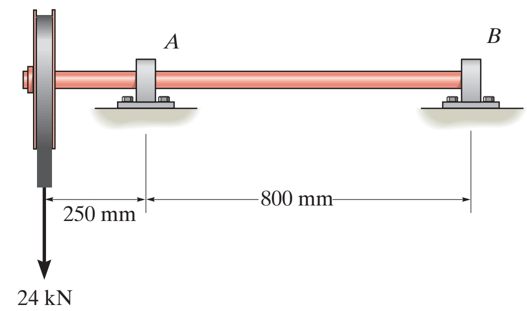
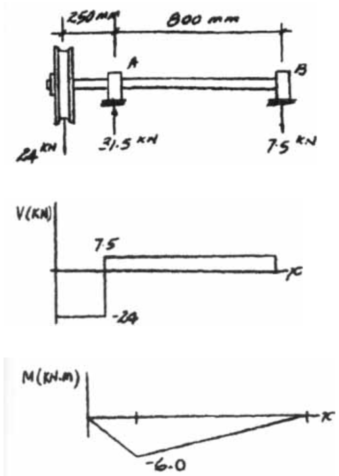
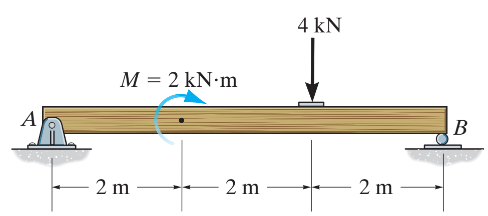
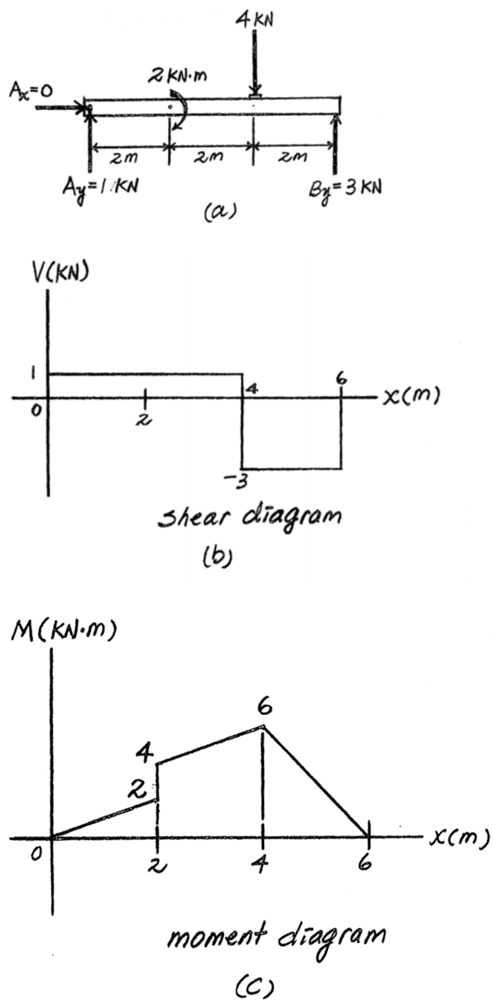
6–1. Draw the shear and moment diagrams for the shaft. The bearings at *A* and *B* exert only vertical reactions on the shaft.



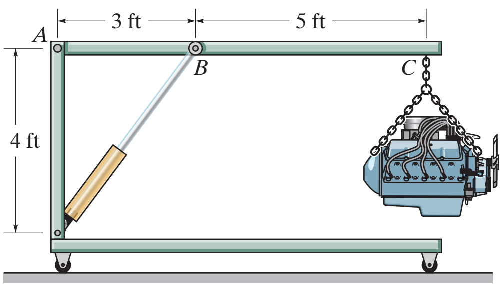


6–2. Draw the shear and moment diagrams for the simply supported beam.





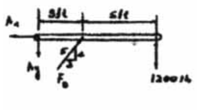
6–3. The engine crane is used to support the engine, which has a weight of 1200 lb. Draw the shear and moment diagrams of the boom ABC when it is in the horizontal position shown.

