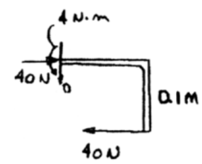
$$= 117 \text{ MPa}$$ **Ans.**



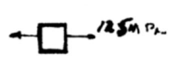
•8–21. The coping saw has an adjustable blade that is tightened with a tension of 40 N. Determine the state of stress in the frame at points *A* and *B*.

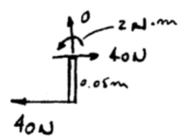


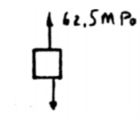
$$\sigma_A = -\frac{P}{A} + \frac{Mc}{I} = -\frac{40}{(0.008)(0.003)} + \frac{4(0.004)}{\frac{1}{12}(0.003)(0.008)^3} = 123 \text{ MPa}$$ **Ans.**



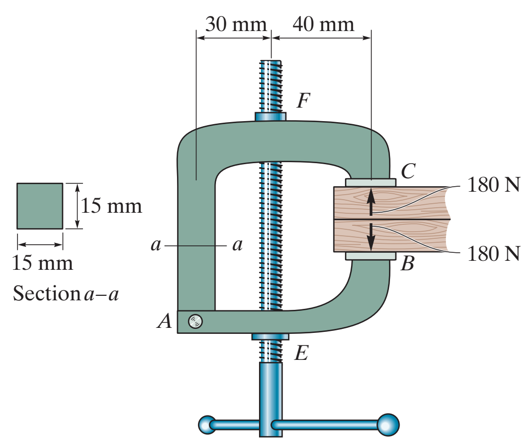
$$\sigma_B = \frac{Mc}{I} = \frac{2(0.004)}{\frac{1}{12}(0.003)(0.008)^3} = 62.5 \text{ MPa}$$ **Ans.**



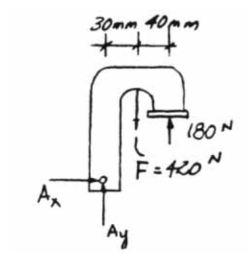




8–22. The clamp is made from members *AB* and *AC*, which are pin connected at *A*. If it exerts a compressive force at *C* and *B* of 180 N, determine the maximum compressive stress in the clamp at section *a–a*. The screw *EF* is subjected only to a tensile force along its axis.

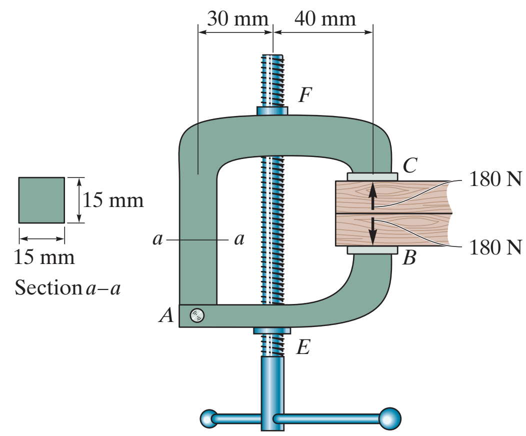


There is no moment in this problem. Therefore, the compressive stress is produced by axial force only.



$$\sigma_{max} = \frac{P}{A} = \frac{240}{(0.015)(0.015)} = 1.07 \text{ MPa}$$ **Ans.**

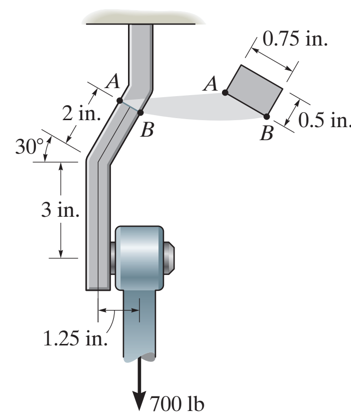
8–23. The clamp is made from members *AB* and *AC*, which are pin connected at *A*. If it exerts a compressive force at *C* and *B* of 180 N, sketch the stress distribution acting over section *a–a*. The screw *EF* is subjected only to a tensile force along its axis.



There is moment in this problem. Therefore, the compressive stress is produced by axial force only.

$$\sigma_{max} = \frac{P}{A} = \frac{240}{(0.015)(0.015)} = 1.07 \text{ MPa}$$

***8–24.** The bearing pin supports the load of 700 lb. Determine the stress components in the support member at point *A*. The support is 0.5 in. thick.



$$\Sigma F_x = 0; \qquad N - 700\cos 30° = 0; \qquad N = 606.218 \text{ lb}$$

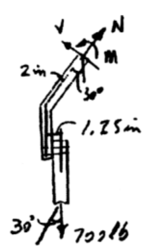
$$\Sigma F_y = 0; \qquad V - 700\sin 30° = 0; \qquad V = 350 \text{ lb}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M - 700(1.25 - 2\sin 30°) = 0; \qquad M = 175 \text{ lb}\cdot\text{in.}$$

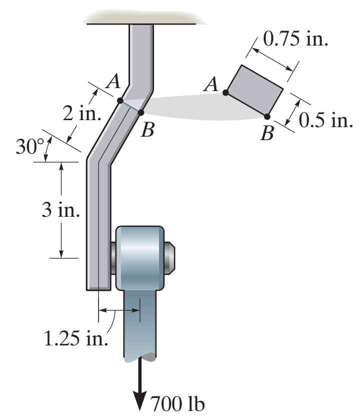
$$\sigma_A = \frac{N}{A} - \frac{Mc}{I} = \frac{606.218}{(0.75)(0.5)} - \frac{(175)(0.375)}{\frac{1}{12}(0.5)(0.75)^3}$$

$$\sigma_A = -2.12 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_A = 0 \qquad (\text{since } Q_A = 0) \qquad \textbf{Ans.}$$



•8–25. The bearing pin supports the load of 700 lb. Determine the stress components in the support member at point *B*. The support is 0.5 in. thick.



$\Sigma F_x = 0; \qquad N - 700 \cos 30° = 0; \qquad N = 606.218 \text{ lb}$

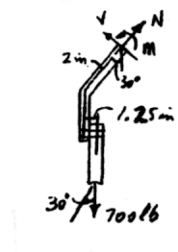
$\Sigma F_y = 0; \qquad V - 700 \sin 30° = 0; \qquad V = 350 \text{ lb}$

$\circlearrowleft + \Sigma M = 0; \qquad M - 700(1.25 - 2 \sin 30°) = 0; \qquad M = 175 \text{ lb} \cdot \text{in.}$

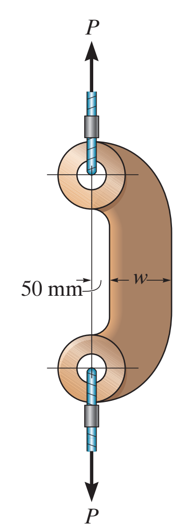
$$\sigma_B = \frac{N}{A} + \frac{Mc}{I} = \frac{606.218}{(0.75)(0.5)} + \frac{175(0.375)}{\frac{1}{12}(0.5)(0.75)^3}$$

$\sigma_B = 5.35 \text{ ksi}$ **Ans.**

$\tau_B = 0 \qquad (\text{since } Q_B = 0)$ **Ans.**



8–26. The offset link supports the loading of $P = 30$ kN. Determine its required width *w* if the allowable normal stress is $\sigma_{allow} = 73$ MPa. The link has a thickness of 40 mm.



σ due to axial force:

$$\sigma_a = \frac{P}{A} = \frac{30(10^3)}{(w)(0.04)} = \frac{750(10^3)}{w}$$

σ due to bending:

$$\sigma_b = \frac{Mc}{I} = \frac{30(10^3)(0.05 + \frac{w}{2})(\frac{w}{2})}{\frac{1}{12}(0.04)(w)^3}$$

$$= \frac{4500\,(10^3)(0.05 + \frac{w}{2})}{w^2}$$

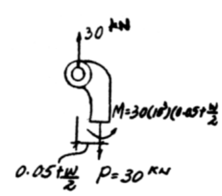
$\sigma_{max} = \sigma_{allow} = \sigma_a + \sigma_b$

$$73(10^6) = \frac{750(10^3)}{w} + \frac{4500(10^3)(0.05 + \frac{w}{2})}{w^2}$$

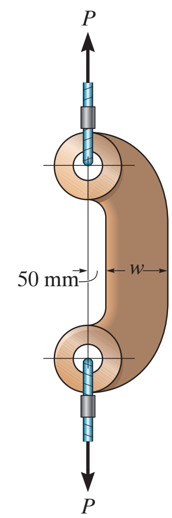
$73\, w^2 = 0.75\, w + 0.225 + 2.25\, w$

$73\, w^2 - 3\, w - 0.225 = 0$

$w = 0.0797 \text{ m} = 79.7 \text{ mm}$ **Ans.**



8–27. The offset link has a width of $w = 200$ mm and a thickness of 40 mm. If the allowable normal stress is $\sigma_{\text{allow}} = 75$ MPa, determine the maximum load P that can be applied to the cables.



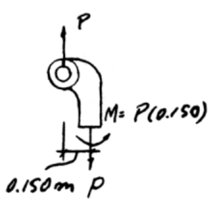
$$A = 0.2(0.04) = 0.008 \text{ m}^2$$

$$I = \frac{1}{12}(0.04)(0.2)^3 = 26.6667(10^{-6}) \text{ m}^4$$

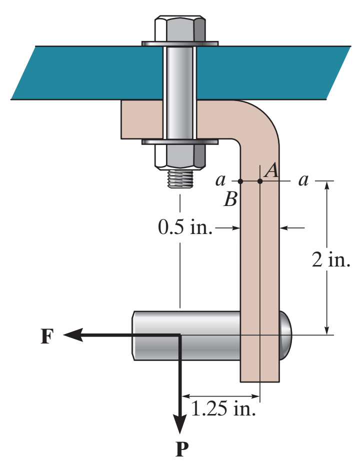
$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$

$$75(10^6) = \frac{P}{0.008} + \frac{0.150\,P(0.1)}{26.6667(10^{-6})}$$

$$P = 109 \text{ kN}$$ **Ans.**



***8–28.** The joint is subjected to a force of $P = 80$ lb and $F = 0$. Sketch the normal-stress distribution acting over section a–a if the member has a rectangular cross-sectional area of width 2 in. and thickness 0.5 in.



σ due to axial force:

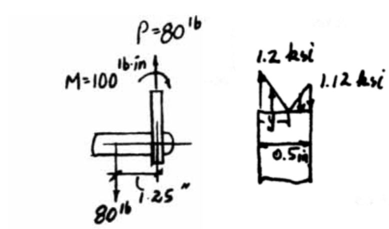
$$\sigma = \frac{P}{A} = \frac{80}{(0.5)(2)} = 80 \text{ psi}$$

σ due to bending:

$$\sigma = \frac{Mc}{I} = \frac{100(0.25)}{\frac{1}{12}(2)(0.5)^3} = 1200 \text{ psi}$$

$$(\sigma_{\max})_t = 80 + 1200 = 1280 \text{ psi} = 1.28 \text{ ksi}$$ **Ans.**

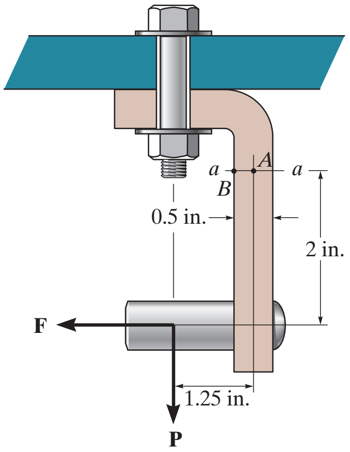
$$(\sigma_{\max})_c = 1200 - 80 = 1120 \text{ psi} = 1.12 \text{ ksi}$$ **Ans.**



$$\frac{y}{1.28} = \frac{(0.5 - y)}{1.12}$$

$$y = 0.267 \text{ in.}$$

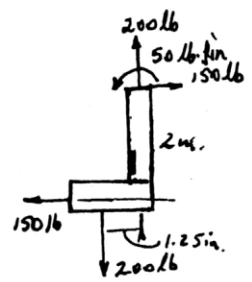
•8–29. The joint is subjected to a force of $P = 200$ lb and $F = 150$ lb. Determine the state of stress at points A and B and sketch the results on differential elements located at these points. The member has a rectangular cross-sectional area of width 0.75 in. and thickness 0.5 in.



$A = 0.5(0.75) = 0.375\ \text{in}^2$

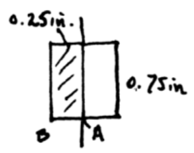
$Q_A = \bar{y}'_A A' = 0.125(0.75)(0.25) = 0.0234375\ \text{in}^3\,; \qquad Q_B = 0$

$I = \frac{1}{12}(0.75)(0.5^3) = 0.0078125\ \text{in}^4$



Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$



$$\sigma_A = \frac{200}{0.375} + 0 = 533\ \text{psi (T)}$$ **Ans.**

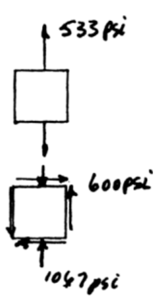
$$\sigma_B = \frac{200}{0.375} - \frac{50(0.25)}{0.0078125} = -1067\ \text{psi} = 1067\ \text{psi (C)}$$ **Ans.**

Shear stress:

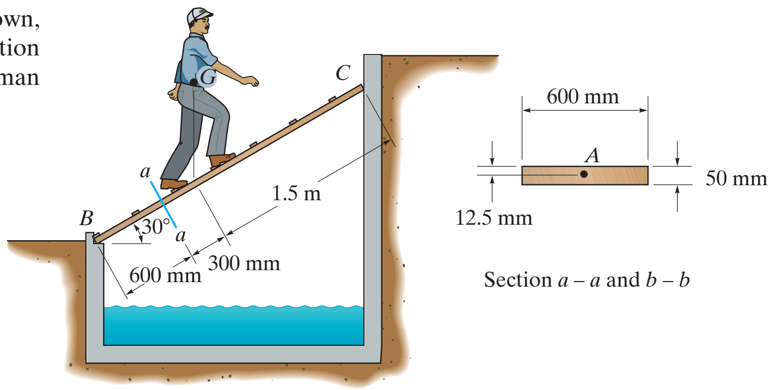
$$\tau = \frac{VQ}{It}$$

$$\tau_A = \frac{150(0.0234375)}{(0.0078125)(0.75)} = 600\ \text{psi}$$ **Ans.**

$\tau_B = 0$ **Ans.**



8–30. If the 75-kg man stands in the position shown, determine the state of stress at point A on the cross section of the plank at section a–a. The center of gravity of the man is at G. Assume that the contact point at C is smooth.



Support Reactions: Referring to the free-body diagram of the entire plank, Fig. a,

$$\circlearrowleft + \Sigma M_B = 0; \qquad F_C \sin 30°(2.4) - 75(9.81)\cos 30°(0.9) = 0$$

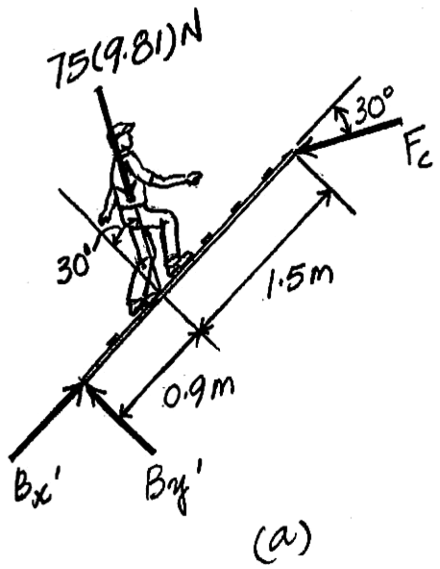
$$F_C = 477.88 \text{ N}$$

$$\Sigma F_{x'} = 0; \quad B_{x'} - 75(9.81)\sin 30° - 477.88 \cos 30° = 0$$

$$B_{x'} = 781.73 \text{ N}$$

$$\Sigma F_{y'} = 0; \quad B_{y'} + 477.88 \sin 30° - 75(9.81)\cos 30° = 0$$

$$B_{y'} = 398.24 \text{ N}$$



Internal Loadings: Consider the equilibrium of the free-body diagram of the plank's lower segment, Fig. b,

$$\Sigma F_{x'} = 0; \quad 781.73 - N = 0 \qquad N = 781.73 \text{ N}$$

$$\Sigma F_{y'} = 0; \quad 398.24 - V = 0 \qquad V = 398.24 \text{ N}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad M - 398.24(0.6) = 0 \qquad M = 238.94 \text{ N}\cdot\text{m}$$

Section Properties: The cross-sectional area and the moment of inertia about the centroidal axis of the plank's cross section are

$$A = 0.6(0.05) = 0.03 \text{ m}^2$$

$$I = \frac{1}{12}(0.6)\left(0.05^3\right) = 6.25\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. c, Q_A is

$$Q_A = \bar{y}'A' = 0.01875(0.0125)(0.6) = 0.140625\left(10^{-3}\right) \text{ m}^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point A, $y = 0.0125$ m. Then

$$\sigma_A = \frac{-781.73}{0.03} - \frac{238.94(0.0125)}{6.25\left(10^{-6}\right)}$$

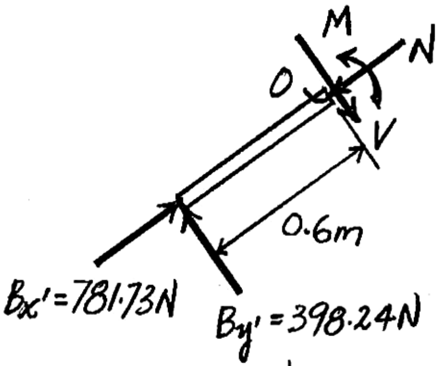
$$= -503.94 \text{ kPa} = 504 \text{ kPa (C)} \qquad \textbf{Ans.}$$

8–30. Continued

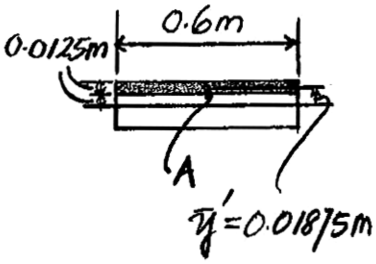
Shear Stress: The shear stress is contributed by transverse shear stress. Thus,

$$\tau_A = \frac{VQ_A}{It} = \frac{398.24\left[0.140625\left(10^{-3}\right)\right]}{6.25\left(10^{-6}\right)(0.6)} = 14.9 \text{ kPa} \qquad \textbf{Ans.}$$

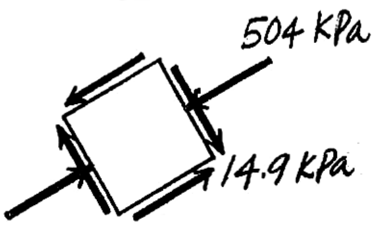
The state of stress at point A is represented on the element shown in Fig. *d*.



(b)

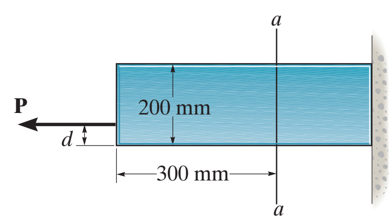


(c)



(d)

8–31. Determine the smallest distance d to the edge of the plate at which the force **P** can be applied so that it produces no compressive stresses in the plate at section a–a. The plate has a thickness of 20 mm and **P** acts along the centerline of this thickness.



Consider the equilibrium of the FBD of the left cut segment in Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N - P = 0 \qquad N = P$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad M - P(0.1 - d) = 0 \qquad M = P(0.1 - d)$$

$$A = 0.2\,(0.02) = 0.004 \text{ m}^4 \qquad I = \frac{1}{12}(0.02)(0.2^3) = 13.3333(10^{-6}) \text{ m}^4$$

The normal stress developed is the combination of axial and bending stress. Thus

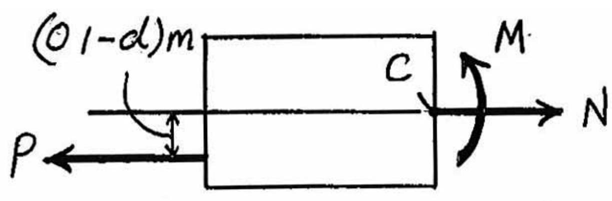
$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

Since no compressive stress is desired, the normal stress at the top edge fiber must be equal to zero. Thus,

$$0 = \frac{P}{0.004} \pm \frac{P(0.1 - d)(0.1)}{13.3333\,(10^{-6})}$$

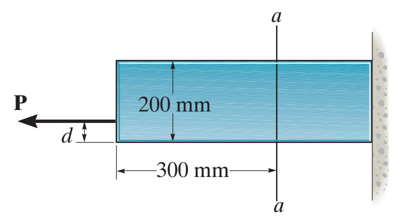
$$0 = 250\,P - 7500\,P\,(0.1 - d)$$

$$d = 0.06667 \text{ m} = 66.7 \text{ mm} \qquad \textbf{Ans.}$$



(a)

***8–32.** The horizontal force of $P = 80$ kN acts at the end of the plate. The plate has a thickness of 10 mm and **P** acts along the centerline of this thickness such that $d = 50$ mm. Plot the distribution of normal stress acting along section *a–a*.



Consider the equilibrium of the FBD of the left cut segment in Fig. *a*,

$\xrightarrow{+} \Sigma F_x = 0;\qquad N - 80 = 0 \qquad N = 80\text{ kN}$

$\circlearrowleft + \Sigma M_C = 0;\qquad M - 80(0.05) = 0 \qquad M = 4.00\text{ kN}\cdot\text{m}$

$A = 0.01(0.2) = 0.002\text{ m}^2 \qquad I = \frac{1}{12}(0.01)(0.2^3) = 6.667(10^{-6})\text{ m}^4$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

At point A, $y = 0.1$ m. Then

$$\sigma_A = \frac{80(10^3)}{0.002} - \frac{4.00(10^3)(0.1)}{6.667(10^{-6})}$$

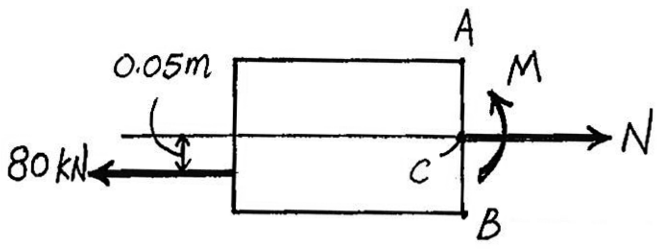
$$= -20.0(10^6)\text{ Pa} = 20.0\text{ Mpa (C)}$$

At point B, $y = 0.1$ m. Then

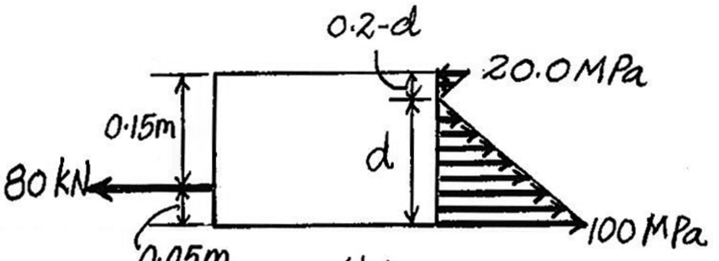
$$\sigma_B = \frac{80(10^3)}{0.002} + \frac{4.00(10^3)(0.1)}{6.667(10^{-6})}$$

$$= 100\,(10^6)\text{ Pa} = 100\text{ MPa (T)}$$

The location of neutral axis can be determined using the similar triangles.