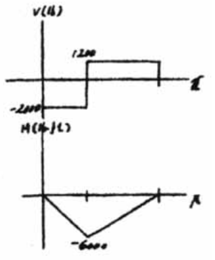


$$\circlearrowleft + \Sigma M_A = 0; \quad \frac{4}{5} F_A(3) - 1200(8) = 0; \qquad F_A = 4000 \text{ lb}$$

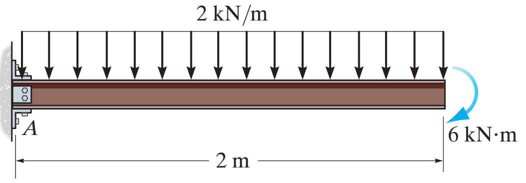
$$+\uparrow \Sigma F_y = 0; \quad -A_y + \frac{4}{5}(4000) - 1200 = 0; \qquad A_y = 2000 \text{ lb}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad A_x - \frac{3}{5}(4000) = 0; \qquad A_x = 2400 \text{ lb}$$





***6–4.** Draw the shear and moment diagrams for the cantilever beam.

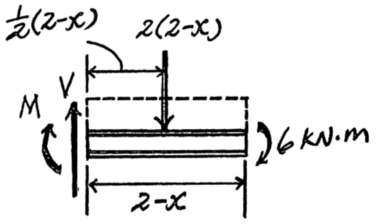


The free-body diagram of the beam's right segment sectioned through an arbitrary point shown in Fig. *a* will be used to write the shear and moment equations of the beam.

$$+\uparrow \Sigma F_y = 0; \qquad V - 2(2 - x) = 0 \qquad V = \{4 - 2x\} \text{ kN}, \quad (1)$$

$$\circlearrowleft + \Sigma M = 0; \; -M - 2(2 - x)\left[\frac{1}{2}(2 - x)\right] - 6 = 0 \quad M = \{-x^2 + 4x - 10\} \text{kN} \cdot \text{m}, (2)$$

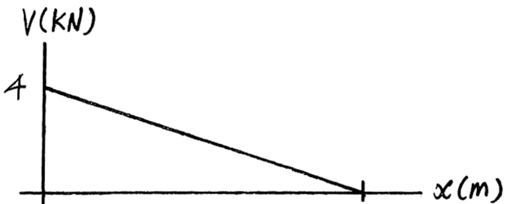
The shear and moment diagrams shown in Figs. *b* and *c* are plotted using Eqs. (1) and (2), respectively. The value of the shear and moment at $x = 0$ is evaluated using Eqs. (1) and (2).



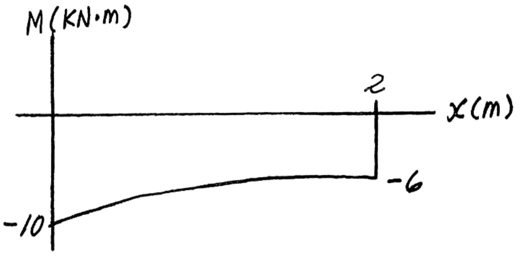
(a)

$$V\big|_{x=0} = 4 - 2(0) = 4 \text{ kN}$$

$$M\big|_{x=0} = \left[-0 + 4(0) - 10\right] = -10 \text{kN} \cdot \text{m}$$

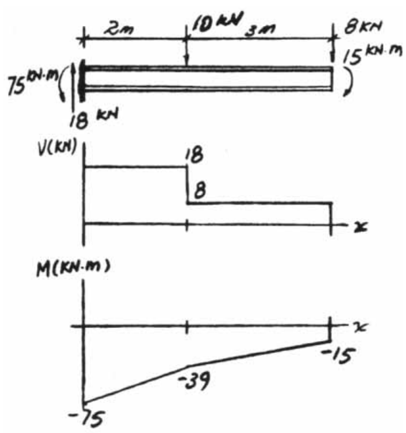


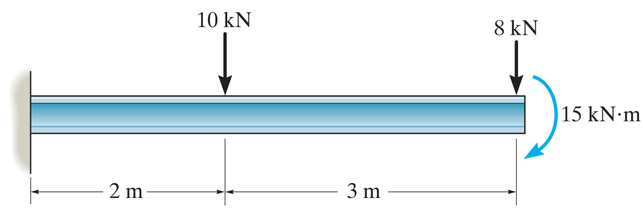
(b)



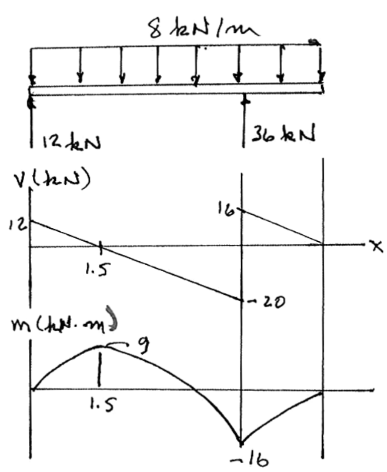
(c)