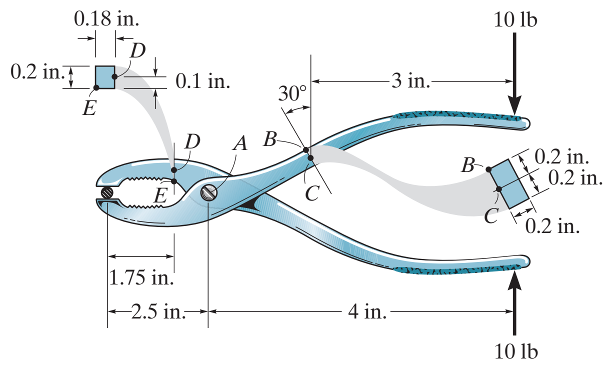


(a)



(b)

•8–33. The pliers are made from two steel parts pinned together at A. If a smooth bolt is held in the jaws and a gripping force of 10 lb is applied at the handles, determine the state of stress developed in the pliers at points B and C. Here the cross section is rectangular, having the dimensions shown in the figure.



$$+\nearrow \Sigma F_x = 0; \qquad N - 10 \sin 30° = 0; \qquad N = 5.0 \text{ lb}$$

$$\nwarrow^+ \Sigma F_y = 0; \qquad V - 10 \cos 30° = 0; \qquad V = 8.660 \text{ lb}$$

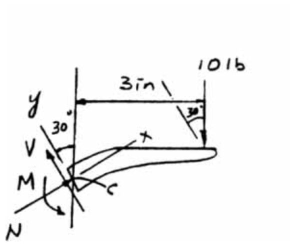
$$\circlearrowleft + \Sigma M_C = 0; \qquad M - 10(3) = 0 \qquad M = 30 \text{ lb} \cdot \text{in.}$$

$$A = 0.2(0.4) = 0.08 \text{ in}^2$$

$$I = \frac{1}{12}(0.2)(0.4^3) = 1.0667(10^{-3}) \text{ in}^4$$

$$Q_B = 0$$

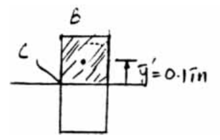
$$Q_C = \bar{y}'A' = 0.1(0.2)(0.2) = 4(10^{-3}) \text{ in}^3$$



Point B:

$$\sigma_B = \frac{N}{A} + \frac{My}{I} = \frac{-5.0}{0.08} + \frac{30(0.2)}{1.0667(10^{-3})} = 5.56 \text{ ksi(T)}$$ **Ans.**

$$\tau_B = \frac{VQ}{It} = 0$$ **Ans.**



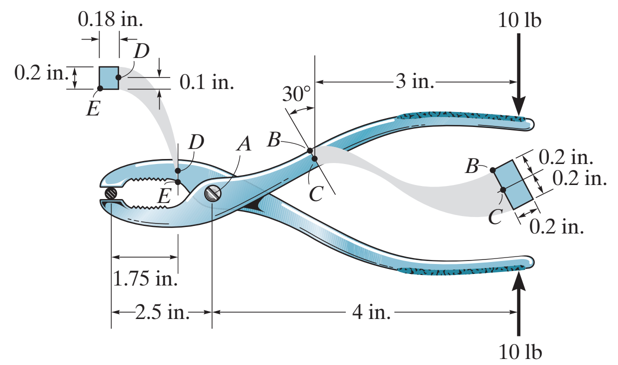
Point C:

$$\sigma_C = \frac{N}{A} + \frac{My}{I} = \frac{-5.0}{0.08} + 0 = -62.5 \text{ psi} = 62.5 \text{ psi(C)}$$ **Ans.**

Shear Stress :

$$\tau_C = \frac{VQ}{It} = \frac{8.660(4)(10^{-3})}{1.0667(10^{-3})(0.2)} = 162 \text{ psi}$$ **Ans.**

8–34. Solve Prob. 8–33 for points D and E.



$$\zeta + \Sigma M_A = 0; \qquad -F(2.5) + 4(10) = 0; \qquad F = 16 \text{ lb}$$

Point D:

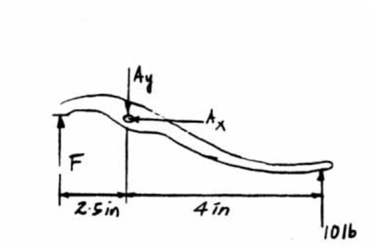
$$\sigma_D = 0 \qquad \textbf{Ans.}$$

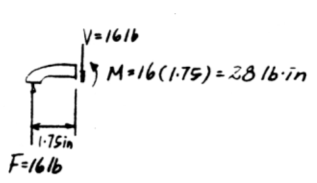
$$\tau_D = \frac{VQ}{It} = \frac{16(0.05)(0.1)(0.18)}{[\frac{1}{12}(0.18)(0.2)^3](0.18)} = 667 \text{ psi} \qquad \textbf{Ans.}$$

Point E:

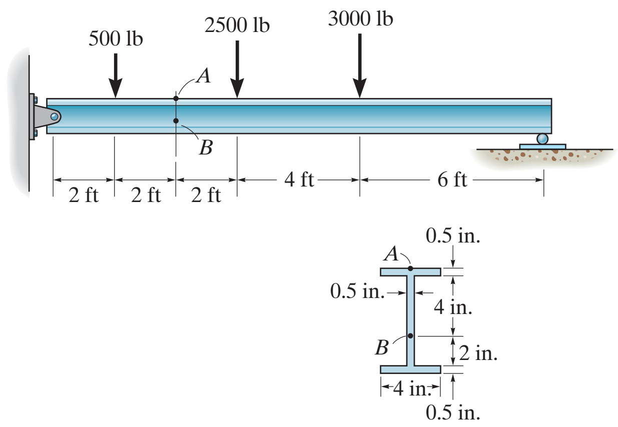
$$\sigma_E = \frac{My}{I} = \frac{28(0.1)}{\frac{1}{12}(0.18)(0.2)^3} = 23.3 \text{ ksi (T)} \qquad \textbf{Ans.}$$

$$\tau_E = 0 \qquad \textbf{Ans.}$$





8–35. The wide-flange beam is subjected to the loading shown. Determine the stress components at points A and B and show the results on a volume element at each of these points. Use the shear formula to compute the shear stress.



$$I = \frac{1}{12}(4)(7^3) - \frac{1}{12}(3.5)(6^3) = 51.33 \text{ in}^4$$

$$A = 2(0.5)(4) + 6(0.5) = 7 \text{ in}^2$$

$$Q_B = \Sigma \bar{y}' A' = 3.25(4)(0.5) + 2(2)(0.5) = 8.5 \text{ in}^3$$

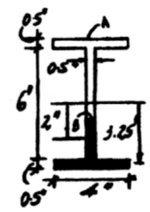
$$Q_A = 0$$

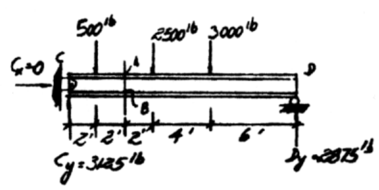
$$\sigma_A = \frac{-Mc}{I} = \frac{-11500\,(12)(3.5)}{51.33} = -9.41 \text{ ksi}$$ **Ans.**

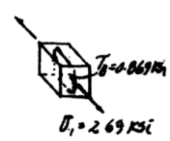
$$\tau_A = 0$$ **Ans.**

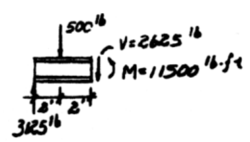
$$\sigma_B = \frac{My}{I} = \frac{11500(12)(1)}{51.33} = 2.69 \text{ ksi}$$ **Ans.**

$$t_B = \frac{VQ_B}{It} = \frac{2625(8.5)}{51.33(0.5)} = 0.869 \text{ ksi}$$ **Ans.**

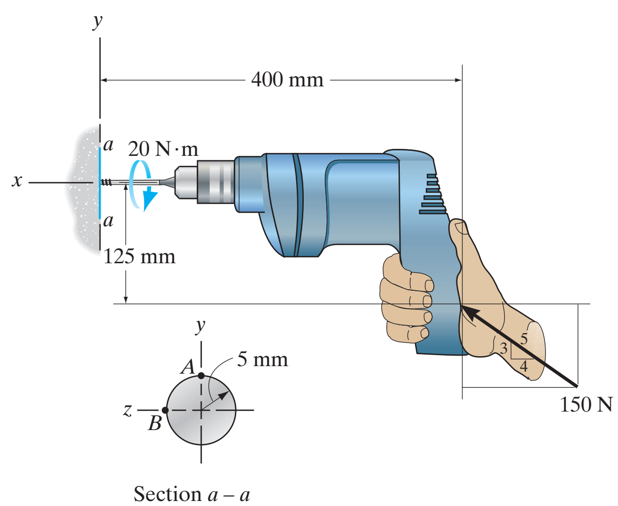








***8–36.** The drill is jammed in the wall and is subjected to the torque and force shown. Determine the state of stress at point A on the cross section of drill bit at section a–a.



Internal Loadings: Consider the equilibrium of the free-body diagram of the drill's right cut segment, Fig. *a*,

$$\Sigma F_x = 0; \quad N - 150\left(\frac{4}{5}\right) = 0 \qquad N = 120\,\text{N}$$

$$\Sigma F_y = 0; \quad 150\left(\frac{3}{5}\right) - V_y = 0 \qquad V_y = 90\,\text{N}$$

$$\Sigma M_x = 0; \; 20 - T = 0 \qquad T = 20\,\text{N}\cdot\text{m}$$

$$\Sigma M_z = 0; \; -150\left(\frac{3}{5}\right)(0.4) + 150\left(\frac{4}{5}\right)(0.125) + M_z = 0$$

$$M_z = 21\,\text{N}\cdot\text{m}$$

Section Properties: The cross-sectional area, the moment of inertia about the z axis, and the polar moment of inertia of the drill's cross section are

$$A = \pi\left(0.005^2\right) = 25\pi\left(10^{-6}\right)\text{m}^2$$

$$I_z = \frac{\pi}{4}\left(0.005^4\right) = 0.15625\pi\left(10^{-9}\right)\text{m}^4$$

$$J = \frac{\pi}{2}\left(0.005^4\right) = 0.3125\pi\left(10^{-9}\right)\text{m}^4$$

Referring to Fig. *b*, Q_A is

$$Q_A = 0$$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z}$$

For point A, $y = 0.005$ m. Then

$$\sigma_A = \frac{-120}{25\pi\left(10^{-6}\right)} - \frac{21(0.005)}{0.15625\pi\left(10^{-9}\right)} = -215.43\text{ MPa} = 215\text{ MPa (C)} \qquad \textbf{Ans.}$$

8–36. Continued

Shear Stress: The transverse shear stress developed at point A is

$$\left[\left(\tau_{xy}\right)_V\right]_A = \frac{V_y Q_A}{I_z t} = 0 \qquad \textbf{Ans.}$$

The torsional shear stress developed at point A is

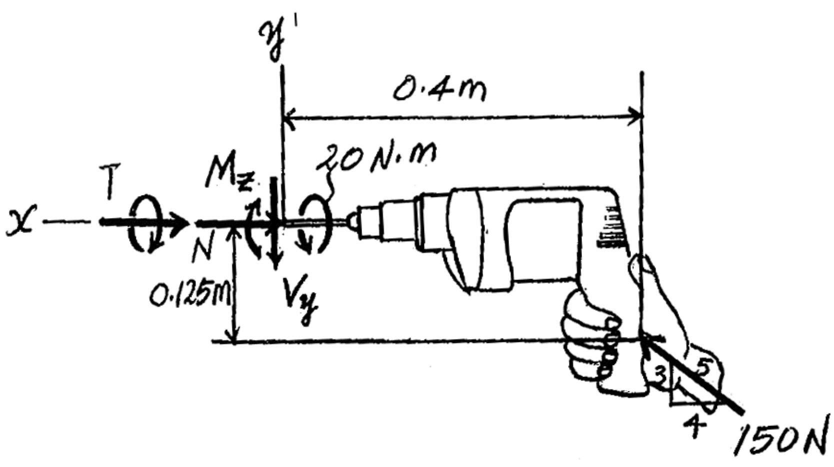
$$\left[(\tau_{xz})_T\right]_A = \frac{Tc}{J} = \frac{20(0.005)}{0.3125\pi\left(10^{-9}\right)} = 101.86 \text{ MPa}$$

Thus,

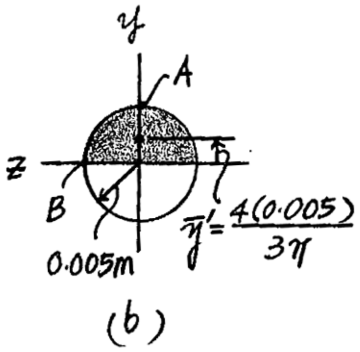
$$\left(\tau_{xy}\right)_A = 0 \qquad \textbf{Ans.}$$

$$\left(\tau_{xz}\right)_A = \left[\left(\tau_{xz}\right)_T\right]_A = 102 \text{ MPa} \qquad \textbf{Ans.}$$

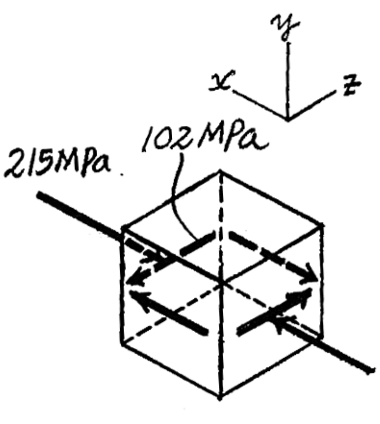
The state of stress at point A is represented on the element shown in Fig c.



(a)

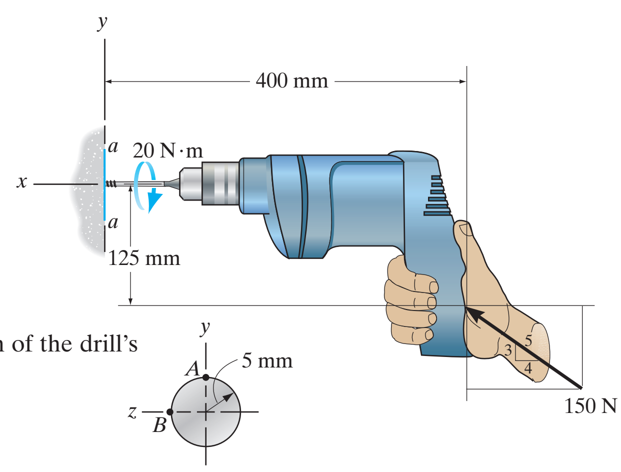


(b)



(C)

•8–37. The drill is jammed in the wall and is subjected to the torque and force shown. Determine the state of stress at point B on the cross section of drill bit at section a–a.



Section $a-a$

Internal Loadings: Consider the equilibrium of the free-body diagram of the drill's right cut segment, Fig. *a,*

$$\Sigma F_x = 0;\quad N - 150\left(\frac{4}{5}\right) = 0 \qquad N = 120\,\text{N}$$

$$\Sigma F_y = 0;\quad 150\left(\frac{3}{5}\right) - V_y = 0 \qquad V_y = 90\,\text{N}$$

$$\Sigma M_x = 0;\quad 20 - T = 0 \qquad T = 20\,\text{N}\cdot\text{m}$$

$$\Sigma M_z = 0;\quad -150\left(\frac{3}{5}\right)(0.4) + 150\left(\frac{4}{5}\right)(0.125) + M_z = 0$$

$$M_z = 21\,\text{N}\cdot\text{m}$$

Section Properties: The cross-sectional area, the moment of inertia about the z axis, and the polar moment of inertia of the drill's cross section are

$$A = \pi\left(0.005^2\right) = 25\pi\left(10^{-6}\right)\text{m}^2$$

$$I_z = \frac{\pi}{4}\left(0.005^4\right) = 0.15625\pi\left(10^{-9}\right)\text{m}^4$$

$$J = \frac{\pi}{2}\left(0.005^4\right) = 0.3125\pi\left(10^{-9}\right)\text{m}^4$$

Referring to Fig. *b*, Q_B is

$$Q_B = \bar{y}'A' = \frac{4(0.005)}{3\pi}\left[\frac{\pi}{2}\left(0.005^2\right)\right] = 83.333\left(10^{-9}\right)\text{m}^3$$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z}$$

For point B, $y = 0$. Then

$$\sigma_B = \frac{-120}{25\pi\left(10^{-6}\right)} - 0 = -1.528\text{ MPa} = 1.53\text{ MPa (C)} \qquad \textbf{Ans.}$$

8–37. Continued

Shear Stress: The transverse shear stress developed at point B is

$$\left[\left(\tau_{xy}\right)_V\right]_B = \frac{V_y Q_B}{I_z t} = \frac{90\left[83.333\left(10^{-9}\right)\right]}{0.15625\pi\left(10^{-9}\right)(0.01)} = 1.528\text{ MPa} \qquad \textbf{Ans.}$$

The torsional shear stress developed at point B is

$$\left[\left(\tau_{xy}\right)_T\right]_B = \frac{T_C}{J} = \frac{20(0.005)}{0.3125\pi\left(10^{-9}\right)} = 101.86\text{ MPa}$$

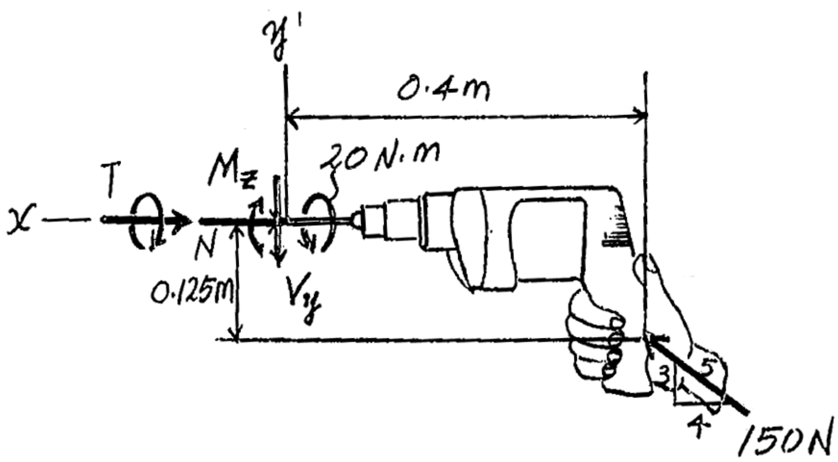
Thus,

$$\left(\tau_C\right)_B = 0 \qquad \textbf{Ans.}$$

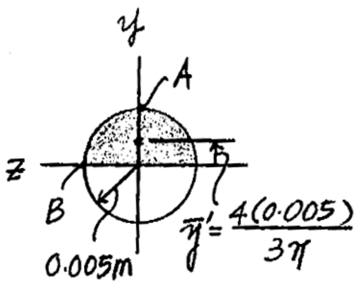
$$\left(\tau_{xy}\right)_B = \left[\left(\tau_{xy}\right)_T\right]_B - \left[\left(\tau_{xy}\right)_V\right]_B$$

$$= 101.86 - 1.528 = 100.33\text{ MPa} = 100\text{ MPa} \qquad \textbf{Ans.}$$

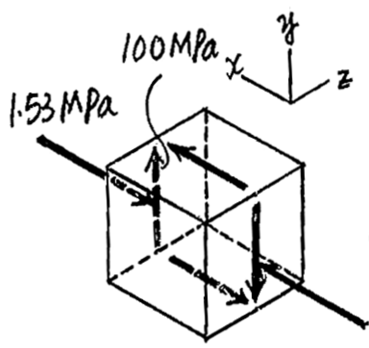
The state of stress at point B is represented on the element shown in Fig. d.



(a)

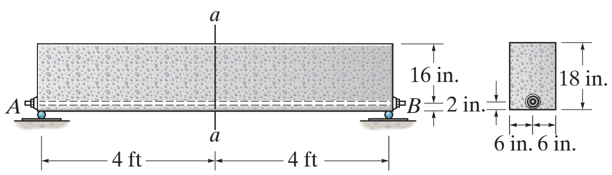


(b)



(d)

8–38. Since concrete can support little or no tension, this problem can be avoided by using wires or rods to *prestress* the concrete once it is formed. Consider the simply supported beam shown, which has a rectangular cross section of 18 in. by 12 in. If concrete has a specific weight of 150 lb/ft^3, determine the required tension in rod AB, which runs through the beam so that no tensile stress is developed in the concrete at its center section a–a. Neglect the size of the rod and any deflection of the beam.



Support Reactions: As shown on FBD.

Internal Force and Moment:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T - N = 0 \qquad N = T$$

$$\circlearrowleft + \Sigma M_o = 0; \qquad M + T(7) - 900(24) = 0$$

$$M = 21600 - 7T$$

Section Properties:

$$A = 18(12) = 216 \text{ in}^2$$

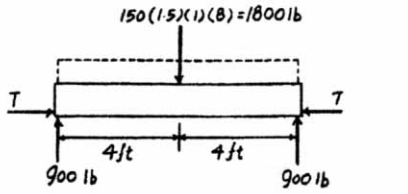
$$I = \frac{1}{12}(12)\left(18^3\right) = 5832 \text{ in}^4$$

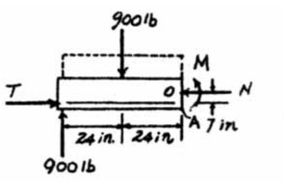
Normal Stress: Requires $\sigma_A = 0$

$$\sigma_A = 0 = \frac{N}{A} + \frac{Mc}{I}$$

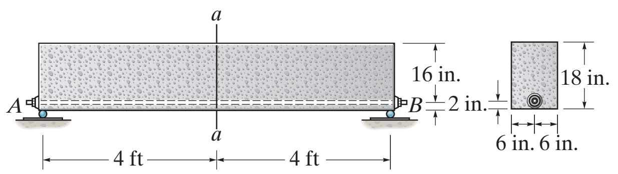
$$0 = \frac{-T}{216} + \frac{(21600 - 7T)(9)}{5832}$$

$$T = 2160 \text{ lb} = 2.16 \text{ kip} \qquad \textbf{Ans.}$$

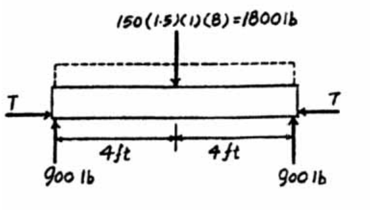




8–39. Solve Prob. 8–38 if the rod has a diameter of 0.5 in. Use the transformed area method discussed in Sec. 6.6. $E_{st} = 29(10^3)$ ksi, $E_c = 3.60(10^3)$ ksi.



Support Reactions: As shown on FBD.

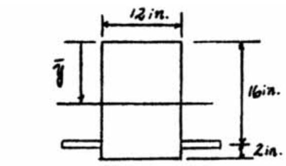


Section Properties:

$$n = \frac{E_{st}}{E_{con}} = \frac{29(10^3)}{3.6(10^3)} = 8.0556$$

$$A_{con} = (n-1)A_{at} = (8.0556 - 1)\left(\frac{\pi}{4}\right)\left(0.5^2\right) = 1.3854 \text{ in}^2$$

$$A = 18(12) + 1.3854 = 217.3854 \text{ in}^2$$

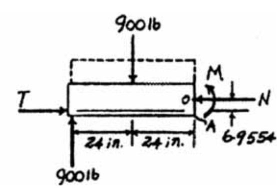


$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{9(18)(12) + 16(1.3854)}{217.3854} = 9.04461 \text{ in.}$$

$$I = \frac{1}{12}(12)\left(18^3\right) + 12(18)(9.04461 - 9)^2 + 1.3854(16 - 9.04461)^2$$

$$= 5899.45 \text{ in}^4$$

Internal Force and Moment:



$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad T - N = 0 \qquad N = T$$

$$\circlearrowleft + \Sigma M_o = 0; \qquad M + T(6.9554) - 900(24) = 0$$

$$M = 21600 - 6.9554T$$

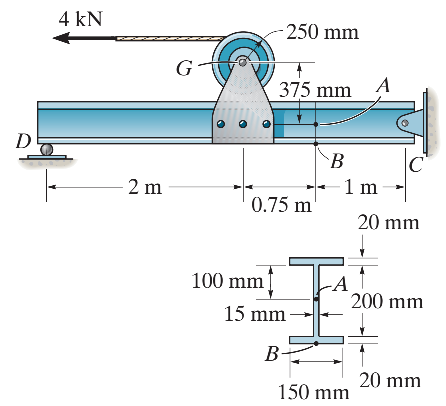
Normal Stress: Requires $\sigma_A = 0$

$$\sigma_A = 0 = \frac{N}{A} + \frac{Mc}{I}$$

$$0 = \frac{-T}{217.3854} + \frac{(21600 - 6.9554T)(8.9554)}{5899.45}$$

$$T = 2163.08 \text{ lb} = 2.16 \text{ kip} \qquad \textbf{Ans.}$$

***8–40.** Determine the state of stress at point A when the beam is subjected to the cable force of 4 kN. Indicate the result as a differential volume element.

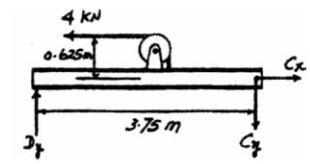


Support Reactions:

$$\zeta+\Sigma M_D = 0; \qquad 4(0.625) - C_y(3.75) = 0$$

$$C_y = 0.6667 \text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad C_x - 4 = 0 \qquad C_x = 4.00 \text{ kN}$$

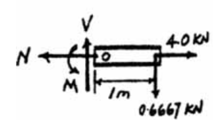


Internal Forces and Moment:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 4.00 - N = 0 \qquad N = 4.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad V - 0.6667 = 0 \qquad V = 0.6667 \text{ kN}$$

$$\zeta+\Sigma M_o = 0; \qquad M - 0.6667(1) = 0 \qquad M = 0.6667 \text{ kN}\cdot\text{m}$$



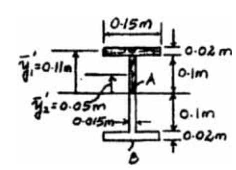
Section Properties:

$$A = 0.24(0.15) - 0.2(0.135) = 9.00\left(10^{-3}\right) \text{ m}^2$$

$$I = \frac{1}{12}(0.15)\left(0.24^3\right) - \frac{1}{12}(0.135)\left(0.2^3\right) = 82.8\left(10^{-6}\right) \text{ m}^4$$

$$Q_A = \Sigma \bar{y}'A' = 0.11(0.15)(0.02) + 0.05(0.1)(0.015)$$

$$= 0.405\left(10^{-3}\right) \text{ m}^3$$

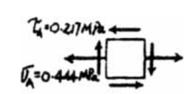


Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{4.00(10^3)}{9.00(10^{-3})} + \frac{0.6667(10^3)(0)}{82.8(10^{-6})}$$

$$= 0.444 \text{ MPa (T)} \qquad \textbf{Ans.}$$

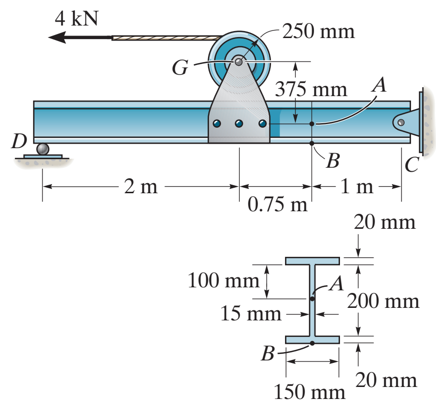


Shear Stress: Applying shear formula.

$$\tau_A = \frac{VQ_A}{It}$$

$$= \frac{0.6667(10^3)\left[0.405(10^{-3})\right]}{82.8(10^{-6})(0.015)} = 0.217 \text{ MPa} \qquad \textbf{Ans.}$$

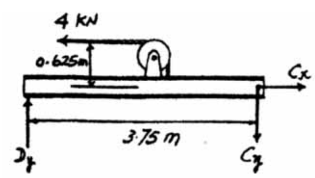
•**8–41.** Determine the state of stress at point B when the beam is subjected to the cable force of 4 kN. Indicate the result as a differential volume element.



Support Reactions:

$$\circlearrowleft + \Sigma M_D = 0; \qquad 4(0.625) - C_y(3.75) = 0$$

$$C_y = 0.6667 \text{ kN}$$



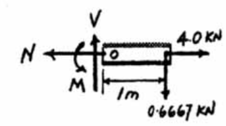
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad C_x - 4 = 0 \qquad C_x = 4.00 \text{ kN}$$

Internal Forces and Moment:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 4.00 - N = 0 \qquad N = 4.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad V - 0.6667 = 0 \qquad V = 0.6667 \text{ kN}$$

$$\circlearrowleft + \Sigma M_o = 0; \qquad M - 0.6667(1) = 0 \qquad M = 0.6667 \text{ kN} \cdot \text{m}$$

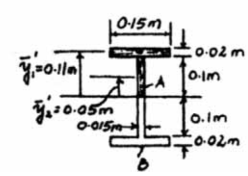


Section Properties:

$$A = 0.24(0.15) - 0.2(0.135) = 9.00\left(10^{-3}\right) \text{m}^2$$

$$I = \frac{1}{12}(0.15)\left(0.24^3\right) - \frac{1}{12}(0.135)\left(0.2^3\right) = 82.8\left(10^{-6}\right) \text{m}$$

$$Q_B = 0$$

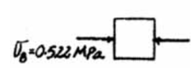


Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_B = \frac{4.00(10^3)}{9.00(10^{-3})} - \frac{0.6667(10^3)(0.12)}{82.8(10^{-6})}$$

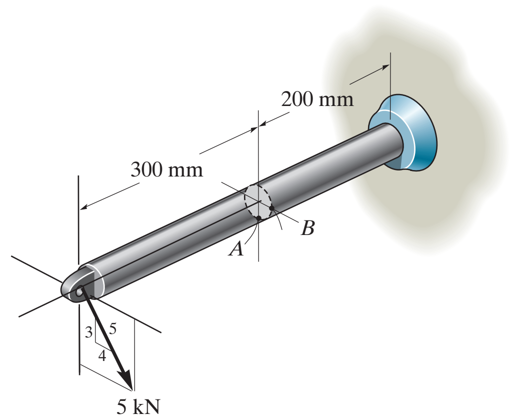
$$= -0.522 \text{ MPa} = 0.522 \text{ MPa (C)} \qquad \textbf{Ans.}$$



Shear Stress: Since $Q_B = 0$, then

$$\tau_B = 0 \qquad \textbf{Ans.}$$

8–42. The bar has a diameter of 80 mm. Determine the stress components that act at point A and show the results on a volume element located at this point.



Consider the equilibrium of the FBD of bar's left cut segment shown in Fig. *a*,

$$\Sigma F_y = 0; \qquad V_y - 5\left(\frac{3}{5}\right) = 0 \qquad V_y = 3\text{ kN}$$

$$\Sigma F_z = 0; \qquad V_z + 5\left(\frac{4}{5}\right) = 0 \qquad V_z = -4\text{ kN}$$

$$\Sigma M_y = 0; \qquad M_y + 5\left(\frac{4}{5}\right)(0.3) = 0 \qquad M_y = -1.2\text{ kN}\cdot\text{m}$$

$$\Sigma M_z = 0; \qquad M_z + 5\left(\frac{3}{5}\right)(0.3) = 0 \qquad M_z = -0.9\text{ kN}\cdot\text{m}$$

$$I_y = I_t = \frac{\pi}{4}(0.04^4) = 0.64(10^{-6})\pi\text{ m}^4$$

Referring to Fig. *b*,

$$(Q_y)_A = 0$$

$$(Q_z)_A = \bar{z}'A' = \frac{4(0.04)}{3\pi}\left[\frac{\pi}{2}(0.04^2)\right] = 42.67(10^{-6})\text{ m}^3$$

The normal stress is contributed by bending stress only. Thus,

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y = -0.04$ m and $z = 0$. Then

$$\sigma = -\frac{-0.9(10^3)(-0.04)}{0.64(10^{-6})\pi} + 0 = -17.90(10^6)\text{Pa} = 17.9\text{ MPa (C)} \qquad \textbf{Ans.}$$

The transverse shear stress developed at point A is

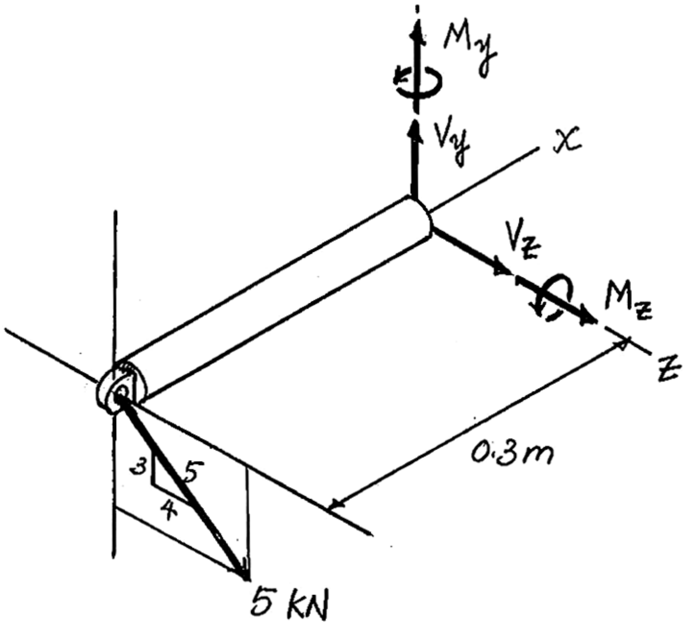
$$\left(\tau_{xy}\right)_v = \frac{V_y(Q_y)_A}{I_z t} = 0 \qquad \textbf{Ans.}$$

$$\left(\tau_{xz}\right)_v = \frac{V_z(Q_z)_A}{I_y t} = \frac{4(10^3)\left[42.67(10^{-6})\right]}{0.64(10^{-6})\pi\,(0.08)}$$

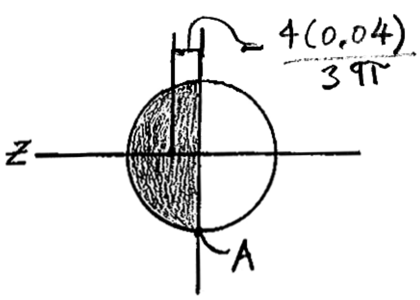
$$= 1.061(10^6)\text{ Pa} = 1.06\text{ MPa} \qquad \textbf{Ans.}$$

The state of stress for point A can be represented by the volume element shown in Fig. *c*,

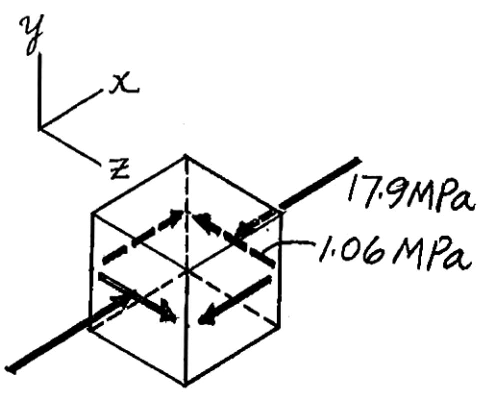
8–42. Continued



(a)

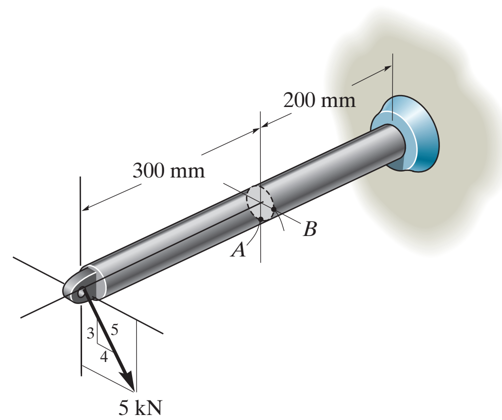


(b)



(c)

8–43. The bar has a diameter of 80 mm. Determine the stress components that act at point B and show the results on a volume element located at this point.



Consider the equilibrium of the FBD of bar's left cut segment shown in Fig. *a*,

$$\Sigma F_y = 0; \qquad V_y - 5\left(\frac{3}{5}\right) = 0 \qquad V_y = 3\text{ kN}$$

$$\Sigma F_z = 0; \qquad V_z + 5\left(\frac{4}{5}\right) = 0 \qquad V_z = -4\text{ kN}$$

$$\Sigma M_y = 0; \qquad M_y + 5\left(\frac{4}{5}\right)(0.3) = 0 \qquad M_y = -1.2\text{ kN}\cdot\text{m}$$

$$\Sigma M_z = 0; \qquad M_z + 5\left(\frac{3}{5}\right)(0.3) = 0 \qquad M_z = -0.9\text{ kN}\cdot\text{m}$$

$$I_y = I_z = \frac{\pi}{4}(0.04^4) = 0.64(10^{-6})\pi\text{ m}^4$$

Referring to Fig. *b*,

$$\left(Q_y\right)_B = \bar{y}'A' = \left[\frac{4(0.04)}{3\cancel{\pi}}\right]\left[\frac{\cancel{\pi}}{2}(0.04^2)\right] = 42.67(10^{-6})\text{ m}^3$$

$$\left(Q_z\right)_B = 0$$

The normal stress is contributed by bending stress only. Thus,

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point B, $y = 0$ and $z = 0.04$ m. Then

$$\sigma = -0 + \frac{-1.2(10^3)(0.04)}{0.64(10^{-6})\pi}$$

$$= -23.87(10^6)\text{ Pa} = 23.9\text{ MPa (C)} \qquad \textbf{Ans.}$$

The transverse shear stress developed at point B is

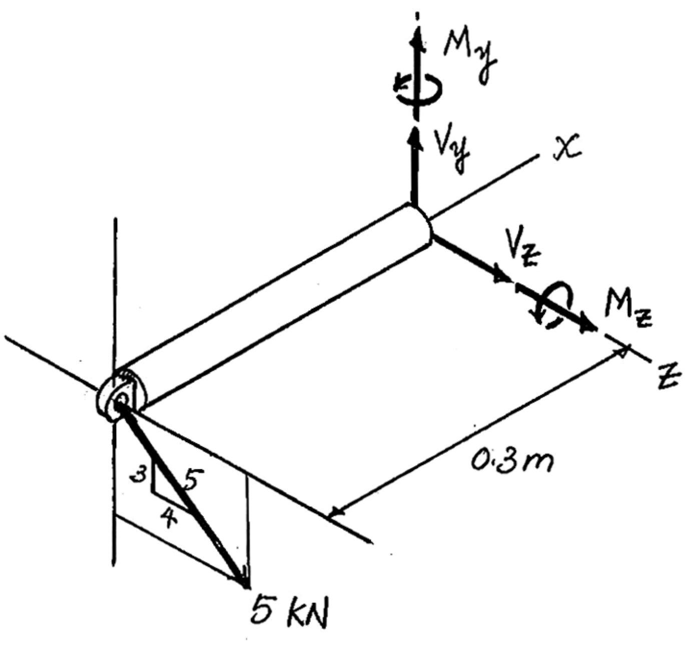
$$\left(\tau_{xy}\right)_v = \frac{V_y (Q_y)_B}{I_z t} = \frac{3(10^3)\left[42.67(10^{-6})\right]}{0.64(10^{-6})\pi\ (0.08)}$$

$$= 0.7958(10^6)\text{ MPa} = 0.796\text{ MPa} \qquad \textbf{Ans.}$$

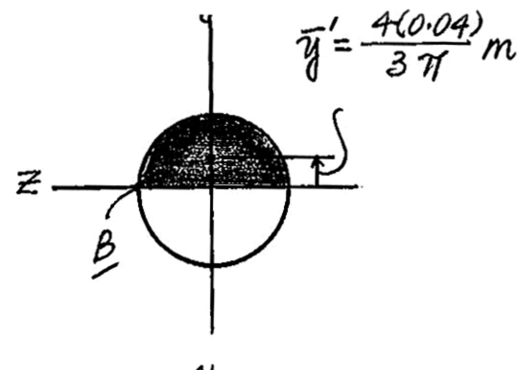
8–43. Continued

$$\left(\tau_{xz}\right)_v = \frac{V_z\,(Q_z)_B}{I_y\,t} = 0 \qquad \textbf{Ans.}$$

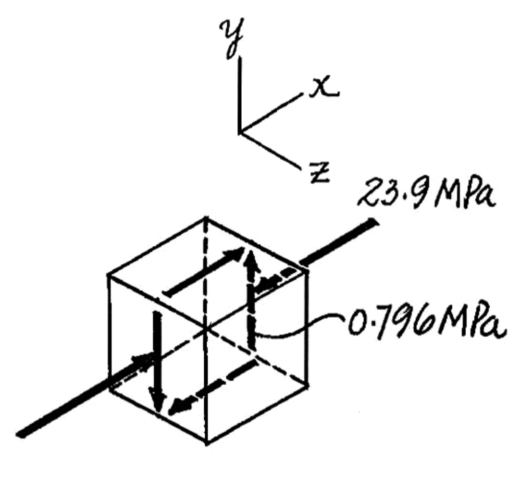
The state of stress for point B can be represented by the volume element shown in Fig. c



(a)

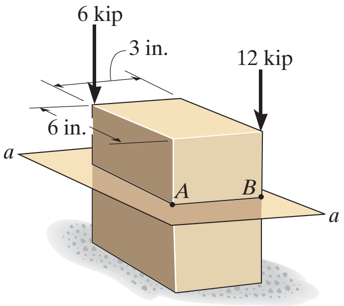


(b)



(c)

***8–44.** Determine the normal stress developed at points A and B. Neglect the weight of the block.



Referring to Fig. *a*,

$\Sigma F_x = (F_R)_x; \qquad -6 - 12 = F \qquad F = -18.0 \text{ kip}$

$\Sigma M_y = (M_R)_y; \qquad 6(1.5) - 12(1.5) = M_y \qquad M_y = -9.00 \text{ kip}\cdot\text{in}$

$\Sigma M_z = (M_R)_z; \qquad 12(3) - 6(3) = M_z \qquad M_z = 18.0 \text{ kip}\cdot\text{in}$

The cross-sectional area and moment of inertia about the y and z axes of the cross-section are

$$A = 6(3) = 18 \text{ in}^2$$

$$I_y = \frac{1}{12}(6)(3)^3 = 13.5 \text{ in}^4$$

$$I_z = \frac{1}{12}(3)(6^3) = 54.0 \text{ in}^4$$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{F}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y = 3$ in. and $z = -1.5$ in.

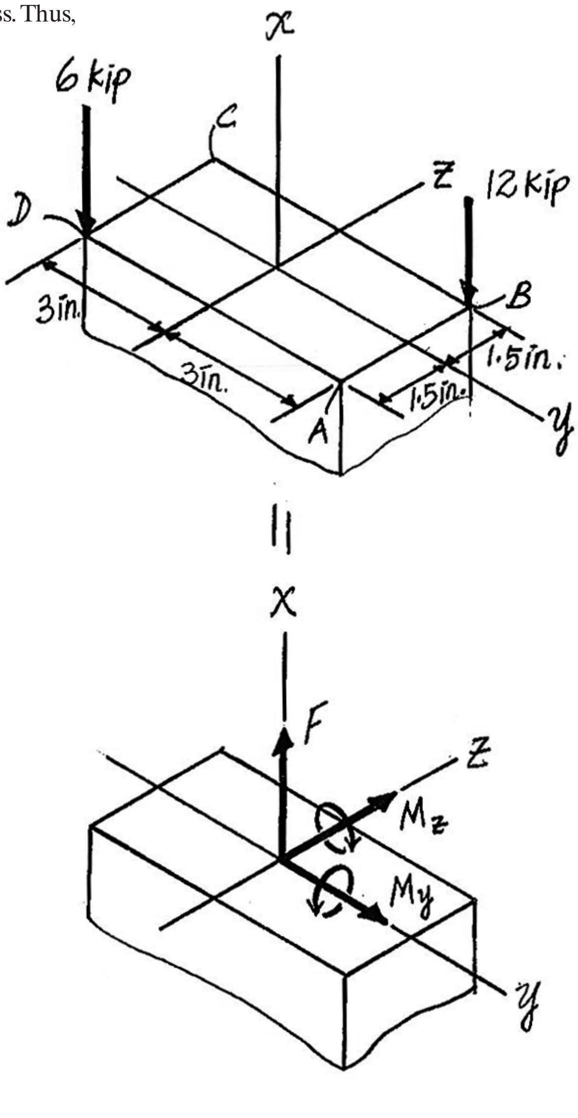
$$\sigma_A = \frac{-18.0}{18.0} - \frac{18.0(3)}{54.0} + \frac{-9.00(-1.5)}{13.5}$$

$$= -1.00 \text{ ksi} = 1.00 \text{ ksi (C)}$$ **Ans.**

For point B, $y = 3$ in and $z = 1.5$ in.

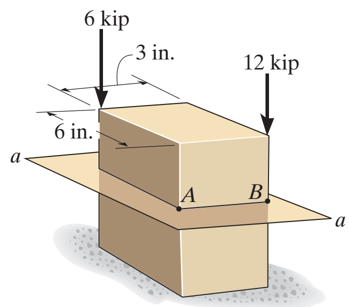
$$\sigma_B = \frac{-18.0}{18.0} - \frac{18.0(3)}{54} + \frac{-9.00(1.5)}{13.5}$$

$$= -3.00 \text{ ksi} = 3.00 \text{ ksi (C)}$$ **Ans.**



(a)

•8–45. Sketch the normal stress distribution acting over the cross section at section *a–a*. Neglect the weight of the block.



Referring to Fig. *a*,

$\Sigma F_x = (F_R)_x; \quad -6 - 12 = F \qquad F = -18.0 \text{ kip}$

$\Sigma M_y = (M_R)_y; \quad 6(1.5) - 12(1.5) = M_y \qquad M_y = -9.00 \text{ kip} \cdot \text{in}$

$\Sigma M_z = (M_R)_z; \quad 12(3) - 6(3) = M_z \qquad M_z = 18.0 \text{ kip} \cdot \text{in}$

The cross-sectional area and the moment of inertia about the y and z axes of the cross-section are

$$A = 3\,(6) = 18.0 \text{ in}^2$$

$$I_y = \frac{1}{12}(6)(3^3) = 13.5 \text{ in}^4$$

$$I_z = \frac{1}{12}(3)(6^3) = 54.0 \text{ in}^4$$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{F}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y = 3$ in. and $z = -1.5$ in.

$$\sigma_A = \frac{-18.0}{18.0} - \frac{18.0(3)}{54.0} + \frac{-9.00(-1.5)}{13.5}$$

$$= -1.00 \text{ ksi} = 1.00 \text{ ksi (C)}$$

For point B, $y = 3$ in. and $z = 1.5$ in.

$$\sigma_B = \frac{-18.0}{18.0} - \frac{18.0(3)}{54.0} + \frac{-9.00(1.5)}{13.5}$$

$$= -3.00 \text{ ksi} = 3.00 \text{ ksi (C)}$$

8–45. Continued

For point C, $y = -3$ in. and $z = 1.5$ in.

$$\sigma_C = \frac{-18.0}{18.0} - \frac{18.0(-3)}{54.0} + \frac{-9.00(1.5)}{13.5}$$

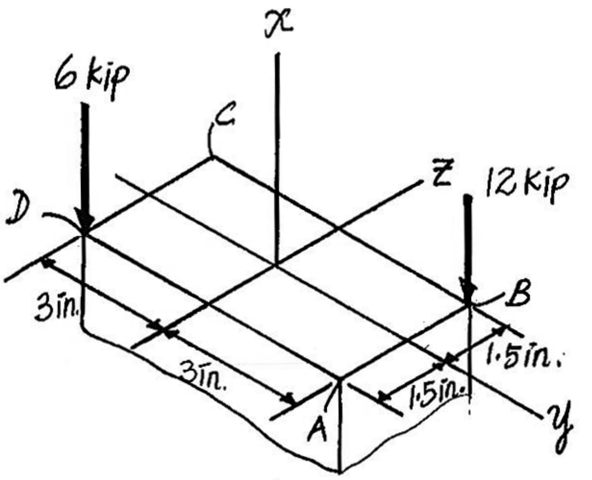
$$= -1.00 \text{ ksi} = 1.00 \text{ ksi (C)}$$

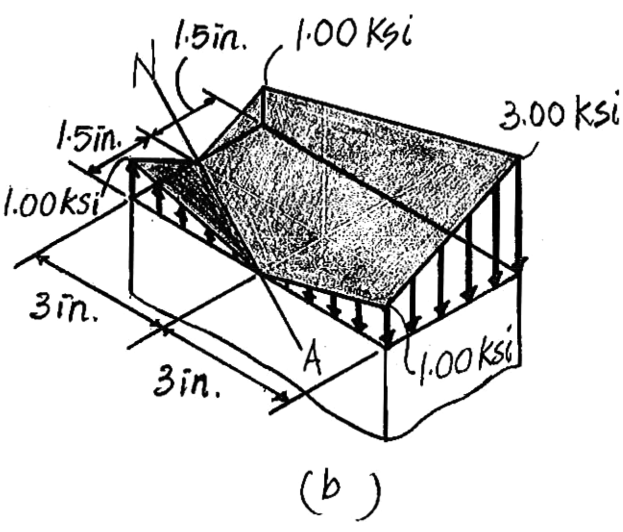
For point D, $y = -3$ in. and $z = -1.5$ in.

$$\sigma_D = \frac{-18.0}{18.0} - \frac{18.0(-3)}{54.0} + \frac{-9.00(-1.5)}{13.5}$$

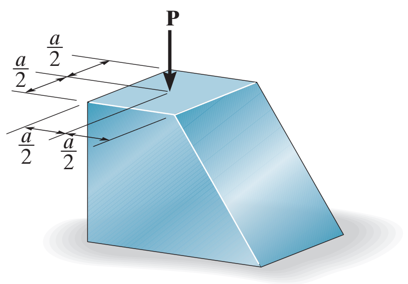
$$= 1.00 \text{ ksi (T)}$$

The normal stress distribution over the cross-section is shown in Fig. *b*





8–46. The support is subjected to the compressive load **P**. Determine the absolute maximum and minimum normal stress acting in the material.



Section Properties:

$$w = a + x$$

$$A = a(a + x)$$

$$I = \frac{1}{12}(a)(a + x)^3 = \frac{a}{12}(a + x)^3$$

Internal Forces and Moment: As shown on FBD.

Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{-P}{a(a + x)} \pm \frac{0.5Px\left[\frac{1}{2}(a + x)\right]}{\frac{a}{12}(a + x)^3}$$

$$= \frac{P}{a}\left[\frac{-1}{a + x} \pm \frac{3x}{(a + x)^2}\right]$$

$$\sigma_A = -\frac{P}{a}\left[\frac{1}{a + x} + \frac{3x}{(a + x)^2}\right]$$

$$= -\frac{P}{a}\left[\frac{4x + a}{(a + x)^2}\right] \quad \textbf{[1]}$$

$$\sigma_B = \frac{P}{a}\left[\frac{-1}{a + x} + \frac{3x}{(a + x)^2}\right]$$

$$= \frac{P}{a}\left[\frac{2x - a}{(a + x)^2}\right] \quad \textbf{[2]}$$

In order to have maximum normal stress, $\frac{d\sigma_A}{dx} = 0$.

$$\frac{d\sigma_A}{dx} = -\frac{P}{a}\left[\frac{(a + x)^2(4) - (4x + a)(2)(a + x)(1)}{(a + x)^4}\right] = 0$$

$$-\frac{P}{a(a + x)^3}(2a - 4x) = 0$$

Since $\frac{P}{a(a + x)^3} \neq 0$, then

$$2a - 4x = 0 \qquad x = 0.500a$$

8–46. Continued

Substituting the result into Eq. [1] yields

$$\sigma_{\max} = -\frac{P}{a}\left[\frac{4(0.500a) + a}{(a + 0.5a)^2}\right]$$

$$= -\frac{1.33P}{a^2} = \frac{1.33P}{a^2}\ \text{(C)} \qquad \textbf{Ans.}$$

In order to have minimum normal stress, $\frac{d\sigma_B}{dx} = 0$.

$$\frac{d\sigma_B}{dx} = \frac{P}{a}\left[\frac{(a + x)^2(2) - (2x - a)(2)(a + x)(1)}{(a + x)^4}\right] = 0$$

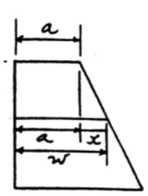
$$\frac{P}{a(a + x)^3}(4a - 2x) = 0$$

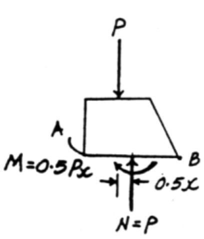
Since $\frac{P}{a(a + x)^3} \neq 0$, then

$$4a - 2x = 0 \qquad x = 2a$$

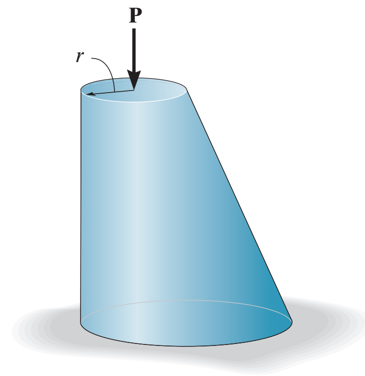
Substituting the result into Eq. [2] yields

$$\sigma_{\min} = \frac{P}{a}\left[\frac{2(2a) - a}{(a + 2a)^2}\right] = \frac{P}{3a^2}\ \textbf{(T)} \qquad \textbf{Ans.}$$





8–47. The support is subjected to the compressive load **P**. Determine the maximum and minimum normal stress acting in the material. All horizontal cross sections are circular.