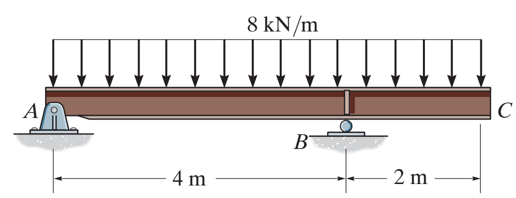
6–5. Draw the shear and moment diagrams for the beam.



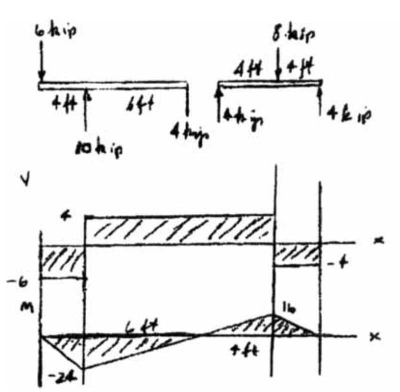


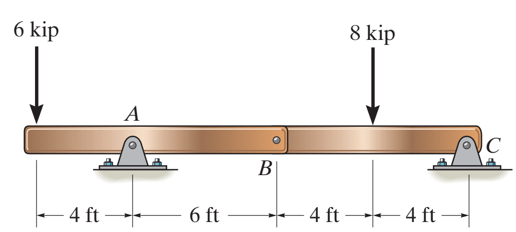
6–6. Draw the shear and moment diagrams for the overhang beam.



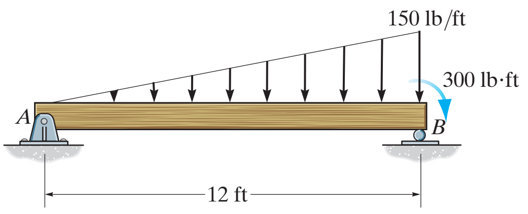


6–7. Draw the shear and moment diagrams for the compound beam which is pin connected at *B*.





***6–8.** Draw the shear and moment diagrams for the simply supported beam.



The free-body diagram of the beam's left segment sectioned through an arbitrary point shown in Fig. *b* will be used to write the shear and moment equations. The intensity of the triangular distributed load at the point of sectioning is

$$w = 150\left(\frac{x}{12}\right) = 12.5x$$

Referring to Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \qquad 275 - \frac{1}{2}(12.5x)(x) - V = 0 \qquad V = \{275 - 6.25x^2\}\text{lb}, \ (1)$$

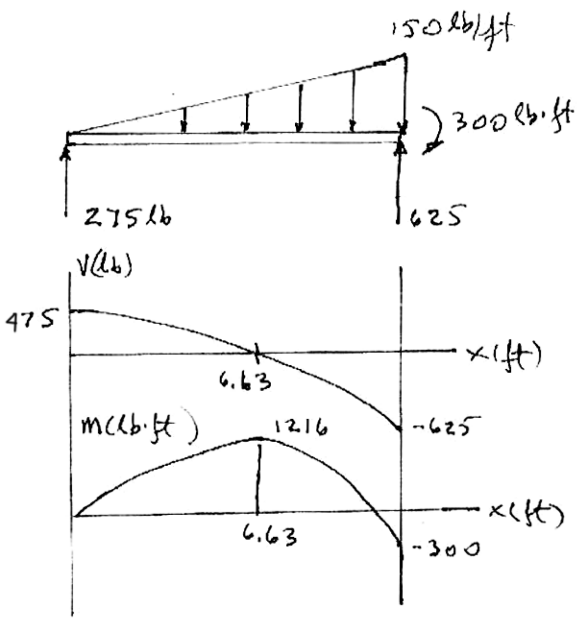
$$\circlearrowleft + \Sigma M = 0; \quad M + \frac{1}{2}(12.5x)(x)\left(\frac{x}{3}\right) - 275x = 0 \quad M = \{275x - 2.083x^3\}\text{lb}\cdot\text{ft}, (2)$$

The shear and moment diagrams shown in Figs. *c* and *d* are plotted using Eqs. (1) and (2), respectively. The location where the shear is equal to zero can be obtained by setting $V = 0$ in Eq. (1).