Section Properties:

$$d' = 2r + x$$

$$A = \pi(r + 0.5x)^2$$

$$I = \frac{\pi}{4}(r + 0.5x)^4$$

Internal Force and Moment: As shown on FBD.

Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{-P}{\pi(r + 0.5x)^2} \pm \frac{0.5Px(r + 0.5x)}{\frac{\pi}{4}(r + 0.5)^4}$$

$$= \frac{P}{\pi}\left[\frac{-1}{(r + 0.5x)^2} \pm \frac{2x}{(r + 0.5x)^3}\right]$$

$$\sigma_A = -\frac{P}{\pi}\left[\frac{1}{(r + 0.5x)^2} + \frac{2x}{(r + 0.5x)^3}\right]$$

$$= -\frac{P}{\pi}\left[\frac{r + 2.5x}{(r + 0.5x)^3}\right] \qquad \textbf{[1]}$$

$$\sigma_B = \frac{P}{\pi}\left[\frac{-1}{(r + 0.5x)^2} + \frac{2x}{(r + 0.5x)^3}\right]$$

$$= \frac{P}{\pi}\left[\frac{1.5x - r}{(r + 0.5x)^3}\right] \qquad \textbf{[2]}$$

In order to have maximum normal stress, $\frac{d\sigma_A}{dx} = 0$.

$$\frac{d\sigma_A}{dx} = -\frac{P}{\pi}\left[\frac{(r + 0.5x)^3(2.5) - (r + 2.5x)(3)(r + 0.5x)^2(0.5)}{(r + 0.5x)^6}\right] = 0$$

$$-\frac{P}{\pi(r + 0.5x)^4}(r - 2.5x) = 0$$

Since $\frac{P}{\pi(r + 0.5x)^4} \neq 0$, then

$$r - 2.5x = 0 \qquad x = 0.400r$$

Substituting the result into Eq. [1] yields

$$\sigma_{\max} = -\frac{\mathrm{P}}{\pi}\left[\frac{\mathrm{r} + 2.5(0.400\mathrm{r})}{[\mathrm{r} + 0.5(0.400\mathrm{r})]^3}\right]$$

$$= -\frac{0.368P}{r^2} = \frac{0.368P}{r^2}\ \text{(C)} \qquad \textbf{Ans.}$$

8–47. Continued

In order to have minimum normal stress, $\dfrac{d\sigma_B}{dx} = 0$.

$$\frac{d\sigma_B}{dx} = \frac{P}{\pi}\left[\frac{(r + 0.5x)^3(1.5) - (1.5x - r)(3)(r + 0.5x)^2(0.5)}{(r + 0.5x)^6}\right] = 0$$

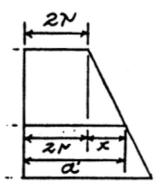
$$\frac{P}{\pi(r + 0.5x)^4}(3r - 1.5x) = 0$$

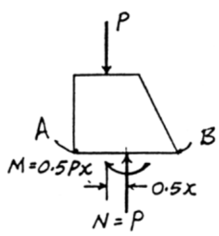
Since $\dfrac{P}{\pi(r + 0.5x)^4} \neq 0$, then

$$3r - 1.5x = 0 \qquad x = 2.00r$$

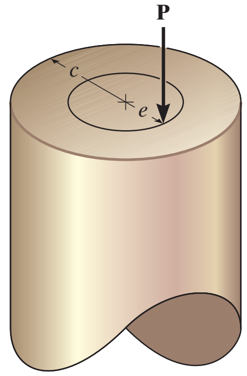
Substituting the result into Eq. [2] yields

$$\sigma_{\min} = \frac{P}{\pi}\left[\frac{1.5(2.00r) - r}{[r + 0.5(2.00r)]^3}\right] = \frac{0.0796P}{r^2} \ \textbf{(T)}$$ **Ans.**





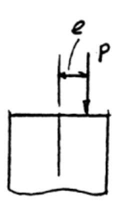
***8–48.** The post has a circular cross section of radius c. Determine the maximum radius e at which the load can be applied so that no part of the post experiences a tensile stress. Neglect the weight of the post.

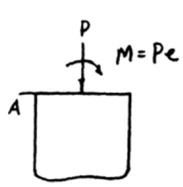


Require $\sigma_A = 0$

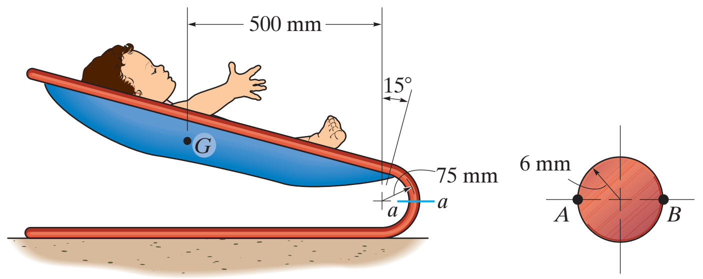
$$\sigma_A = 0 = \frac{P}{A} + \frac{Mc}{I}; \qquad 0 = \frac{-P}{\pi c^2} + \frac{(Pe)c}{\frac{\pi}{4}c^4}$$

$$e = \frac{c}{4}$$ **Ans.**



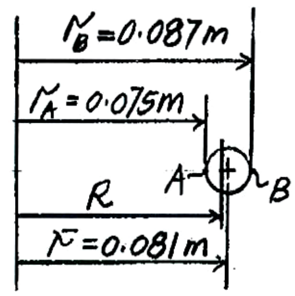


•8–49. If the baby has a mass of 5 kg and his center of mass is at G, determine the normal stress at points A and B on the cross section of the rod at section a–a. There are two rods, one on each side of the cradle.



Section a–a

Section Properties: The location of the neutral surface from the center of curvature of the rod, Fig. a, can be determined from



(a)

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}}$$

where $A = \pi\left(0.006^2\right) = 36\pi\left(10^{-6}\right)\ \text{m}^2$

$$\Sigma \int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right) = 2\pi\left(0.081 - \sqrt{0.081^2 - 0.006^2}\right) = 1.398184\left(10^{-3}\right)\text{m}$$

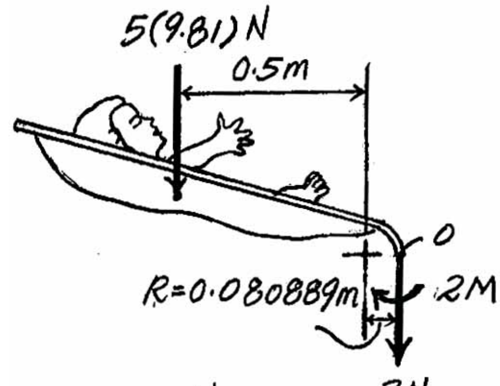
Thus,

$$R = \frac{36\pi\left(10^{-6}\right)}{1.398184\left(10^{-3}\right)} = 0.080889\ \text{m}$$

Then

$$e = \bar{r} - R = 0.081 - 0.080889 = 0.111264\left(10^{-3}\right)\text{m}$$

Internal Loadings: Consider the equilibrium of the free-body diagram of the cradle's upper segment, Fig. b,



(b)

$+\uparrow \Sigma F_y = 0;\qquad -5(9.81) - 2N = 0 \qquad N = -24.525\ \text{N}$

$\circlearrowleft + \Sigma M_O = 0;\qquad 5(9.81)(0.5 + 0.080889) - 2M = 0 \qquad M = 14.2463\ \text{N}\cdot\text{m}$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{M(R - r)}{Aer}$$

Here, $M = -14.1747$ (negative) since it tends to increase the curvature of the rod. For point A, $r = r_A = 0.075\ \text{m}$. Then,

$$\sigma_A = \frac{-24.525}{36\pi\left(10^{-6}\right)} + \frac{-14.2463(0.080889 - 0.075)}{36\pi\left(10^{-6}\right)(0.111264)\left(10^{-3}\right)(0.075)}$$

$$= -89.1\ \text{MPa} = 89.1\ \text{MPa (C)}$$ **Ans.**

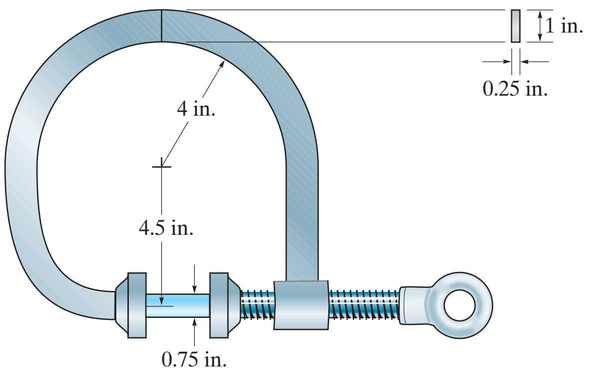
For point B, $r = r_B = 0.087$ m. Then,

$$\sigma_B = \frac{-24.525}{36\pi\left(10^{-6}\right)} + \frac{-14.2463(0.080889 - 0.087)}{36\pi\left(10^{-6}\right)(0.111264)\left(10^{-3}\right)(0.087)}$$

$$= 79.3\ \text{kPa (T)}$$ **Ans.**

$$\int_A \frac{dA}{r} = 0.25 \ln \frac{5}{4} = 0.055786$$

8–50. The C-clamp applies a compressive stress on the cylindrical block of 80 psi. Determine the maximum normal stress developed in the clamp.



$$R = \frac{A}{\int \frac{dA}{r}} = \frac{1(0.25)}{0.055786} = 4.48142$$

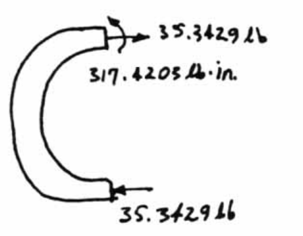
$$P = \sigma_b A = 80\pi(0.375)^2 = 35.3429 \text{ lb}$$

$$M = 35.3429(8.98142) = 317.4205 \text{ lb} \cdot \text{in.}$$

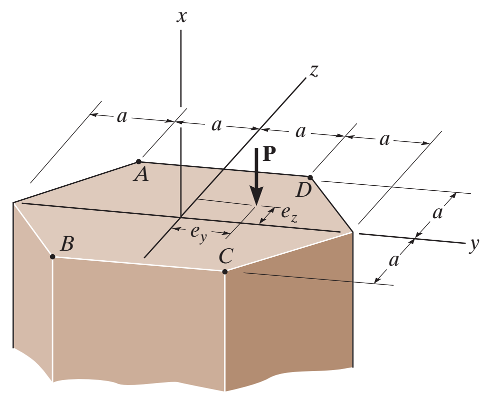
$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)} + \frac{P}{A}$$

$$(\sigma_t)_{max} = \frac{317.4205(4.48142 - 4)}{(1)(0.25)(4)(4.5 - 4.48142)} + \frac{35.3429}{(1)(0.25)} = 8.37 \text{ ksi}$$ **Ans.**

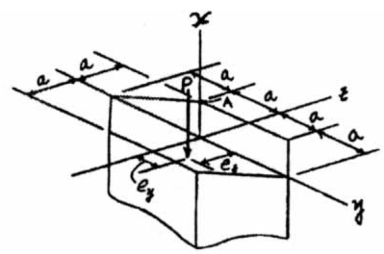
$$(\sigma_c)_{max} = \frac{317.4205(4.48142 - 5)}{1(0.25)(5)(4.5 - 4.48142)} + \frac{35.3429}{(1)(0.25)} = -6.95 \text{ ksi}$$



8–51. A post having the dimensions shown is subjected to the bearing load **P**. Specify the region to which this load can be applied without causing tensile stress to be developed at points A, B, C, and D.



Equivalent Force System: As shown on FBD.



Section Properties:

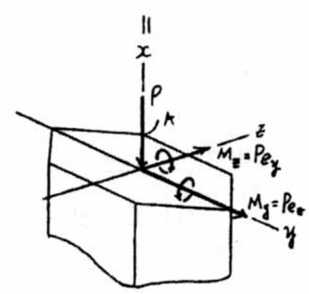
$$A = 2a(2a) + 2\left[\frac{1}{2}(2a)a\right] = 6a^2$$

$$I_z = \frac{1}{12}(2a)(2a)^3 + 2\left[\frac{1}{36}(2a)a^3 + \frac{1}{2}(2a)a\left(a + \frac{a}{3}\right)^2\right]$$

$$= 5a^4$$

$$I_y = \frac{1}{12}(2a)(2a)^3 + 2\left[\frac{1}{36}(2a)a^3 + \frac{1}{2}(2a)a\left(\frac{a}{3}\right)^2\right]$$

$$= \frac{5}{3}a^4$$

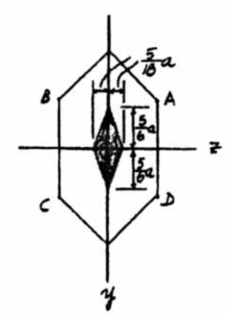


Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$= \frac{-P}{6a^2} - \frac{Pe_y y}{5a^4} + \frac{Pe_z z}{\frac{5}{3}a^4}$$

$$= \frac{P}{30a^4}\left(-5a^2 - 6e_y y + 18e_z z\right)$$



At point A where $y = -a$ and $z = a$, we require $\sigma_A < 0$.

$$0 > \frac{P}{30a^4}\left[-5a^2 - 6(-a)\,e_y + 18(a)\,e_z\right]$$

$$0 > -5a + 6e_y + 18e_z$$

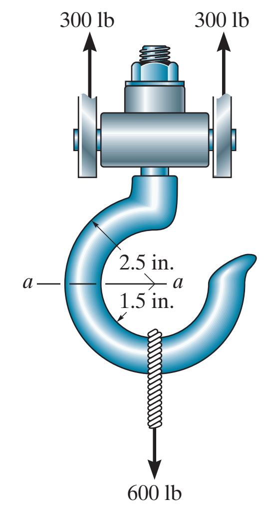
$$6e_y + 18e_z < 5a \qquad \textbf{Ans.}$$

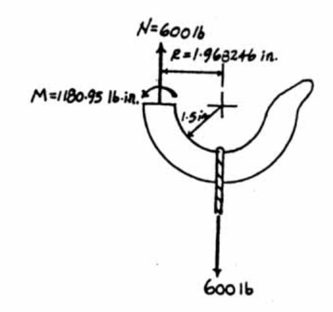
When $\quad e_z = 0, \qquad e_y < \frac{5}{6}a$

When $\quad e_y = 0, \qquad e_z < \frac{5}{18}a$

Repeat the same procedures for point B, C and D. The region where P can be applied without creating tensile stress at points A, B, C and D is shown shaded in the diagram.

***8–52.** The hook is used to lift the force of 600 lb. Determine the maximum tensile and compressive stresses at section *a–a*. The cross section is circular and has a diameter of 1 in. Use the curved-beam formula to compute the bending stress.





Section Properties:

$$\bar{r} = 1.5 + 0.5 = 2.00 \text{ in.}$$

$$\int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$$

$$= 2\pi\left(2.00 - \sqrt{2.00^2 - 0.5^2}\right)$$

$$= 0.399035 \text{ in.}$$

$$A = \pi\left(0.5^2\right) = 0.25\pi \text{ in}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.25\pi}{0.399035} = 1.968246 \text{ in.}$$

$$\bar{r} - R = 2 - 1.968246 = 0.031754 \text{ in.}$$

Internal Force and Moment: As shown on FBD. The internal moment must be computed about the neutral axis. $M = 1180.95 \text{ lb} \cdot \text{in.}$ is positive since it tends to increase the beam's radius of curvature.

Normal Stress: Applying the curved-beam formula.

For tensile stress

$$(\sigma_t)_{\max} = \frac{N}{A} + \frac{M(R - r_1)}{Ar_1(\bar{r} - R)}$$

$$= \frac{600}{0.25\pi} + \frac{1180.95(1.968246 - 1.5)}{0.25\pi(1.5)(0.031754)}$$

$$= 15546 \text{ psi} = 15.5 \text{ ksi (T)}$$ **Ans.**

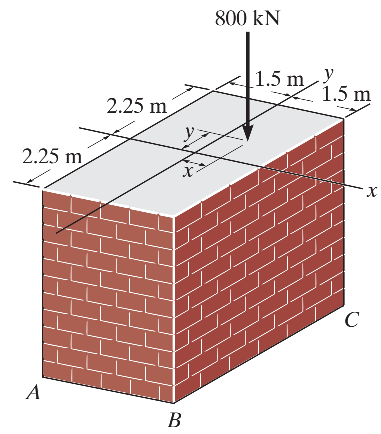
For compressive stress,

$$(\sigma_c)_{\max} = \frac{N}{A} + \frac{M(R - r_2)}{Ar_2(\bar{r} - R)}$$

$$= \frac{600}{0.25\pi} + \frac{1180.95(1.968246 - 2.5)}{0.25\pi(2.5)(0.031754)}$$

$$= -9308 \text{ psi} = 9.31 \text{ ksi (C)}$$ **Ans.**

•8–53. The masonry pier is subjected to the 800-kN load. Determine the equation of the line $y = f(x)$ along which the load can be placed without causing a tensile stress in the pier. Neglect the weight of the pier.



$A = 3(4.5) = 13.5\ \text{m}^2$

$I_x = \dfrac{1}{12}(3)(4.5^3) = 22.78125\ \text{m}^4$

$I_y = \dfrac{1}{12}(4.5)(3^3) = 10.125\ \text{m}^4$

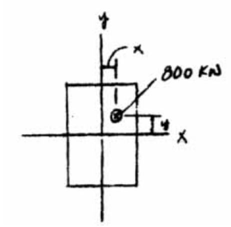
Normal Stress: Require $\sigma_A = 0$

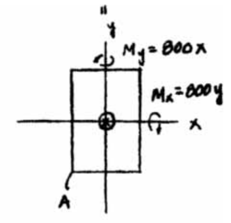
$$\sigma_A = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

$$0 = \frac{-800(10^3)}{13.5} + \frac{800(10^3)y(2.25)}{22.78125} + \frac{800(10^3)x(1.5)}{10.125}$$

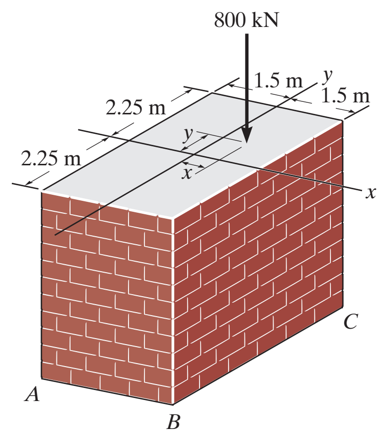
$0 = 0.148x + 0.0988y - 0.0741$

$y = 0.75 - 1.5\,x$ **Ans.**





8–54. The masonry pier is subjected to the 800-kN load. If $x = 0.25$ m and $y = 0.5$ m, determine the normal stress at each corner A, B, C, D (not shown) and plot the stress distribution over the cross section. Neglect the weight of the pier.



$$A = 3(4.5) = 13.5 \text{ m}^2$$

$$I_x = \frac{1}{12}(3)(4.5^3) = 22.78125 \text{ m}^4$$

$$I_y = \frac{1}{12}(4.5)(3^3) = 10.125 \text{ m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

$$\sigma_A = \frac{-800(10^3)}{13.5} + \frac{400(10^3)(2.25)}{22.78125} + \frac{200(10^3)(1.5)}{10.125}$$

$$= 9.88 \text{ kPa (T)} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{-800(10^3)}{13.5} + \frac{400(10^3)(2.25)}{22.78125} - \frac{200(10^3)(1.5)}{10.125}$$

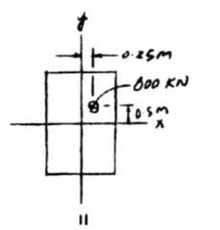
$$= -49.4 \text{ kPa} = 49.4 \text{ kPa (C)} \qquad \textbf{Ans.}$$

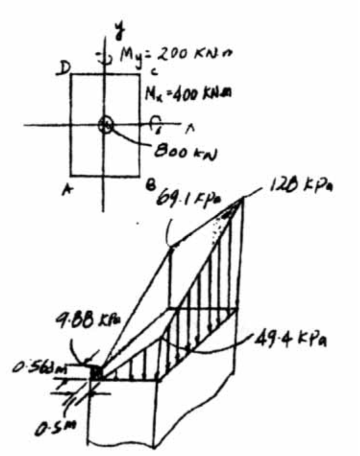
$$\sigma_C = \frac{-800(10^3)}{13.5} - \frac{400(10^3)(2.25)}{22.78125} + \frac{200(10^3)(1.5)}{10.125}$$

$$= -128 \text{ kPa} = 128 \text{ kPa (C)} \qquad \textbf{Ans.}$$

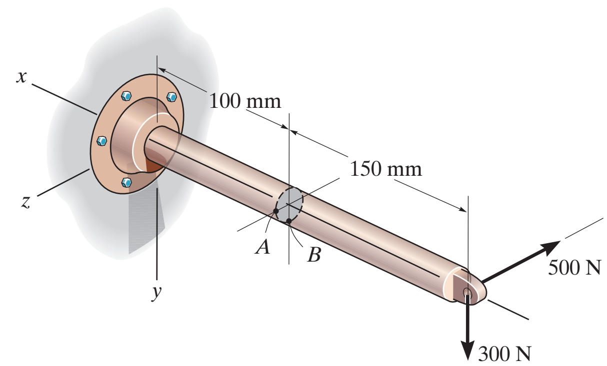
$$\sigma_D = \frac{-800(10^3)}{13.5} - \frac{400(10^3)(2.25)}{22.78125} + \frac{200(10^3)(1.5)}{10.125}$$

$$= -69.1 \text{ kPa} = 69.1 \text{ kPa (C)} \qquad \textbf{Ans.}$$

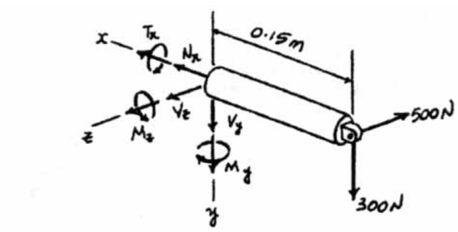




8–55. The bar has a diameter of 40 mm. If it is subjected to the two force components at its end as shown, determine the state of stress at point A and show the results on a differential volume element located at this point.



Internal Forces and Moment:



$$\Sigma F_x = 0; \qquad N_x = 0$$

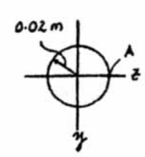
$$\Sigma F_y = 0; \qquad V_y + 300 = 0 \qquad V_y = -300 \text{ N}$$

$$\Sigma F_z = 0; \qquad V_z - 500 = 0 \qquad V_z = 500 \text{ N}$$

$$\Sigma M_x = 0; \qquad T_x = 0$$

$$\Sigma M_y = 0; \qquad M_y - 500(0.15) = 0 \qquad M_y = 75.0 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0; \qquad M_z - 300(0.15) = 0 \qquad M_z = 45.0 \text{ N}\cdot\text{m}$$



Section Properties:

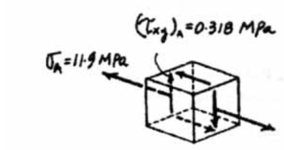
$$A = \pi\left(0.02^2\right) = 0.400\left(10^{-3}\right)\pi \text{ m}^2$$

$$I_x = I_y = \frac{\pi}{4}\left(0.02^4\right) = 40.0\left(10^{-9}\right)\pi \text{ m}^4$$

$$J = \frac{\pi}{2}\left(0.02^4\right) = 80.0\left(10^{-9}\right)\pi \text{ m}^4$$

$$(Q_A)_z = 0$$

$$(Q_A)_y = \frac{4(0.02)}{3\pi}\left[\frac{1}{2}\pi\left(0.02^2\right)\right] = 5.333\left(10^{-6}\right) \text{ m}^3$$



Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = 0 - \frac{45.0(0)}{40.0(10^{-9})\pi} + \frac{75.0(0.02)}{40.0(10^{-9})\pi}$$

$$= 11.9 \text{ MPa (T)} \qquad \textbf{Ans.}$$

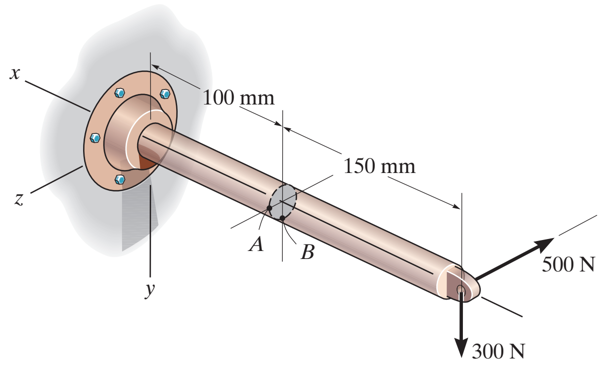
Shear Stress: The tranverse shear stress in the z and y directions can be obtained using the shear formula, $\tau_V = \dfrac{VQ}{It}$.

$$(\tau_{xy})_A = -\tau_{V_y} = -\frac{300\left[5.333(10^{-6})\right]}{40.0(10^{-9})\pi\ (0.04)}$$

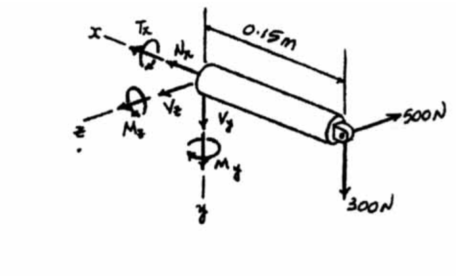
$$= -0.318 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{xz})_A = \tau_{V_z} = 0 \qquad \textbf{Ans.}$$

***8–56.** Solve Prob. 8–55 for point *B*.



Internal Forces and Moment:



$$\Sigma F_x = 0; \qquad N_x = 0$$

$$\Sigma F_y = 0; \qquad V_y + 300 = 0 \qquad V_y = -300\text{ N}$$

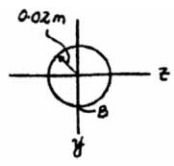
$$\Sigma F_z = 0; \qquad V_z - 500 = 0 \qquad V_z = 500\text{ N}$$

$$\Sigma M_x = 0; \qquad T_x = 0$$

$$\Sigma M_y = 0; \qquad M_y - 500(0.15) = 0 \qquad M_y = 75.0\text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0; \qquad M_z - 300(0.15) = 0 \qquad M_z = 45.0\text{ N}\cdot\text{m}$$

Section Properties:



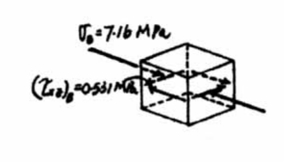
$$A = \pi\left(0.02^2\right) = 0.400\left(10^{-3}\right)\pi\text{ m}^2$$

$$I_x = I_y = \frac{\pi}{4}\left(0.02^4\right) = 40.0\left(10^{-9}\right)\pi\text{ m}^4$$

$$J = \frac{\pi}{2}\left(0.02^4\right) = 80.0\left(10^{-9}\right)\pi\text{ m}^4$$

$$(Q_B)_y = 0$$

$$(Q_B)_z = \frac{4(0.02)}{3\pi}\left[\frac{1}{2}\pi\left(0.02^2\right)\right] = 5.333\left(10^{-6}\right)\text{ m}^3$$



Normal Strees:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_B = 0 - \frac{45.0(0.02)}{40.0(10^{-9})\pi} + \frac{75.0(0)}{40.0(10^{-9})\pi}$$

$$= -7.16\text{ MPa} = 7.16\text{ MPa (C)}$$ **Ans.**

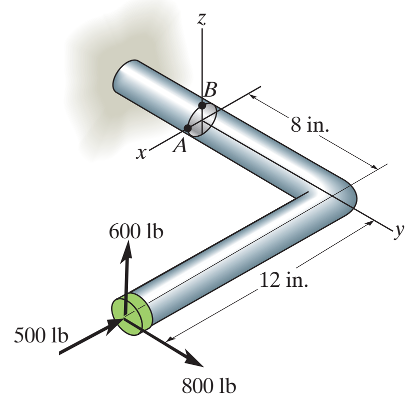
Shear Stress: The tranverse shear stress in the z and y directions can be obtained using the shear formula, $\tau_V = \frac{VQ}{It}$.

$$(\tau_{xz})_B = \tau_{V_z} = \frac{500\left[5.333(10^{-6})\right]}{40.0(10^{-9})\pi\ (0.04)}$$

$$= 0.531\text{ MPa}$$ **Ans.**

$$(\tau_{xy})_B = \tau_{V_y} = 0$$ **Ans.**

•8–57. The 2-in.-diameter rod is subjected to the loads shown. Determine the state of stress at point A, and show the results on a differential element located at this point.



Consider the equilibrium of the FBD of the right cut segment, Fig. *a*,

$\Sigma F_y = 0; \quad N_y + 800 = 0 \quad N_y = -800 \text{ lb}$

$\Sigma F_z = 0; \quad V_z + 600 = 0 \quad V_z = -600 \text{ lb}$

$\Sigma F_x = 0; \quad V_x - 500 = 0 \quad V_x = 500 \text{ lb}$

$\Sigma M_y = 0; \quad T_y - 600(12) = 0 \quad T_y = 7200 \text{ lb} \cdot \text{in}$

$\Sigma M_z = 0; \quad M_z + 800(12) + 500(8) = 0 \quad M_z = -13600 \text{ lb} \cdot \text{in}$

$\Sigma M_x = 0; \quad M_x + 600(8) = 0 \quad M_x = -4800 \text{ lb} \cdot \text{in}$

$$I_x = I_z = \frac{\pi}{4}(1^4) = \frac{\pi}{4} \text{ in}^4 \qquad A = \pi(1^2) = \pi \text{ in}^2$$

$$J = \frac{\pi}{2}(1^4) = \frac{\pi}{2} \text{ in}^4$$

Referring to Fig. *b*,

$$(Q_x)_A = 0 \qquad (Q_z)_A = \bar{y}'A' = \frac{4(1)}{3\pi}\left[\frac{\pi}{2}(1^2)\right] = 0.6667 \text{ in}^3$$

The normal stress is contributed by axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{M_x z}{I_x} - \frac{M_z x}{I_z}$$

For point A, $z = 0$ and $x = 1$ in.

$$\sigma = \frac{800}{\pi} - \frac{4800(0)}{\pi/4} - \frac{-13600(1)}{\pi/4}$$

$$= 17.57(10^3) \text{ psi} = 17.6 \text{ ksi (T)} \qquad \textbf{Ans.}$$

The torsional shear stress developed at point A is

$$(\tau_{yz})_T = \frac{T_y C}{J} = \frac{7200(1)}{\pi/2} = 4.584(10^3) \text{ psi} = 4.584 \text{ ksi} \downarrow$$

The transverse shear stress developed at point A is.

$$(\tau_{yz})_\gamma = \frac{V_z(Q_z)_A}{I_x t} = \frac{600(0.6667)}{\frac{\pi}{4}(2)} = 254.64 \text{ psi} = 0.2546 \text{ ksi} \downarrow$$

$$(\tau_{xy})_\gamma = \frac{V_x(Q_x)_A}{I_z t} = \frac{500(0)}{\frac{\pi}{4}(2)} = 0$$

8–57. Continued

Combining these two shear stress components,

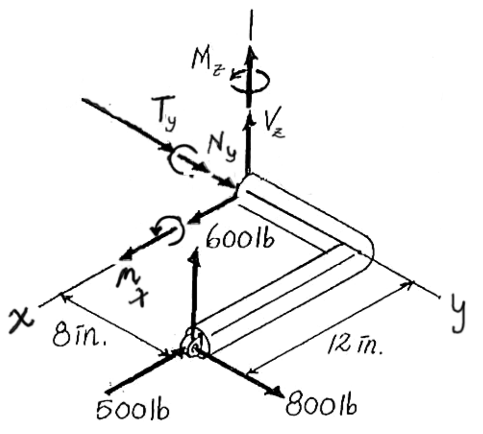
$$\tau_{yz} = \left(\tau_{yz}\right)_T + \left(\tau_{yz}\right)_V$$

$$= 4.584 + 0.2546$$

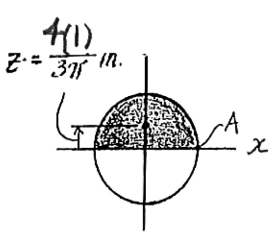
$$= 4.838 \text{ ksi} = 4.84 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_{xy} = 0 \qquad \textbf{Ans.}$$

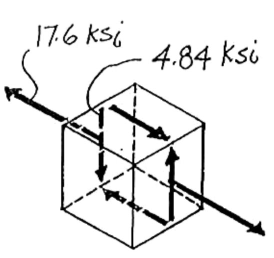
The state of stress of point A can be represented by the volume element shown in Fig. c.



(a)

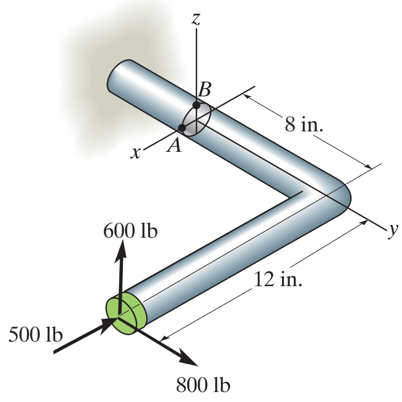


(b)



(c)

8–58. The 2-in.-diameter rod is subjected to the loads shown. Determine the state of stress at point B, and show the results on a differential element located at this point.



Consider the equilibrium of the FBD of the right cut segment, Fig. *a*,

$$\Sigma F_y = 0; \quad N_y + 800 = 0 \quad N_y = -800 \text{ lb}$$

$$\Sigma F_z = 0; \quad V_z + 600 = 0 \quad V_z = -600 \text{ lb}$$

$$\Sigma F_x = 0; \quad V_x - 500 = 0 \quad V_x = 500 \text{ lb}$$

$$\Sigma M_y = 0; \quad T_y - 600(12) = 0 \quad T_y = 7200 \text{ lb} \cdot \text{in}$$

$$\Sigma M_z = 0; \quad M_z + 800(12) + 500(8) = 0 \quad M_z = -13600 \text{ lb} \cdot \text{in}$$

$$\Sigma M_x = 0; \quad M_x + 600(8) = 0 \quad M_x = -4800 \text{ lb} \cdot \text{in.}$$

The cross-sectional area the moment of inertia about x and Z axes and polar moment of inertia of the rod are

$$A = \pi(1^2) = \pi \text{ in}^2 \qquad I_x = I_z = \frac{\pi}{4}(1^4) = \frac{\pi}{4} \text{ in}^4 \quad J = \frac{\pi}{2}(1^4) = \frac{\pi}{2} \text{ in}^4$$

Referring to Fig. *b*,

$$(Q_z)_B = 0 \quad (Q_x)_B = \bar{z}'A' = \frac{4(1)}{3\pi}\left[\frac{\pi}{2}(1^2)\right] = 0.6667 \text{ in}^4$$

The normal stress is contributed by axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{M_x z}{I_x} - \frac{M_z x}{I_z}$$

For point B, $x = 0$ and $z = 1$ in.

$$\sigma = \frac{800}{\pi} - \frac{4800\,(1)}{\pi/4} + \frac{13600\,(0)}{\pi/4}$$

$$= 5.86 \text{ ksi (C)}$$

The torsional shear stress developed at point B is

$$(\tau_{xy})_T = \frac{T_y C}{J} = \frac{7200(1)}{\pi/2} = 4.584(10^3) \text{ psi} = 4.584 \text{ ksi} \;\rightarrow$$

The transverse shear stress developed at point B is.

$$(\tau_{xy})_v = \frac{V_x (Q_x)_B}{I_z t} = \frac{500\,(0.6667)}{\frac{\pi}{4}(2)} = 212.21 \text{ psi} = 0.2122 \text{ ksi} \;\rightarrow$$

$$(\tau_{yz})_v = \frac{V_z (Q_z)_B}{I_x t} = \frac{600\,(0)}{\frac{\pi}{4}(2)} = 0$$

8–58. Continued

Combining these two shear stress components,

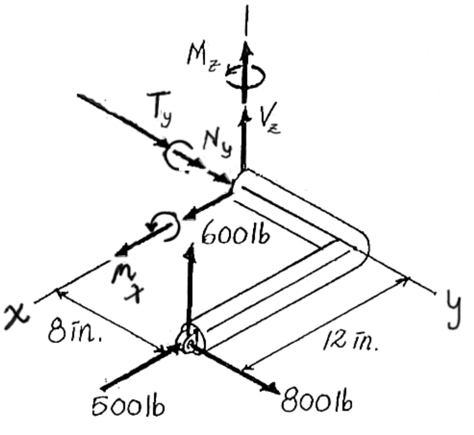
$$\tau_{xy} = (\tau_{xy})_T + (\tau_{xy})_v$$

$$= 4.584 + 0.2122$$

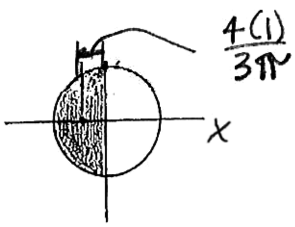
$$= 4.796 \text{ ksi} = 4.80 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_{yz} = 0 \qquad \textbf{Ans.}$$

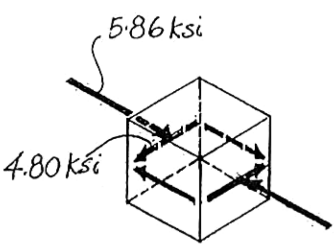
The state of stress of point *B* can be represented by the volume element shown in Fig. *c*.



(a)

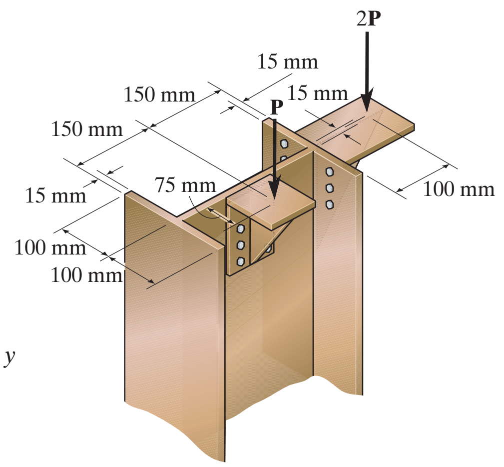


(b)



(c)

8–59. If $P = 60$ kN, determine the maximum normal stress developed on the cross section of the column.



Equivalent Force System: Referring to Fig. *a*,

$+\uparrow \Sigma F_x = (F_R)_x; \qquad -60 - 120 = -F \qquad F = 180 \text{ kN}$

$\Sigma M_y = (M_R)_y; \qquad -60(0.075) = -M_y \qquad M_y = 4.5 \text{kN} \cdot \text{m}$

$\Sigma M_z = (M_R)_z; \qquad -120(0.25) = -M_z \qquad M_z = 30 \text{kN} \cdot \text{m}$

Section Properties: The cross-sectional area and the moment of inertia about the y and z axes of the cross section are

$$A = 0.2(0.3) - 0.185(0.27) = 0.01005 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.2)(0.3^3) - \frac{1}{12}(0.185)(0.27^3) = 0.14655(10^{-3}) \text{ m}^4$$

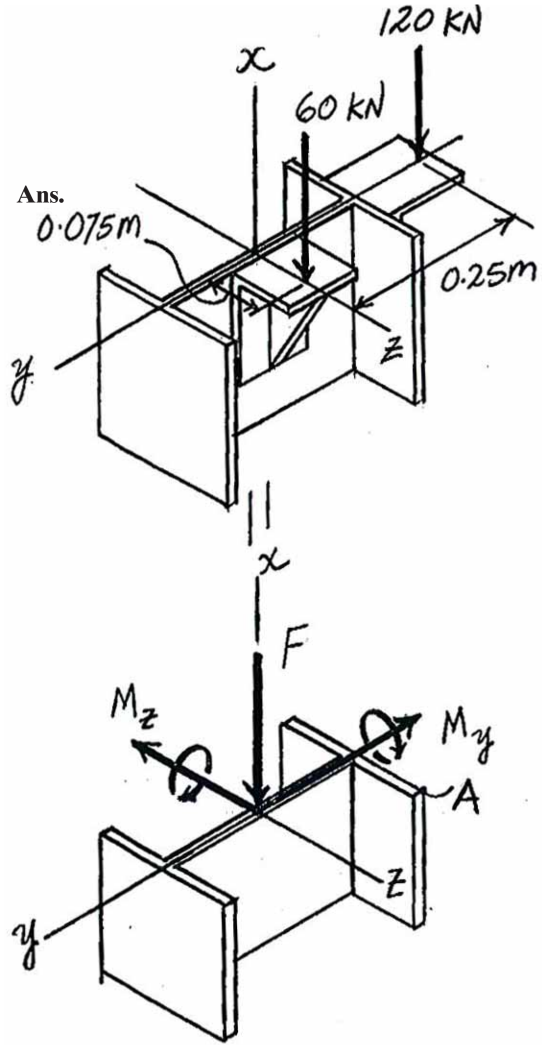
$$I_y = 2\left[\frac{1}{12}(0.015)(0.2^3)\right] + \frac{1}{12}(0.27)(0.015^3) = 20.0759(10^{-6})\text{m}^4$$

Normal Stress: The normal stress is the combination of axial and bending stress. Here, **F** is negative since it is a compressive force. Also, $\mathbf{M}_y$ and $\mathbf{M}_z$ are negative since they are directed towards the negative sense of their respective axes. By inspection, point A is subjected to a maximum normal stress. Thus,

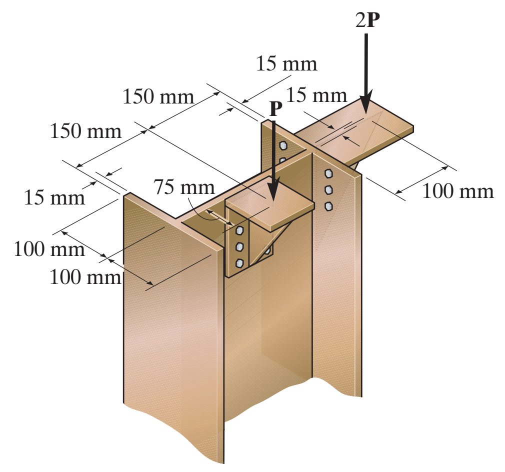
$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_{\max} = \sigma_A = \frac{-180(10^3)}{0.01005} - \frac{[-30(10^3)](-0.15)}{0.14655(10^{-3})} + \frac{[-4.5(10^3)](0.1)}{20.0759(10^{-6})}$$

$$= -71.0 \text{ MPa} = 71.0 \text{ MPa(C)} \qquad \textbf{Ans.}$$



***8–60.** Determine the maximum allowable force **P**, if the column is made from material having an allowable normal stress of $\sigma_{\text{allow}} = 100\text{ MPa}$.



Equivalent Force System: Referring to Fig. *a*,

$$+\uparrow \Sigma F_x = (F_R)_x; \qquad -P - 2P = -F$$

$$F = 3P$$

$$\Sigma M_y = (M_R)_y; \qquad -P(0.075) = -M_y$$

$$M_y = 0.075\,P$$

$$\Sigma M_z = (M_R)_z; \qquad -2P(0.25) = -M_z$$

$$M_z = 0.5P$$

Section Properties: The cross-sectional area and the moment of inertia about the y and z axes of the cross section are

$$A = 0.2(0.3) - 0.185(0.27) = 0.01005\text{ m}^2$$

$$I_z = \frac{1}{12}(0.2)\left(0.3^3\right) - \frac{1}{12}(0.185)\left(0.27^3\right) = 0.14655\left(10^{-3}\right)\text{m}^4$$

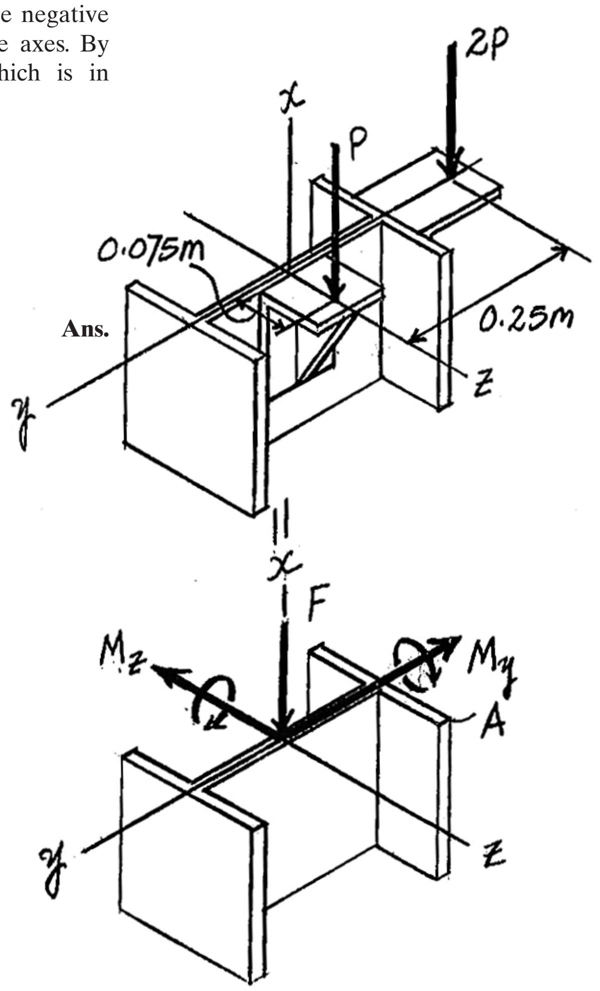
$$I_y = 2\left[\frac{1}{12}(0.15)\left(0.2^3\right)\right] + \frac{1}{12}(0.27)\left(0.015^3\right) = 20.0759\left(10^{-6}\right)\text{m}^4$$

Normal Stress: The normal stress is the combination of axial and bending stress. Here, **F** is negative since it is a compressive force. Also, $\mathbf{M}_y$ and $\mathbf{M}_z$ are negative since they are directed towards the negative sense of their respective axes. By inspection, point A is subjected to a maximum normal stress, which is in compression. Thus,

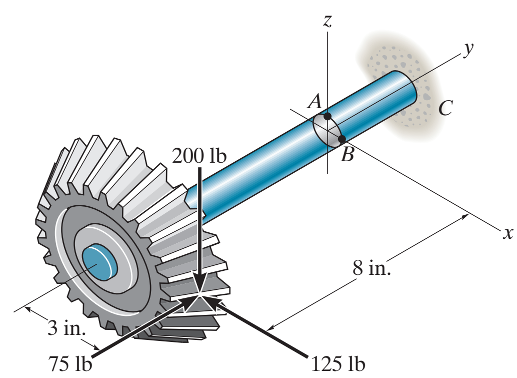
$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$-100\left(10^6\right) = -\frac{3P}{0.01005} - \frac{(-0.5P)(-0.15)}{0.14655\left(10^{-3}\right)} + \frac{-0.075P(0.1)}{20.0759\left(10^{-6}\right)}$$

$$P = 84470.40\text{ N} = 84.5\text{ kN} \qquad \textbf{Ans.}$$



•8–61. The beveled gear is subjected to the loads shown. Determine the stress components acting on the shaft at point A, and show the results on a volume element located at this point. The shaft has a diameter of 1 in. and is fixed to the wall at C.



$\Sigma F_x = 0; \quad V_x - 125 = 0; \quad V_x = 125 \text{ lb}$

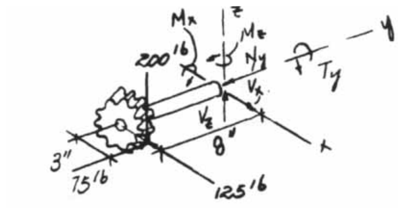
$\Sigma F_y = 0; \quad 75 - N_y = 0; \quad N_y = 75 \text{ lb}$

$\Sigma F_z = 0; \quad V_z - 200 = 0; \quad V_z = 200 \text{ lb}$

$\Sigma M_x = 0; \quad 200(8) - M_x = 0; \quad M_x = 1600 \text{ lb} \cdot \text{in.}$

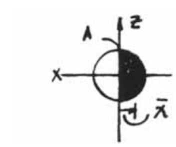
$\Sigma M_y = 0; \quad 200(3) - T_y = 0; \quad T_y = 600 \text{ lb} \cdot \text{in.}$

$\Sigma M_z = 0; \quad M_z + 75(3) - 125(8) = 0; \quad M_z = 775 \text{ lb} \cdot \text{in.}$



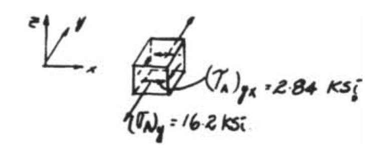
$A = \pi(0.5^2) = 0.7854 \text{ in}^2$

$J = \frac{\pi}{2}(0.5^4) = 0.098175 \text{ in}^4$



$I = \frac{\pi}{4}(0.5^4) = 0.049087 \text{ in}^4$

$(Q_A)_x = 0$



$$(Q_A)_z = \frac{4(0.5)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.5^2) = 0.08333 \text{ in}^3$$

$$(\sigma_A)_y = -\frac{N_y}{A} + \frac{M_x c}{I}$$

$$= -\frac{75}{0.7854} + \frac{1600(0.5)}{0.049087}$$

$= 16202 \text{ psi} = 16.2 \text{ ksi (T)}$ **Ans.**

$$(\tau_A)_{yx} = (\tau_A)_V - (\tau_A)_{\text{twist}}$$

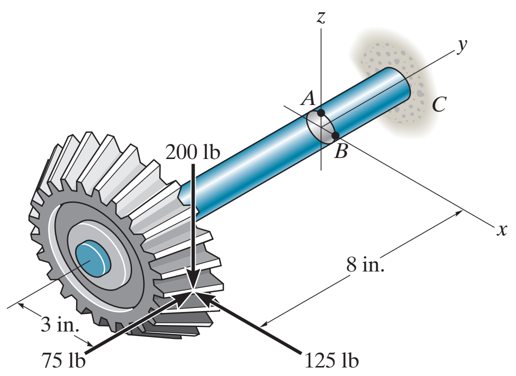
$$= \frac{V_x (Q_A)_z}{It} - \frac{T_y c}{J}$$

$$= \frac{125(0.08333)}{0.049087\,(1)} - \frac{600(0.5)}{0.098175}$$

$= -2843 \text{ psi} = -2.84 \text{ ksi}$ **Ans.**

$(\tau_A)_{yz} = \dfrac{V_z (Q_A)_x}{It} = 0$ **Ans.**

8–62. The beveled gear is subjected to the loads shown. Determine the stress components acting on the shaft at point B, and show the results on a volume element located at this point. The shaft has a diameter of 1 in. and is fixed to the wall at C.



$\Sigma F_x = 0; \qquad V_x - 125 = 0; \qquad V_x = 125 \text{ lb}$

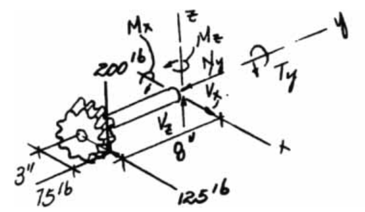
$\Sigma F_y = 0; \qquad 75 - N_y = 0; \qquad N_y = 75 \text{ lb}$

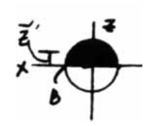
$\Sigma F_z = 0; \qquad V_z - 200 = 0; \qquad V_z = 200 \text{ lb}$

$\Sigma M_x = 0; \qquad 200(8) - M_x = 0; \qquad M_x = 1600 \text{ lb} \cdot \text{in.}$

$\Sigma M_y = 0; \qquad 200(3) - T_y = 0; \qquad T_y = 600 \text{ lb} \cdot \text{in.}$

$\Sigma M_z = 0; \qquad M_z + 75(3) - 125(8) = 0; \qquad M_z = 775 \text{ lb} \cdot \text{in.}$





$A = \pi(0.5^2) = 0.7854 \text{ in}^2$

$$J = \frac{\pi}{2}(0.5^4) = 0.098175 \text{ in}^4$$

$$I = \frac{\pi}{4}(0.5^4) = 0.049087 \text{ in}^4$$

$(Q_B)_z = 0$

$$(Q_B)_x = \frac{4(0.5)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.5^2) = 0.08333 \text{ in}^3$$

$$(\sigma_B)_y = -\frac{N_y}{A} + \frac{M_z c}{I}$$

$$= -\frac{75}{0.7854} + \frac{775(0.5)}{0.049087}$$

$= 7.80 \text{ ksi (T)}$ **Ans.**

$(\tau_B)_{yz} = (\tau_B)_V + (\tau_B)_{\text{twist}}$

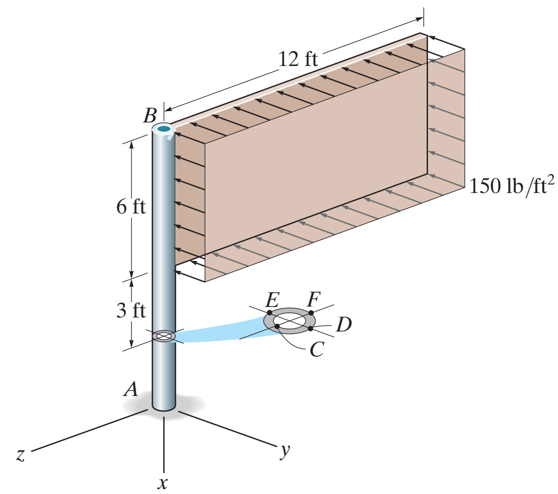
$$= \frac{V_z (Q_B)_x}{It} + \frac{T_y c}{J}$$

$$= \frac{200(0.08333)}{0.049087\,(1)} + \frac{600(0.5)}{0.098175}$$

$= 3395 \text{ psi} = 3.40 \text{ ksi}$ **Ans.**

$$(\tau_B)_{yx} = \frac{V_x (Q_B)_z}{It} = 0$$ **Ans.**

8–63. The uniform sign has a weight of 1500 lb and is supported by the pipe AB, which has an inner radius of 2.75 in. and an outer radius of 3.00 in. If the face of the sign is subjected to a uniform wind pressure of $p = 150\ \text{lb/ft}^2$, determine the state of stress at points C and D. Show the results on a differential volume element located at each of these points. Neglect the thickness of the sign, and assume that it is supported along the outside edge of the pipe.



Section Properties:

$$A = \pi\left(3^2 - 2.75^2\right) = 1.4375\pi\ \text{in}^2$$

$$I_y = I_z = \frac{\pi}{4}\left(3^4 - 2.75^4\right) = 18.6992\ \text{in}^4$$

$$(Q_C)_z = (Q_D)_y = 0$$

$$(Q_C)_y = (Q_D)_z = \frac{4(3)}{3\pi}\left[\frac{1}{2}(\pi)\left(3^2\right)\right] - \frac{4(2.75)}{3\pi}\left[\frac{1}{2}(\pi)\left(2.75^2\right)\right]$$

$$= 4.13542\ \text{in}^3$$

$$J = \frac{\pi}{2}\left(3^4 - 2.75^4\right) = 37.3984\ \text{in}^4$$

Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_C = \frac{-1.50}{1.4375\pi} - \frac{(-64.8)(12)(0)}{18.6992} + \frac{9.00(12)(2.75)}{18.6992}$$

$$= 15.6\ \text{ksi}\ (\mathbf{T})$$ **Ans.**

$$\sigma_D = \frac{-1.50}{1.4375\pi} - \frac{(-64.8)(12)(3)}{18.6992} + \frac{9.00(12)(0)}{18.6992}$$

$$= 124\ \text{ksi}\ (\mathbf{T})$$ **Ans.**

Shear Stress: The tranverse shear stress in the z and y directions and the torsional shear stress can be obtained using the shear formula and the torsion formula, $\tau_V = \frac{VQ}{It}$ and $\tau_{\text{twist}} = \frac{T\rho}{J}$, respectively.

$$(\tau_{xz})_D = \tau_{\text{twist}} = \frac{64.8(12)(3)}{37.3984} = 62.4\ \text{ksi}$$ **Ans.**

$$(\tau_{xy})_D = \tau_{V_y} = 0$$ **Ans.**

8–63. Continued

$$(\tau_{xy})_C = \tau_{V_y} - \tau_{\text{twist}}$$

$$= \frac{10.8(4.13542)}{18.6992(2)(0.25)} - \frac{64.8(12)(2.75)}{37.3984}$$

$$= -52.4 \text{ ksi}$$ **Ans.**

$$(\tau_{xz})_C = \tau_{V_z} = 0$$ **Ans.**

Internal Forces and Moments: As shown on FBD.

$\Sigma F_x = 0;\quad 1.50 + N_x = 0\qquad N_x = -15.0 \text{ kip}$

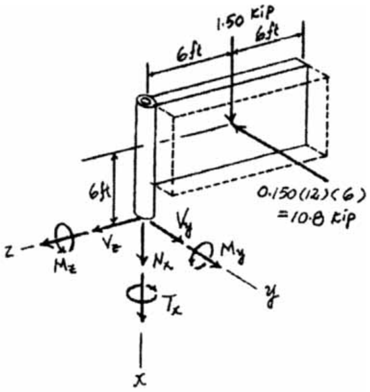
$\Sigma F_y = 0;\quad V_y - 10.8 = 0\qquad V_y = 10.8 \text{ kip}$

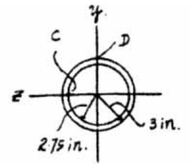
$\Sigma F_z = 0;\quad V_z = 0$

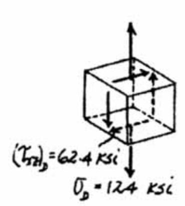
$\Sigma M_x = 0;\quad T_x - 10.8(6) = 0\qquad T_x = 64.8 \text{ kip}\cdot\text{ft}$

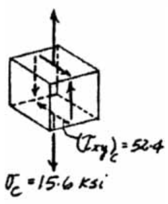
$\Sigma M_y = 0;\quad M_y - 1.50(6) = 0\qquad M_y = 9.00 \text{ kip}\cdot\text{ft}$

$\Sigma M_z = 0;\quad 10.8(6) + M_z = 0\qquad M_z = -64.8 \text{ kip}\cdot\text{ft}$

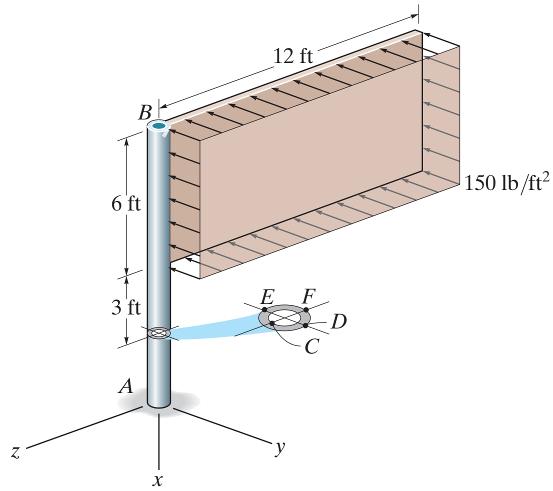








***8–64.** Solve Prob. 8–63 for points *E* and *F*.



Internal Forces and Moments: As shown on FBD.

$$\Sigma F_x = 0; \quad 1.50 + N_x = 0 \quad N_x = -1.50 \text{ kip}$$

$$\Sigma F_y = 0; \quad V_y - 10.8 = 0 \quad V_y = 10.8 \text{ kip}$$

$$\Sigma F_z = 0; \quad V_z = 0$$

$$\Sigma M_x = 0; \quad T_x - 10.8(6) = 0 \quad T_x = 64.8 \text{ kip} \cdot \text{ft}$$

$$\Sigma M_y = 0; \quad M_y - 1.50(6) = 0 \quad M_y = 9.00 \text{ kip} \cdot \text{ft}$$

$$\Sigma M_z = 0; \quad 10.8(6) + M_z = 0 \quad M_z = -64.8 \text{ kip} \cdot \text{ft}$$

Section Properties:

$$A = \pi\left(3^2 - 2.75^2\right) = 1.4375\pi \text{ in}^2$$

$$I_y = I_z = \frac{\pi}{4}\left(3^4 - 2.75^4\right) = 18.6992 \text{ in}^4$$

$$(Q_C)_z = (Q_D)_y = 0$$

$$(Q_C)_y = (Q_D)_z = \frac{4(3)}{3\pi}\left[\frac{1}{2}(\pi)\left(3^2\right)\right] - \frac{4(2.75)}{3\pi}\left[\frac{1}{2}(\pi)\left(2.75^2\right)\right]$$

$$= 4.13542 \text{ in}^3$$

$$J = \frac{\pi}{2}\left(3^4 - 2.75^4\right) = 37.3984 \text{ in}^4$$

Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_F = \frac{-1.50}{1.4375\pi} - \frac{(-64.8)(12)(0)}{18.6992} + \frac{9.00(12)(-3)}{18.6992}$$

$$= -17.7 \text{ ksi} = 17.7 \text{ ksi (C)}$$ **Ans.**

$$\sigma_E = \frac{-1.50}{1.4375\pi} - \frac{(-64.8)(12)(-3)}{18.6992} + \frac{9.00(12)(0)}{18.6992}$$

$$= -125 \text{ ksi} = 125 \text{ ksi (C)}$$ **Ans.**

8–64. Continued

Shear Stress: The tranverse shear stress in the z and y directions and the torsional shear stress can be obtained using the shear formula and the torsion formula, $\tau_V = \dfrac{VQ}{It}$ and $\tau_{\text{twist}} = \dfrac{T\rho}{J}$, respectively.

$$(\tau_{xz})_E = -\tau_{\text{twist}} = -\frac{64.8(12)(3)}{37.3984} = -62.4 \text{ ksi} \qquad \textbf{Ans.}$$

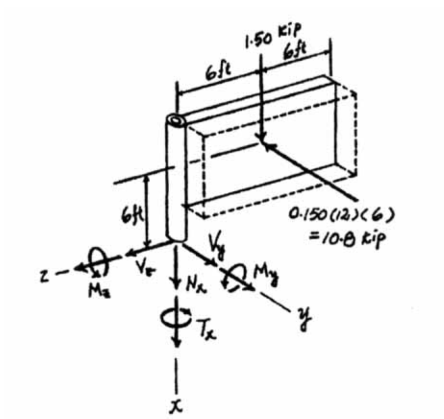
$$(\tau_{xy})_E = \tau_{V_y} = 0 \qquad \textbf{Ans.}$$

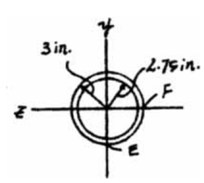
$$(\tau_{xy})_F = \tau_{V_y} + \tau_{\text{twist}}$$

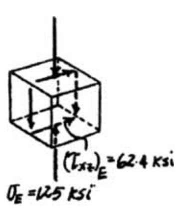
$$= \frac{10.8(4.13542)}{18.6992(2)(0.25)} + \frac{64.8(12)(3)}{37.3984}$$

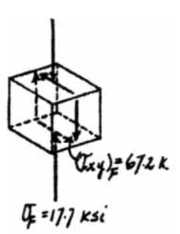
$$= 67.2 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{xy})_F = \tau_{V_y} = 0 \qquad \textbf{Ans.}$$

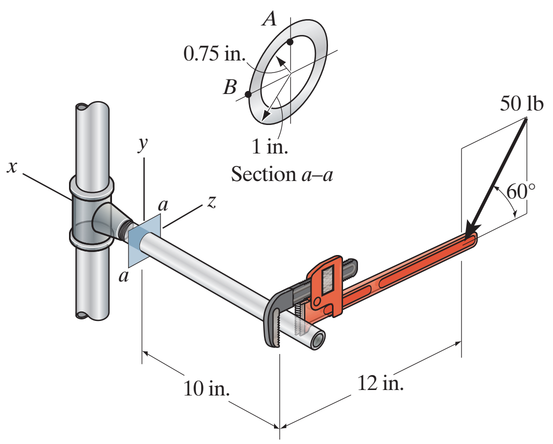








•8–65. Determine the state of stress at point A on the cross section of the pipe at section a–a.



Internal Loadings: Referring to the free - body diagram of the pipe's right segment, Fig. a,

$\Sigma F_y = 0;\quad V_y - 50\sin 60° = 0 \qquad V_y = 43.30 \text{ lb}$

$\Sigma F_z = 0;\quad V_z - 50\cos 60° = 0 \qquad V_z = 25 \text{ lb}$

$\Sigma M_x = 0;\ T + 50\sin 60°(12) = 0 \qquad T = -519.62 \text{ lb}\cdot\text{in}$

$\Sigma M_y = 0;\ M_y - 50\cos 60°(10) = 0 \qquad M_y = 250 \text{ lb}\cdot\text{in}$

$\Sigma M_z = 0;\ M_z + 50\sin 60°(10) = 0 \qquad M_z = -433.01 \text{ lb}\cdot\text{in}$

Section Properties: The moment of inertia about the y and z axes and the polar moment of inertia of the pipe are

$$I_y = I_z = \frac{\pi}{4}\left(1^4 - 0.75^4\right) = 0.53689 \text{ in}^4$$

$$J = \frac{\pi}{2}\left(1^4 - 0.75^4\right) = 1.07379 \text{ in}^4$$

Referring to Fig. b,

$$\left(Q_y\right)_A = 0$$

$$\left(Q_z\right)_A = \bar{y}_1'A_1' - \bar{y}_2'A_2' = \frac{4(1)}{3\pi}\left[\frac{\pi}{2}\left(1^2\right)\right] - \frac{4(0.75)}{3\pi}\left[\frac{\pi}{2}\left(0.75^2\right)\right] = 0.38542 \text{ in}^3$$

Normal Stress: The normal stress is contributed by bending stress only. Thus,

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y = 0.75$ in and $z = 0$. Then

$$\sigma_A = \frac{-433.01(0.75)}{0.53689} + 0 = 604.89 \text{ psi} = 605 \text{ psi (T)} \qquad \textbf{Ans.}$$

Shear Stress: The torsional shear stress developed at point A is

$$\left[\left(\tau_{xz}\right)_T\right]_A = \frac{T\rho_A}{J} = \frac{519.62(0.75)}{1.07379} = 362.93 \text{ psi}$$

8–65. Continued

The transverse shear stress developed at point A is

$$\left[\left(\tau_{xy}\right)_V\right]_A = 0$$

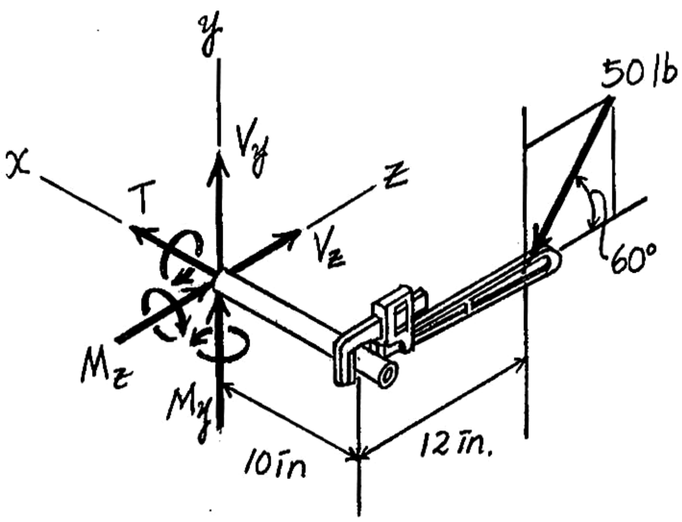
$$\left[\left(\tau_{xz}\right)_V\right]_A = \frac{V_z\left(Q_z\right)_A}{I_y t} = \frac{25(0.38542)}{0.53689(2 - 1.5)} = 35.89 \text{ psi}$$

Combining these two shear stress components,

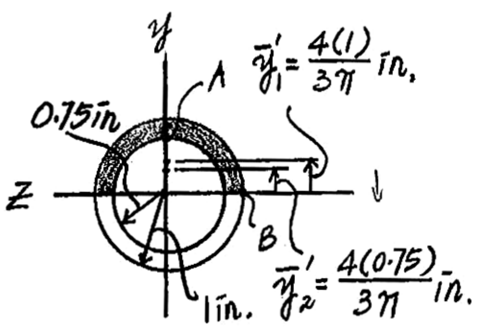
$$\left(\tau_{xy}\right)_A = 0 \qquad \textbf{Ans.}$$

$$\left(\tau_{xz}\right)_A = \left[\left(\tau_{xz}\right)_T\right]_A - \left[\left(\tau_{xz}\right)_V\right]_A$$

$$= 362.93 - 35.89 = 327 \text{ psi} \qquad \textbf{Ans.}$$

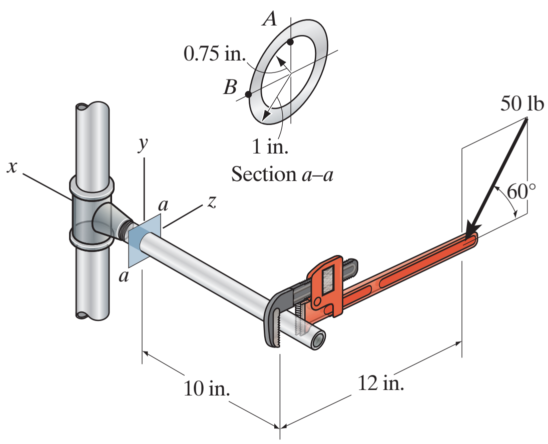


(a)



(b)

8–66. Determine the state of stress at point B on the cross section of the pipe at section a–a.



Internal Loadings: Referring to the free - body diagram of the pipe's right segment, Fig. *a*,

$\Sigma F_y = 0;\quad V_y - 50\sin 60^\circ = 0 \qquad V_y = 43.30\text{ lb}$

$\Sigma F_z = 0;\quad V_z - 50\cos 60^\circ = 0 \qquad V_z = 25\text{ lb}$

$\Sigma M_x = 0;\quad T + 50\sin 60^\circ(12) = 0 \qquad T = -519.62\text{ lb}\cdot\text{in}$

$\Sigma M_y = 0;\quad M_y - 50\cos 60^\circ(10) = 0 \qquad M_y = 250\text{ lb}\cdot\text{in}$

$\Sigma M_z = 0;\quad M_z + 50\sin 60^\circ(10) = 0 \qquad M_z = -433.01\text{ lb}\cdot\text{in}$

Section Properties: The moment of inertia about the y and z axes and the polar moment of inertia of the pipe are

$$I_y = I_z = \frac{\pi}{4}\left(1^4 - 0.75^4\right) = 0.53689\text{ in}^4$$

$$J = \frac{\pi}{2}\left(1^4 - 0.75^4\right) = 1.07379\text{ in}^4$$

Referring to Fig. *b*,

$$\left(Q_z\right)_B = 0$$

$$\left(Q_y\right)_B = \bar{y}_1'A_1' - \bar{y}_2'A_2' = \frac{4(1)}{3\pi}\left[\frac{\pi}{2}\left(1^2\right)\right] - \frac{4(0.75)}{3\pi}\left[\frac{\pi}{2}\left(0.75^2\right)\right] = 0.38542\text{ in}^3$$

Normal Stress: The normal stress is contributed by bending stress only. Thus,

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point B, $y = 0$ and $z = -1$. Then

$$\sigma_B = -0 + \frac{250(1)}{0.53689} = -465.64\text{ psi} = 466\text{ psi (C)} \qquad \textbf{Ans.}$$

Shear Stress: The torsional shear stress developed at point B is

$$\left[\left(\tau_{xy}\right)_T\right]_B = \frac{T\rho_C}{J} = \frac{519.62(1)}{1.07379} = 483.91\text{ psi}$$

8–66. Continued

The transverse shear stress developed at point B is

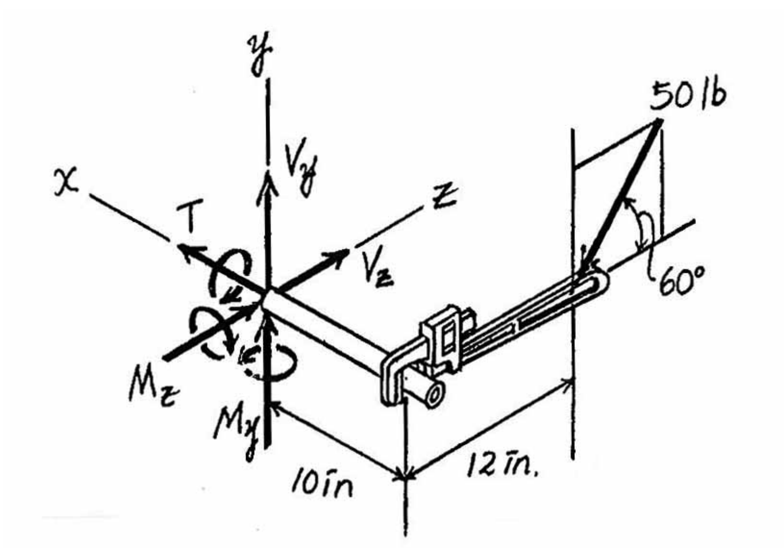
$$\left[\left(\tau_{xz}\right)_V\right]_B = 0$$

$$\left[\left(\tau_{xy}\right)_V\right]_B = \frac{V_y\left(Q_y\right)_B}{I_z t} = \frac{43.30(0.38542)}{0.53689(2 - 1.5)} = 62.17 \text{ psi}$$

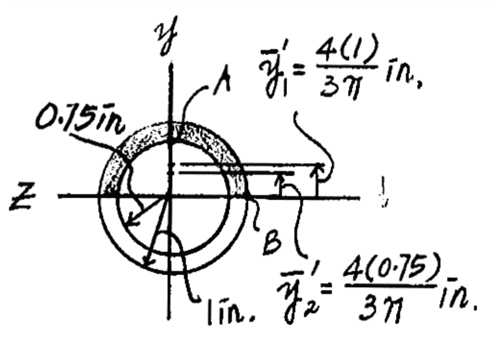
Combining these two shear stress components,

$$\left(\tau_{xy}\right)_B = \left[\left(\tau_{xy}\right)_T\right]_B - \left[\left(\tau_{xy}\right)_V\right]_B$$
$$= 483.91 - 62.17 = 422 \text{ psi} \qquad \textbf{Ans.}$$

$$\left(\tau_{xz}\right)_B = 0 \qquad \textbf{Ans.}$$

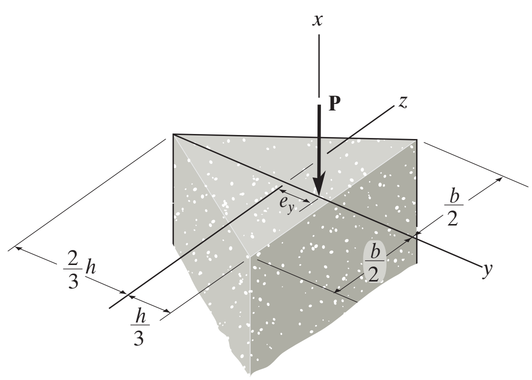


(a)

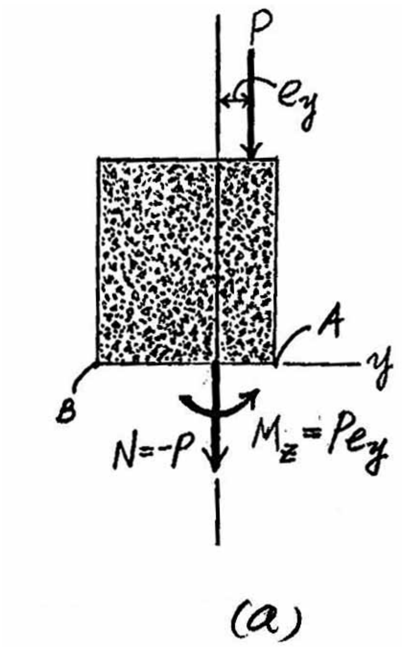


(b)

•8–67. The eccentric force **P** is applied at a distance e_y from the centroid on the concrete support shown. Determine the range along the y axis where **P** can be applied on the cross section so that no tensile stress is developed in the material.



Internal Loadings: As shown on the free - body diagram, Fig. *a*.



Section Properties: The cross-sectional area and moment of inertia about the z axis of the triangular concrete support are

$$A = \frac{1}{2}bh \qquad I_z = \frac{1}{36}bh^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z}$$

$$\sigma = \frac{-P}{\frac{1}{2}bh} - \frac{(Pe_y)y}{\frac{1}{36}bh^3}$$

$$\sigma = -\frac{2P}{bh^3}\left(h^2 + 18e_y y\right) \tag{1}$$

Here, it is required that $\sigma_A \leq 0$ and $\sigma_B \leq 0$. For point A, $y = \frac{h}{3}$, Then. Eq. (1) gives

$$0 \geq -\frac{2P}{bh^3}\left[h^2 + 18e_y\left(\frac{h}{3}\right)\right]$$

$$0 \leq h^2 + 6he_y$$

$$e_y \geq -\frac{h}{6}$$

For Point B, $y = -\frac{2}{3}h$. Then. Eq. (1) gives

$$0 \geq -\frac{2P}{bh^3}\left[h^2 + 18e_y\left(-\frac{2}{3}h\right)\right]$$

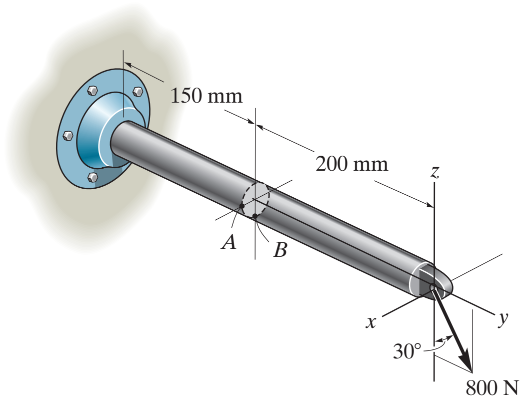
$$0 \leq h^2 - 12he_y$$

$$e_y \leq \frac{h}{12}$$

Thus, in order that no tensile stress be developed in the concrete support, e_y must be in the range of

$$-\frac{h}{6} \leq e_y \leq \frac{h}{12} \qquad \textbf{Ans.}$$

***8–68.** The bar has a diameter of 40 mm. If it is subjected to a force of 800 N as shown, determine the stress components that act at point A and show the results on a volume element located at this point.



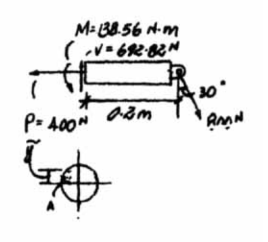
$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}(\pi)(0.02^4) = 0.1256637\,(10^{-6})\,\text{m}^4$$

$$A = \pi r^2 = \pi(0.02^2) = 1.256637\,(10^{-3})\,\text{m}^2$$

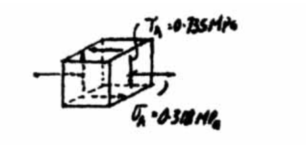
$$Q_A = \bar{y}'A' = \left(\frac{4\,(0.02)}{3\pi}\right)\left(\frac{\pi\,(0.02)^2}{2}\right) = 5.3333\,(10^{-6})\,\text{m}^3$$

$$\sigma_A = \frac{P}{A} + \frac{Mz}{I}$$

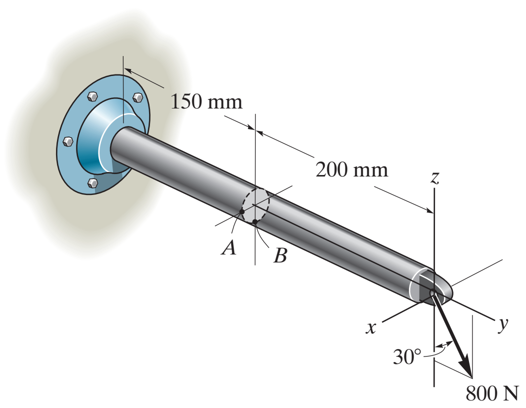
$$= \frac{400}{1.256637\,(10^{-3})} + 0 = 0.318\text{ MPa} \qquad \textbf{Ans.}$$

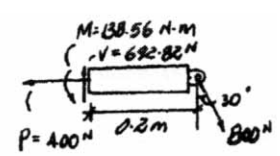


$$\tau_A = \frac{VQ_A}{I\,t} = \frac{692.82\,(5.3333)\,(10^{-6})}{0.1256637\,(10^{-6})(0.04)} = 0.735\text{ MPa} \qquad \textbf{Ans.}$$



•8–69. Solve Prob. 8–68 for point B.



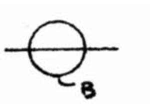


$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}(\pi)(0.02^4) = 0.1256637\,(10^{-6})\,\text{m}^4$$

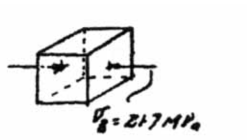
$$A = \pi r^2 = \pi(0.02^2) = 1.256637\,(10^{-3})\,\text{m}^2$$

$$Q_B = 0$$

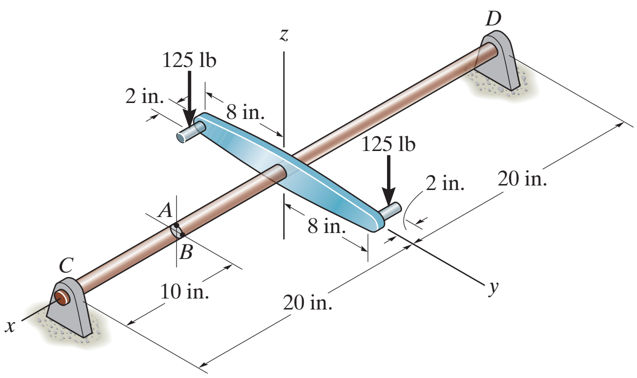
$$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{400}{1.256637\,(10^{-3})} - \frac{138.56\,(0.02)}{0.1256637\,(10^{-6})} = -21.7\text{ MPa} \qquad \textbf{Ans.}$$



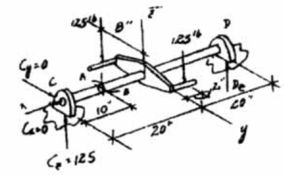
$$\tau_B = 0 \qquad \textbf{Ans.}$$



8–70. The $\frac{3}{4}$-in.-diameter shaft is subjected to the loading shown. Determine the stress components at point A. Sketch the results on a volume element located at this point. The journal bearing at C can exert only force components $\mathbf{C}_y$ and $\mathbf{C}_z$ on the shaft, and the thrust bearing at D can exert force components $\mathbf{D}_x$, $\mathbf{D}_y$, and $\mathbf{D}_z$ on the shaft.



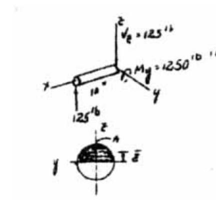
$$A = \frac{\pi}{4}(0.75^2) = 0.44179 \text{ in}^2$$



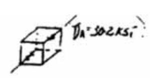
$$I = \frac{\pi}{4}(0.375^4) = 0.015531 \text{ in}^4$$

$$Q_A = 0$$

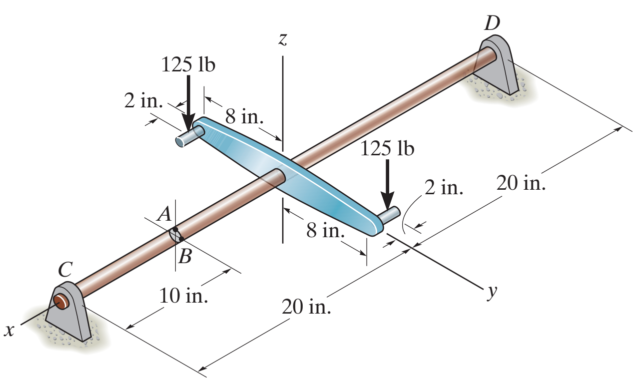
$$\tau_A = 0 \qquad \textbf{Ans.}$$



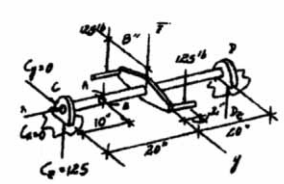
$$\sigma_A = \frac{M_y c}{I} = \frac{-1250(0.375)}{0.015531} = -30.2 \text{ ksi} = 30.2 \text{ ksi (C)} \qquad \textbf{Ans.}$$



8–71. Solve Prob. 8–70 for the stress components at point B.



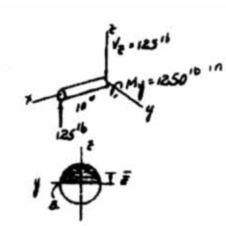
$$A = \frac{\pi}{4}(0.75^2) = 0.44179 \text{ in}^2$$



$$I = \frac{\pi}{4}(0.375^4) = 0.015531 \text{ in}^4$$

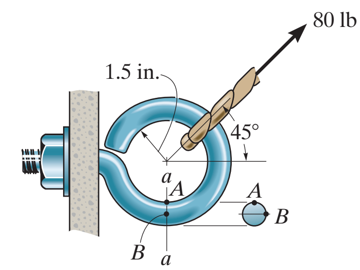
$$Q_B = y'A' = \frac{4(0.375)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.375^2) = 0.035156 \text{ in}^3$$

$$\sigma_B = 0 \qquad \textbf{Ans.}$$



$$\tau_B = \frac{V_z Q_B}{I\,t} = \frac{125(0.035156)}{0.015531(0.75)} = 0.377 \text{ ksi} \qquad \textbf{Ans.}$$

***8–72.** The hook is subjected to the force of 80 lb. Determine the state of stress at point A at section $a–a$. The cross section is circular and has a diameter of 0.5 in. Use the curved-beam formula to compute the bending stress.



The location of the neutral surface from the center of curvature of the hook, Fig. a, can be determined from

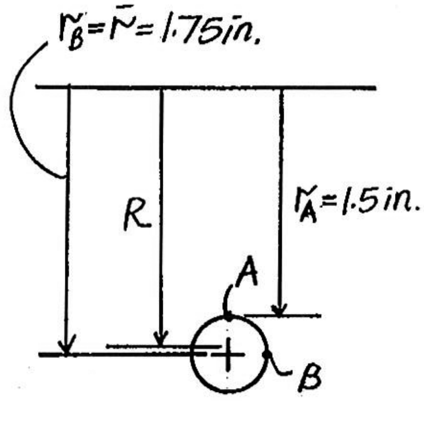
$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}}$$

where $A = \pi(0.25^2) = 0.0625\pi \text{ in}^2$

$$\Sigma \int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right) = 2\pi\left(1.75 - \sqrt{1.75^2 - 0.25^2}\right) = 0.11278 \text{ in.}$$

Thus,

$$R = \frac{0.0625\pi}{0.11278} = 1.74103 \text{ in.}$$



(a)

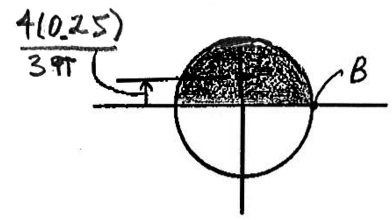
Then

$$e = \bar{r} - R = 1.75 - 1.74103 = 0.0089746 \text{ in.}$$

Referring to Fig. b, I and Q_A are

$$I = \frac{\pi}{4}(0.25^4) = 0.9765625(10^{-3})\pi \text{ in}^4$$

$$Q_A = 0$$



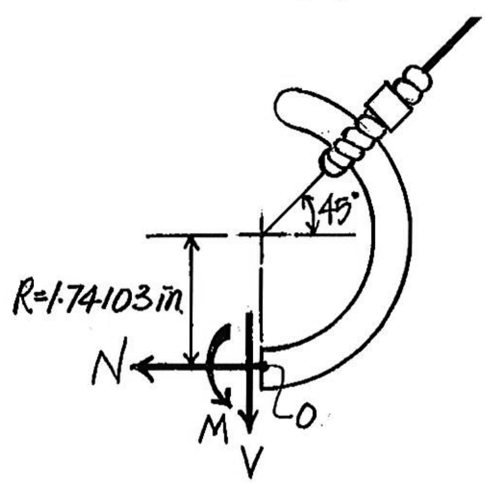
(b)

Consider the equilibrium of the FBD of the hook's cut segment, Fig. c,

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad N - 80\cos 45° = 0 \qquad N = 56.57 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad 80\sin 45° - V = 0 \qquad V = 56.57 \text{ lb}$$

$$\circlearrowleft + \Sigma M_o = 0; \qquad M - 80\cos 45°(1.74103) = 0 \qquad M = 98.49 \text{ lb}\cdot\text{in}$$



(c)

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{M(R - r)}{Ae\,r}$$

Here, $M = 98.49 \text{ lb}\cdot\text{in}$ since it tends to reduce the curvature of the hook. For point A, $r = 1.5$ in. Then

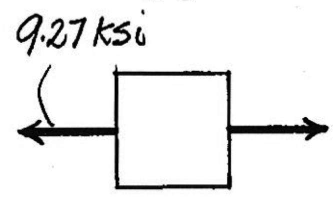
$$\sigma = \frac{56.57}{0.0625\pi} + \frac{(98.49)(1.74103 - 1.5)}{0.0625\pi(0.0089746)(1.5)}$$

$$= 9.269(10^3) \text{ psi} = 9.27 \text{ ksi (T)} \qquad \textbf{Ans.}$$

The shear stress in contributed by the transverse shear stress only. Thus

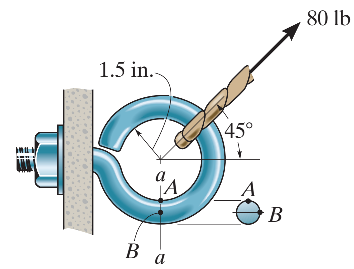
$$\tau = \frac{VQ_A}{It} = 0 \qquad \textbf{Ans.}$$

The state of strees of point A can be represented by the element shown in Fig. d.



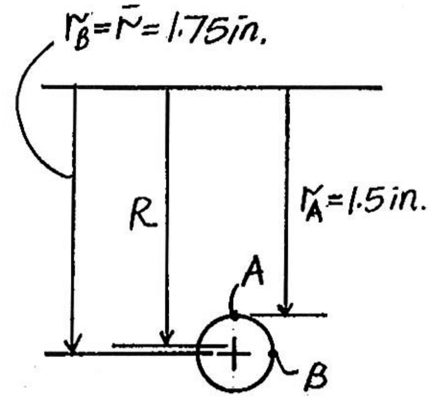
(d)

•8–73. The hook is subjected to the force of 80 lb. Determine the state of stress at point B at section a–a. The cross section has a diameter of 0.5 in. Use the curved-beam formula to compute the bending stress.



The location of the neutral surface from the center of curvature of the the hook, Fig. a, can be determined from

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}}$$



Where $A = \pi(0.25^2) = 0.0625\pi \text{ in}^2$

$$\Sigma \int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right) = 2\pi\left(1.75 - \sqrt{1.75^2 - 0.25^2}\right) = 0.11278 \text{ in.}$$

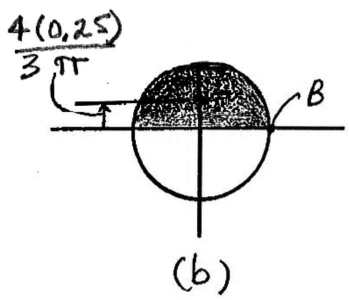
Thus,

$$R = \frac{0.0625\pi}{0.11278} = 1.74103 \text{ in}$$

Then

$$e = \bar{r} - R = 1.75 - 1.74103 = 0.0089746 \text{ in}$$

Referring to Fig. b, I and Q_B are computed as



$$I = \frac{\pi}{4}(0.25^4) = 0.9765625(10^{-3})\pi \text{ in}^4$$

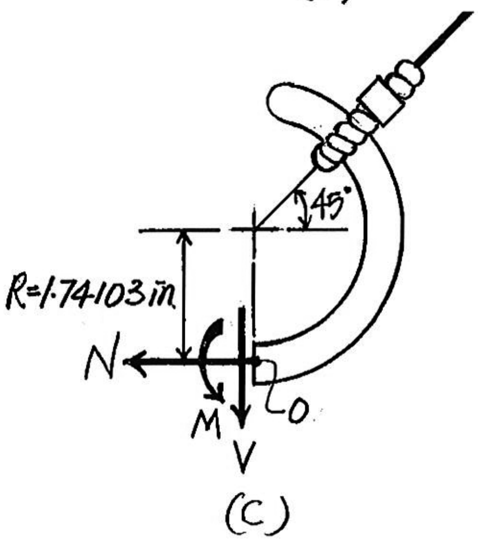
$$Q_B = \bar{y}'A' = \frac{4(0.25)}{3\cancel{\pi}}\left[\frac{\cancel{\pi}}{2}(0.25^2)\right] = 0.0104167 \text{ in}^3$$

Consider the equilibrium of the FBD of the hook's cut segment, Fig. c,

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad N - 80\cos 45° = 0 \qquad N = 56.57 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad 80\sin 45° - V = 0 \qquad V = 56.57 \text{ lb}$$

$$\circlearrowleft + \Sigma M_o = 0; \qquad M - 80\cos 45°\,(1.74103) = 0 \qquad M = 98.49 \text{ lb}\cdot\text{in}$$



The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{M(R - r)}{Aer}$$

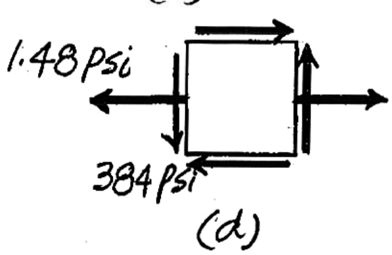
Here, $M = 98.49 \text{ lb}\cdot\text{in}$ since it tends to reduce. the curvature of the hook. For point B, $r = 1.75$ in. Then

$$\sigma = \frac{56.57}{0.0625\pi} + \frac{(98.49)(1.74103 - 1.75)}{0.0625\,\pi\,(0.0089746)(1.75)}$$

$$= 1.62 \text{ psi (T)} \qquad \textbf{Ans.}$$

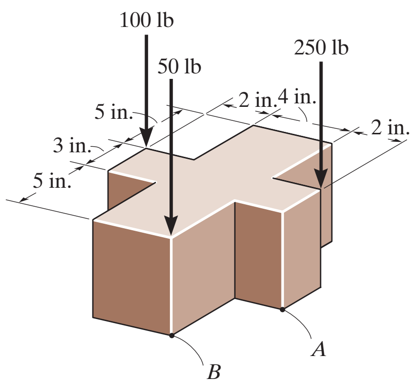
The shear stress is contributed by the transverse shear stress only. Thus,

$$\tau = \frac{VQ_B}{It} = \frac{56.57\,(0.0104167)}{0.9765625(10^{-3})\pi\,(0.5)} = 3.84 \text{ psi} \qquad \textbf{Ans.}$$



The state of stress of point B can be represented by the element shown in Fig. d.

8–74. The block is subjected to the three axial loads shown. Determine the normal stress developed at points A and B. Neglect the weight of the block.



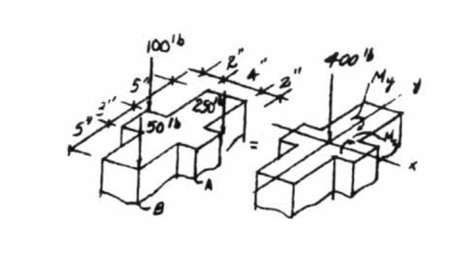
$$M_x = -250(1.5) - 100(1.5) + 50(6.5) = -200\ \text{lb}\cdot\text{in.}$$

$$M_y = 250(4) + 50(2) - 100(4) = 700\ \text{lb}\cdot\text{in.}$$

$$I_x = \frac{1}{12}(4)(13^3) + 2\left(\frac{1}{12}\right)(2)(3^3) = 741.33\ \text{in}^4$$

$$I_y = \frac{1}{12}(3)(8^3) + 2\left(\frac{1}{12}\right)(5)(4^3) = 181.33\ \text{in}^4$$

$$A = 4(13) + 2(2)(3) = 64\ \text{in}^2$$



$$\sigma = \frac{P}{A} - \frac{M_y\,x}{I_y} + \frac{M_x\,y}{I_x}$$

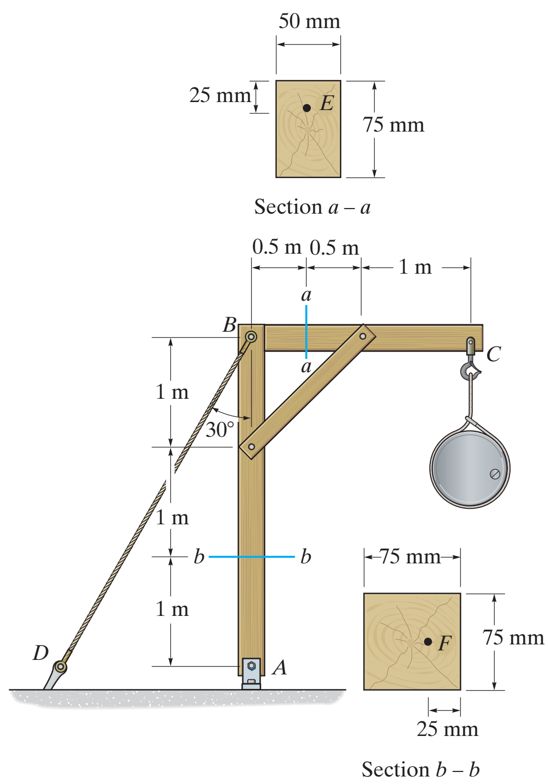
$$\sigma_A = -\frac{400}{64} - \frac{700(4)}{181.33} + \frac{-200\,(-1.5)}{741.33}$$

$$= -21.3\ \text{psi}$$ **Ans.**

$$\sigma_B = -\frac{400}{64} - \frac{700(2)}{181.33} + \frac{-200\,(-6.5)}{741.33}$$

$$= -12.2\ \text{psi}$$ **Ans.**

8–75. The 20-kg drum is suspended from the hook mounted on the wooden frame. Determine the state of stress at point E on the cross section of the frame at section a–a. Indicate the results on an element.



Support Reactions: Referring to the free-body diagram of member BC shown in Fig. a,

$$\circlearrowleft + \Sigma M_B = 0; \quad F \sin 45°(1) - 20(9.81)(2) = 0 \qquad F = 554.94\,\text{N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 554.94 \cos 45° - B_x = 0 \qquad B_x = 392.4\,\text{N}$$

$$+\uparrow \Sigma F_y = 0; \quad 554.94 \sin 45° - 20(9.81) - B_y = 0 \qquad B_y = 196.2\,\text{N}$$

Internal Loadings: Consider the equilibrium of the free - body diagram of the right segment shown in Fig. b.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N - 392.4 = 0 \qquad N = 392.4\,\text{N}$$

$$+\uparrow \Sigma F_y = 0; \quad V - 196.2 = 0 \qquad V = 196.2\,\text{N}$$

$$\circlearrowleft + \Sigma M_C = 0; \quad 196.2(0.5) - M = 0 \qquad M = 98.1\,\text{N}\cdot\text{m}$$

Section Properties: The cross -sectional area and the moment of inertia of the cross section are

$$A = 0.05(0.075) = 3.75\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.05)\left(0.075^3\right) = 1.7578\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. c, Q_E is

$$Q_E = \bar{y}'A' = 0.025(0.025)(0.05) = 3.125\left(10^{-6}\right)\text{m}^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point A, $y = 0.0375 - 0.025 = 0.0125\,\text{m}$. Then

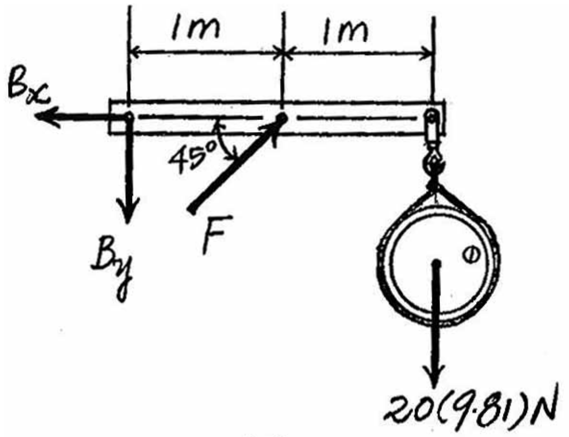
$$\sigma_E = \frac{392.4}{3.75\left(10^{-3}\right)} + \frac{98.1(0.0125)}{1.7578\left(10^{-6}\right)} = 802\text{ kPa} \qquad \textbf{Ans.}$$

Shear Stress: The shear stress is contributed by transverse shear stress only. Thus,

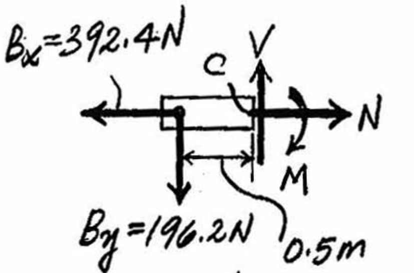
$$\tau_E = \frac{VQ_A}{It} = \frac{196.2\left[31.25\left(10^{-6}\right)\right]}{1.7578\left(10^{-6}\right)(0.05)} = 69.8\text{ kPa} \qquad \textbf{Ans.}$$

The state of stress at point E is represented on the element shown in Fig. d.

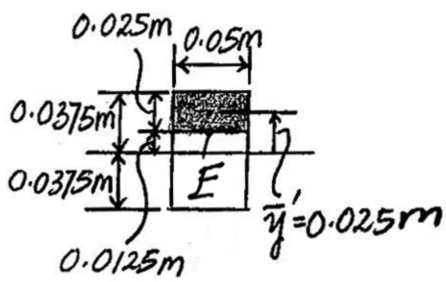
8–75. Continued



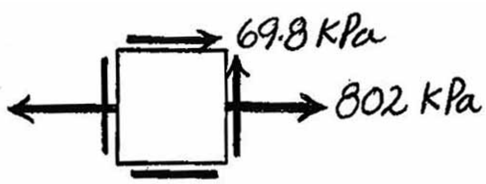
(a)



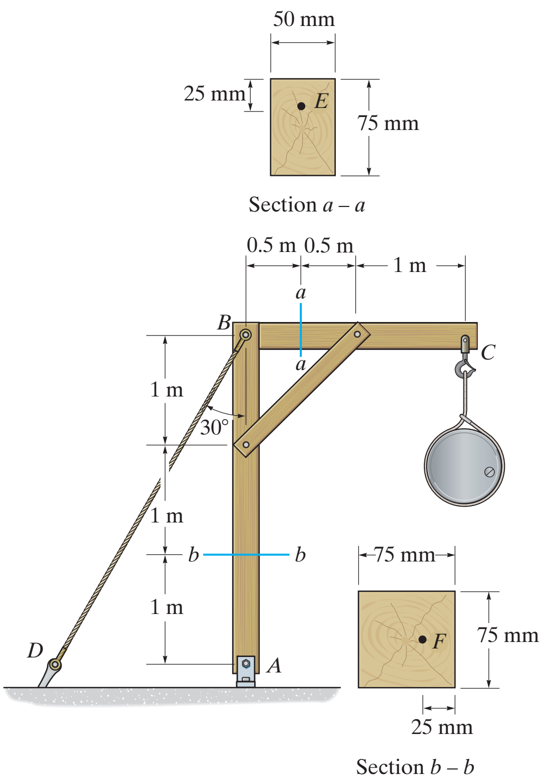
(b)



(c)



***8–76.** The 20-kg drum is suspended from the hook mounted on the wooden frame. Determine the state of stress at point F on the cross section of the frame at section b–b. Indicate the results on an element.



Support Reactions: Referring to the free-body diagram of the entire frame shown in Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \quad F_{BD} \sin 30°(3) - 20(9.81)(2) = 0 \quad F_{BD} = 261.6\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 261.6\cos 30° - 20(9.81) = 0 \quad A_y = 422.75\text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 261.6 \sin 30° = 0 \quad A_x = 130.8\text{ N}$$

Internal Loadings: Consider the equilibrium of the free - body diagram of the lower cut segment, Fig. b,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 130.8 - V = 0 \quad V = 130.8\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad 422.75 - N = 0 \quad N = 422.75\text{ N}$$

$$\circlearrowleft + \Sigma M_C = 0; \quad 130.8(1) - M = 0 \quad M = 130.8\text{ N}\cdot\text{m}$$

Section Properties: The cross -sectional area and the moment of inertia about the centroidal axis of the cross section are

$$A = 0.075(0.075) = 5.625\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.075)\left(0.075^3\right) = 2.6367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. c, Q_E is

$$Q_F = \bar{y}'A' = 0.025(0.025)(0.075) = 46.875\left(10^{-6}\right)\text{m}^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point F, $y = 0.0375 - 0.025 = 0.0125$ m. Then

$$\sigma_F = \frac{-422.75}{5.625\left(10^{-3}\right)} - \frac{130.8(0.0125)}{2.6367\left(10^{-6}\right)}$$

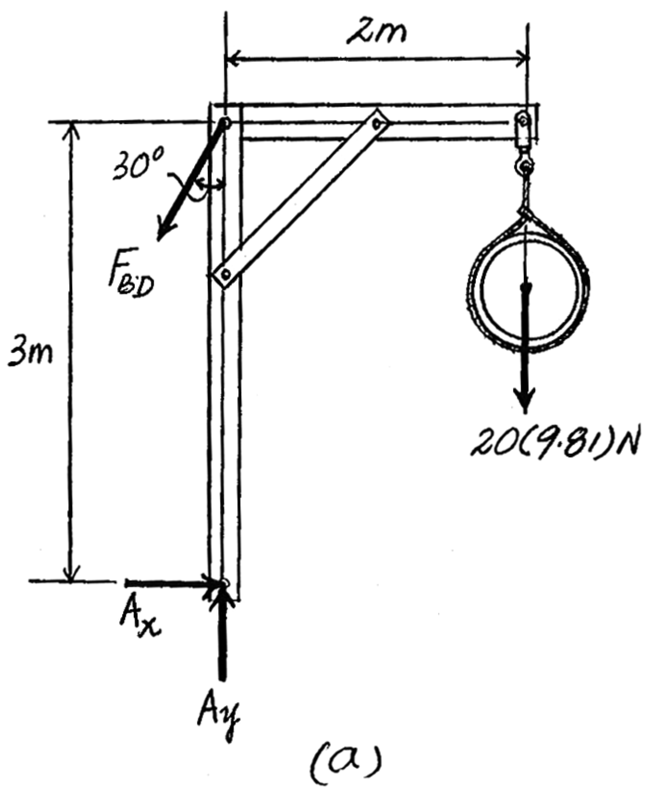
$$= -695.24\text{ kPa} = 695\text{ kPa (C)} \qquad \textbf{Ans.}$$

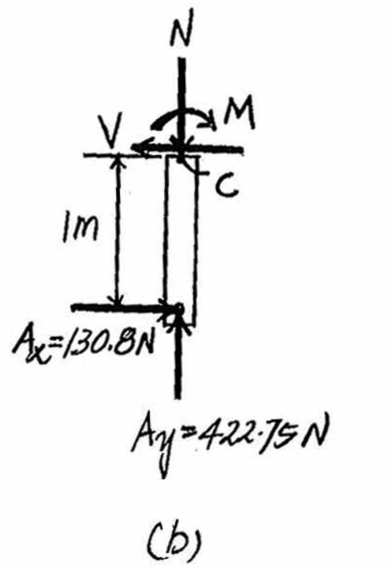
8–76. Continued

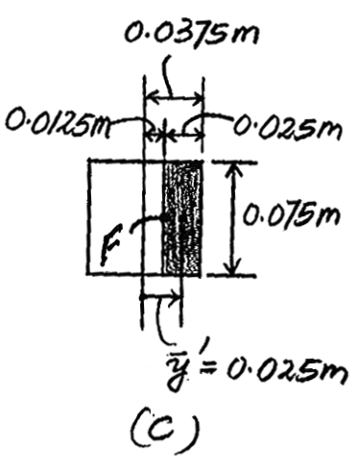
Shear Stress: The shear stress is contributed by transverse shear stress only. Thus,

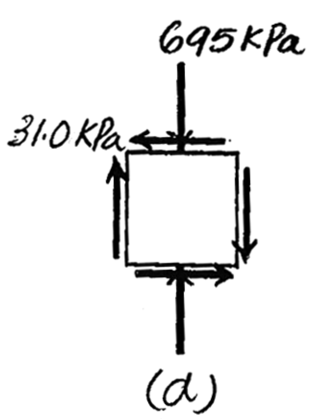
$$\tau_A = \frac{VQ_A}{It} = \frac{130.8\left[46.875\left(10^{-6}\right)\right]}{2.6367\left(10^{-6}\right)(0.075)} = 31.0 \text{ kPa} \qquad \textbf{Ans.}$$

The state of stress at point A is represented on the element shown in Fig. *d*.

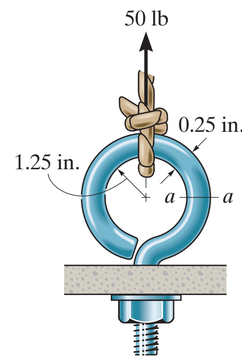




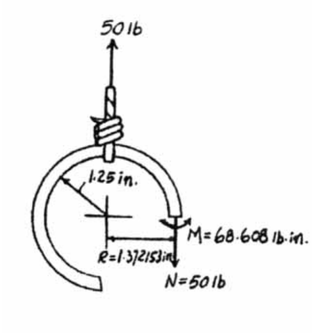




•**8–77.** The eye is subjected to the force of 50 lb. Determine the maximum tensile and compressive stresses at section $a-a$. The cross section is circular and has a diameter of 0.25 in. Use the curved-beam formula to compute the bending stress.



Section Properties:



$$\bar{r} = 1.25 + \frac{0.25}{2} = 1.375 \text{ in.}$$

$$\int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$$

$$= 2\pi\left(1.375 - \sqrt{1.375^2 - 0.125^2}\right)$$

$$= 0.035774 \text{ in.}$$

$$A = \pi\left(0.125^2\right) = 0.049087 \text{ in}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.049087}{0.035774} = 1.372153 \text{ in.}$$

$$\bar{r} - R = 1.375 - 1.372153 = 0.002847 \text{ in.}$$

Internal Force and Moment: As shown on FBD. The internal moment must be computed about the neutral axis. $M = 68.608 \text{ lb} \cdot \text{in}$ is positive since it tends to increase the beam's radius of curvature.

Normal Stress: Applying the curved - beam formula, For tensile stress

$$(\sigma_t)_{\max} = \frac{N}{A} + \frac{M(R - r_1)}{Ar_1(\bar{r} - R)}$$

$$= \frac{50.0}{0.049087} + \frac{68.608(1.372153 - 1.25)}{0.049087(1.25)(0.002847)}$$

$$= 48996 \text{ psi} = 49.0 \text{ ksi (T)} \qquad \textbf{Ans.}$$

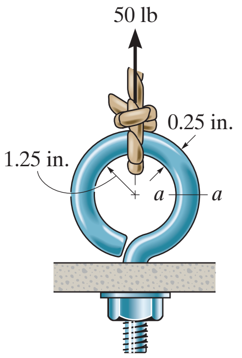
For compressive stress

$$(\sigma_c)_{\max} = \frac{N}{A} + \frac{M(R - r_2)}{Ar_2(\bar{r} - R)}$$

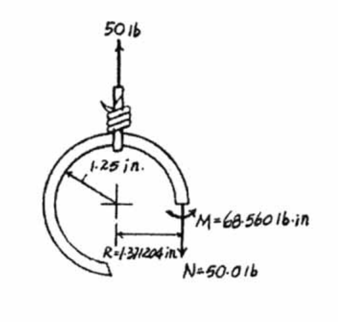
$$= \frac{50.0}{0.049087} + \frac{68.608(1.372153 - 1.50)}{0.049087(1.50)(0.002847)}$$

$$= -40826 \text{ psi} = 40.8 \text{ ksi (C)} \qquad \textbf{Ans.}$$

8–78. Solve Prob. 8–77 if the cross section is square, having dimensions of 0.25 in. by 0.25 in.



Section Properties:



$$\bar{r} = 1.25 + \frac{0.25}{2} = 1.375 \text{ in.}$$

$$\int_A \frac{dA}{r} = b\ln\frac{r_2}{r_1} = 0.25\ln\frac{1.5}{1.25} = 0.45580 \text{ in.}$$

$$A = 0.25(0.25) = 0.0625 \text{ in}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.0625}{0.045580} = 1.371204 \text{ in.}$$

$$\bar{r} - R = 1.375 - 1.371204 = 0.003796 \text{ in.}$$

Internal Force and Moment: As shown on FBD. The internal moment must be computed about the neutral axis. $M = 68.560 \text{ lb}\cdot\text{in.}$ is positive since it tends to increase the beam's radius of curvature.

Normal Stress: Applying the curved -beam formula, For tensile stress

$$(\sigma_t)_{\max} = \frac{N}{A} + \frac{M(R - r_1)}{Ar_1(\bar{r} - R)}$$

$$= \frac{50.0}{0.0625} + \frac{68.560(1.371204 - 1.25)}{0.0625(1.25)(0.003796)}$$

$$= 28818 \text{ psi} = 28.8 \text{ ksi (T)} \qquad \textbf{Ans.}$$

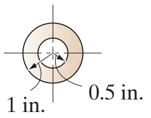
For Compressive stress

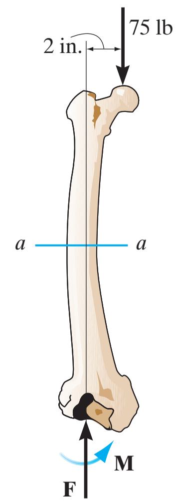
$$(\sigma_c)_{\max} = \frac{N}{A} + \frac{M(R - r_2)}{Ar_2(\bar{r} - R)}$$

$$= \frac{50.0}{0.0625} + \frac{68.560(1.371204 - 1.5)}{0.0625(1.5)(0.003796)}$$

$$= -24011 \text{ psi} = 24.0 \text{ ksi (C)} \qquad \textbf{Ans.}$$

8–79. If the cross section of the femur at section *a–a* can be approximated as a circular tube as shown, determine the maximum normal stress developed on the cross section at section *a–a* due to the load of 75 lb.

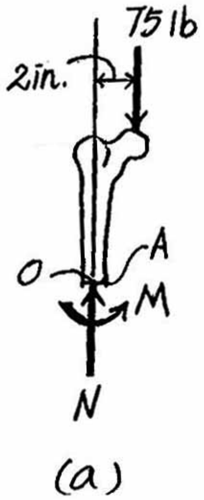




Internal Loadings: Considering the equilibrium for the free-body diagram of the femur's upper segment, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad N - 75 = 0 \qquad N = 75\,\text{lb}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad M - 75(2) = 0 \qquad M = 150\,\text{lb}\cdot\text{in}$$



Section Properties: The cross-sectional area, the moment of inertia about the centroidal axis of the femur's cross section are

$$A = \pi\left(1^2 - 0.5^2\right) = 0.75\pi\ \text{in}^2$$

$$I = \frac{\pi}{4}\left(1^4 - 0.5^4\right) = 0.234375\pi\ \text{in}^4$$

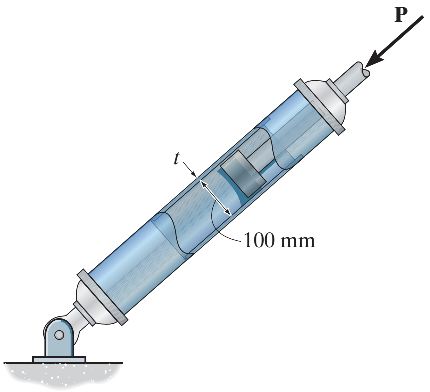
Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

By inspection, the maximum normal stress is in compression.

$$\sigma_{\max} = \frac{-75}{0.75\pi} - \frac{150(1)}{0.234375\pi} = -236\ \text{psi} = 236\ \text{psi (C)} \qquad \textbf{Ans.}$$

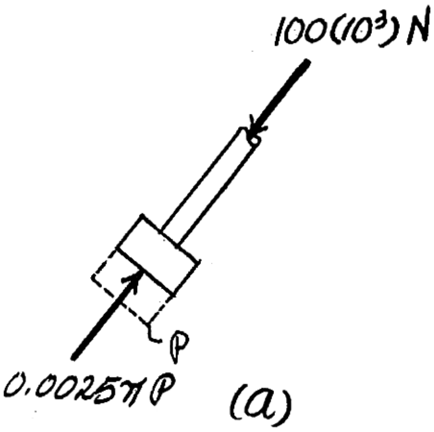
***8–80.** The hydraulic cylinder is required to support a force of $P = 100\text{ kN}$. If the cylinder has an inner diameter of 100 mm and is made from a material having an allowable normal stress of $\sigma_{\text{allow}} = 150\text{ MPa}$, determine the required minimum thickness t of the wall of the cylinder.



Equation of Equilibrium: The absolute pressure developed in the hydraulic cylinder can be determined by considering the equilibrium of the free-body diagram of the piston shown in Fig. *a*. The resultant force of the pressure on the piston is $F = pA = p\left[\frac{\pi}{4}\left(0.1^2\right)\right] = 0.0025\pi p$. Thus,

$$\Sigma F_{x'} = 0; \quad 0.0025\pi p - 100\left(10^3\right) = 0$$

$$p = 12.732\left(10^6\right)\text{ Pa}$$



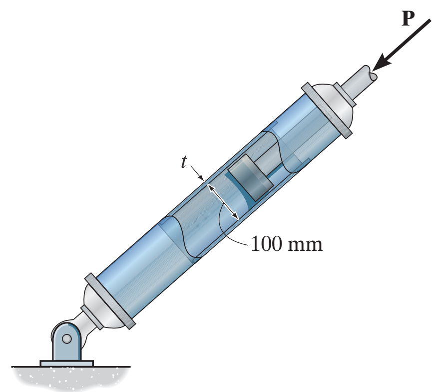
Normal Stress: For the cylinder, the hoop stress is twice as large as the longitudinal stress,

$$\sigma_{\text{allow}} = \frac{pr}{t}; \qquad 150\left(10^6\right) = \frac{12.732\left(10^6\right)(50)}{t}$$

$$t = 4.24\text{ mm} \qquad \textbf{Ans.}$$

Since $\frac{r}{t} = \frac{50}{4.24} = 11.78 > 10$, thin -wall analysis is valid.

•8–81. The hydraulic cylinder has an inner diameter of 100 mm and wall thickness of $t = 4$ mm. If it is made from a material having an allowable normal stress of $\sigma_{\text{allow}} = 150$ MPa, determine the maximum allowable force **P**.



Normal Stress: For the hydraulic cylinder, the hoop stress is twice as large as the longitudinal stress.

Since $\dfrac{r}{t} = \dfrac{50}{4} = 12.5 > 10$, thin-wall analysis can be used.

$$\sigma_{\text{allow}} = \frac{pr}{t}; \qquad 150\left(10^6\right) = \frac{p(50)}{4}$$

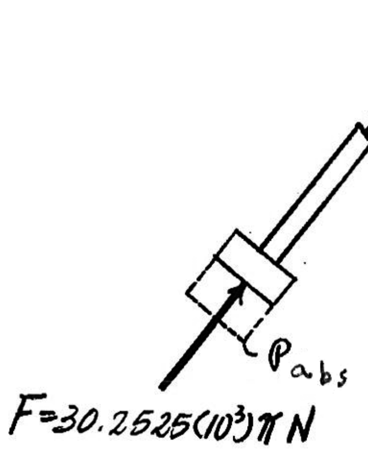
$$p = 12\left(10^6\right) \text{ MPa} \qquad \textbf{Ans.}$$

Equation of Equilibrium: The resultant force on the piston is

$F = pA = 12\left(10^6\right)\left[\dfrac{\pi}{4}\left(0.1^2\right)\right] = 30\left(10^3\right)\pi$. Referring to the free-body diagram of the piston shown in Fig. a,

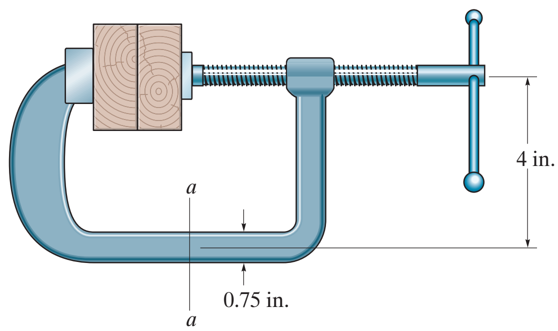
$$\Sigma F_{x'} = 0; \quad 30\left(10^3\right)\pi - P = 0$$

$$P = 94.247\left(10^3\right)\text{N} = 94.2 \text{ kN} \qquad \textbf{Ans.}$$



(a)

8–82. The screw of the clamp exerts a compressive force of 500 lb on the wood blocks. Determine the maximum normal stress developed along section $a-a$. The cross section there is rectangular, 0.75 in. by 0.50 in.

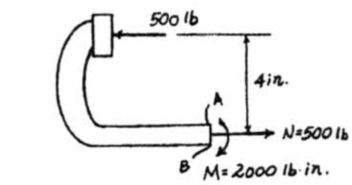


Internal Force and Moment: As shown on FBD.

Section Properties:

$$A = 0.5(0.75) = 0.375 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)\left(0.75^3\right) = 0.017578 \text{ in}^4$$



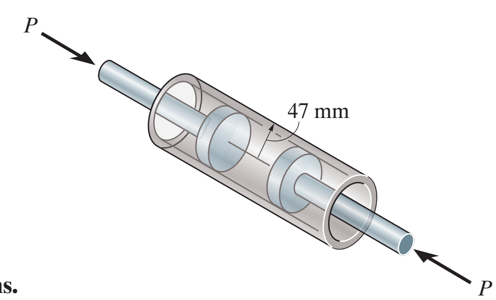
Maximum Normal Stress: Maximum normal stress occurs at point A.

$$\sigma_{\max} = \sigma_A = \frac{N}{A} + \frac{Mc}{I}$$

$$= \frac{500}{0.375} + \frac{2000(0.375)}{0.017578}$$

$$= 44000 \text{ psi} = 44.0 \text{ ksi } (\mathbf{T})$$ **Ans.**

8–83. Air pressure in the cylinder is increased by exerting forces $P = 2$ kN on the two pistons, each having a radius of 45 mm. If the cylinder has a wall thickness of 2 mm, determine the state of stress in the wall of the cylinder.



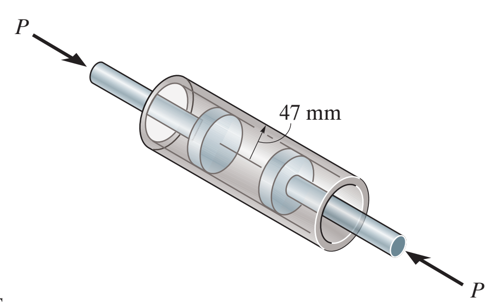
$$p = \frac{P}{A} = \frac{2(10^3)}{\pi(0.045^2)} = 314\,380.13 \text{ Pa}$$

$$\sigma_1 = \frac{p\,r}{t} = \frac{314\,380.13(0.045)}{0.002} = 7.07 \text{ MPa}$$ **Ans.**

$$\sigma_2 = 0$$ **Ans.**

The pressure P is supported by the surface of the pistons in the longitudinal direction.

***8–84.** Determine the maximum force P that can be exerted on each of the two pistons so that the circumferential stress component in the cylinder does not exceed 3 MPa. Each piston has a radius of 45 mm and the cylinder has a wall thickness of 2 mm.

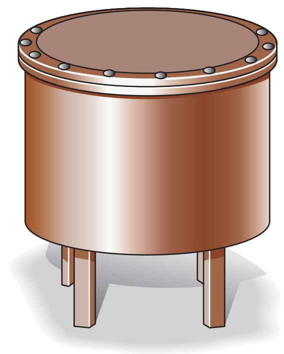


$$\sigma = \frac{p\,r}{t}; \qquad 3(10^6) = \frac{p(0.045)}{0.002}$$

$$P = 133.3 \text{ kPa}$$ **Ans.**

$$P = pA = 133.3\left(10^3\right)(\pi)(0.045)^2 = 848 \text{ N}$$ **Ans.**

•**8–85.** The cap on the cylindrical tank is bolted to the tank along the flanges. The tank has an inner diameter of 1.5 m and a wall thickness of 18 mm. If the largest normal stress is not to exceed 150 MPa, determine the maximum pressure the tank can sustain. Also, compute the number of bolts required to attach the cap to the tank if each bolt has a diameter of 20 mm. The allowable stress for the bolts is $(\sigma_{\text{allow}})_b = 180$ MPa.



Hoop Stress for Cylindrical Tank: Since $\frac{r}{t} = \frac{750}{18} = 41.7 > 10$, then thin wall analysis can be used. Applying Eq. 8–1

$$\sigma_1 = \sigma_{\text{allow}} = \frac{pr}{t}$$

$$150\left(10^6\right) = \frac{p(750)}{18}$$

$$p = 3.60 \text{ MPa} \qquad \textbf{Ans.}$$

Force Equilibrium for the Cap:

$$+\uparrow \Sigma F_y = 0; \qquad 3.60\left(10^6\right)\left[\pi\left(0.75^2\right)\right] - F_b = 0$$

$$F_b = 6.3617\left(10^6\right) \text{ N}$$

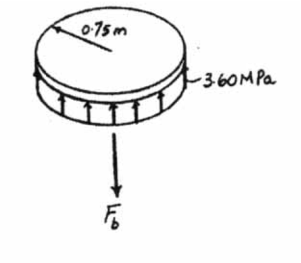
Allowable Normal Stress for Bolts:

$$(\sigma_{\text{allow}})_b = \frac{P}{A}$$

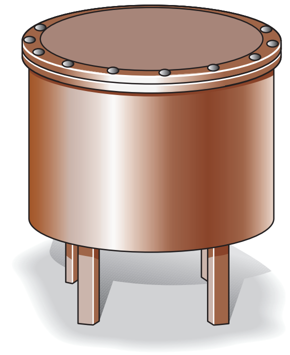
$$180\left(10^6\right) = \frac{6.3617(10^6)}{n\left[\frac{\pi}{4}(0.02^2)\right]}$$

$$n = 112.5$$

Use $n = 113$ bolts **Ans.**



8–86. The cap on the cylindrical tank is bolted to the tank along the flanges. The tank has an inner diameter of 1.5 m and a wall thickness of 18 mm. If the pressure in the tank is $p = 1.20$ MPa, determine the force in each of the 16 bolts that are used to attach the cap to the tank. Also, specify the state of stress in the wall of the tank.



Hoop Stress for Cylindrical Tank: Since $\frac{r}{t} = \frac{750}{18} = 41.7 > 10$, then thin wall analysis can be used. Applying Eq. 8–1

$$\sigma_1 = \frac{pr}{t} = \frac{1.20(10^6)(750)}{18} = 50.0 \text{ MPa} \qquad \textbf{Ans.}$$

Longitudinal Stress for Cylindrical Tank:

$$\sigma_2 = \frac{pr}{2t} = \frac{1.20(10^6)(750)}{2(18)} = 25.0 \text{ MPa} \qquad \textbf{Ans.}$$

Force Equilibrium for the Cap:

$$+\uparrow \Sigma F_y = 0; \qquad 1.20\left(10^6\right)\left[\pi\left(0.75^2\right)\right] - 16F_b = 0$$

$$F_b = 132536 \text{ N} = 133 \text{ kN} \qquad \textbf{Ans.}$$