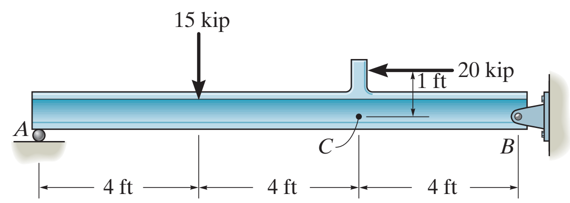
$$0 = 275 - 6.25x^2 \qquad\qquad x = 6.633 \text{ ft}$$

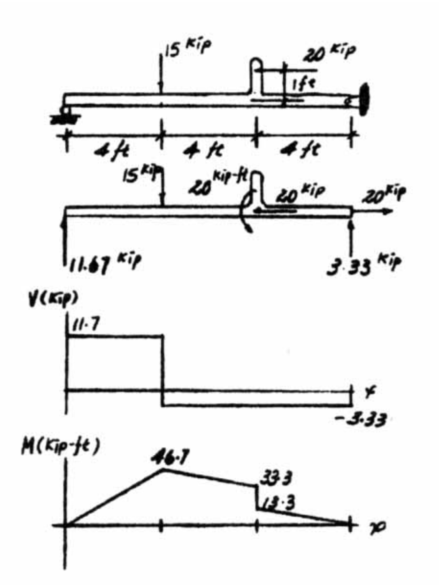
The value of the moment at $x = 6.633$ ft ($V = 0$) is evaluated using Eq. (2).

$$M\big|_{x=6.633\text{ ft}} = 275(6.633) - 2.083(6.633)^3 = 1216 \text{ lb}\cdot\text{ft}$$

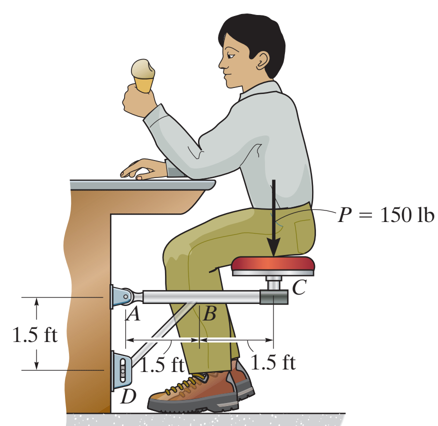


6–9. Draw the shear and moment diagrams for the beam. *Hint:* The 20-kip load must be replaced by equivalent loadings at point C on the axis of the beam.





6–10. Members ABC and BD of the counter chair are rigidly connected at B and the smooth collar at D is allowed to move freely along the vertical slot. Draw the shear and moment diagrams for member ABC.



Equations of Equilibrium: Referring to the free-body diagram of the frame shown in Fig. *a*,

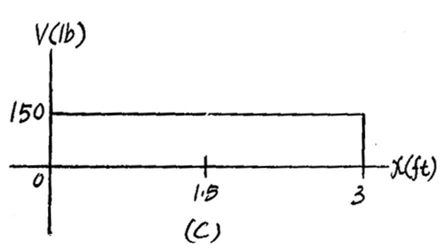
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 150 = 0$$

$$A_y = 150 \text{ lb}$$

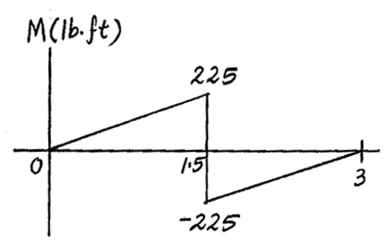
$$\circlearrowleft + \Sigma M_A = 0; \qquad N_D(1.5) - 150(3) = 0$$

$$N_D = 300 \text{ lb}$$

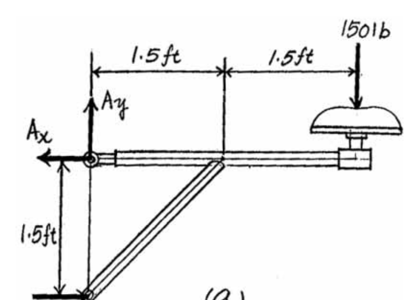
Shear and Moment Diagram: The couple moment acting on B due to N_D is $M_B = 300(1.5) = 450 \text{ lb} \cdot \text{ft}$. The loading acting on member ABC is shown in Fig. *b* and the shear and moment diagrams are shown in Figs. *c* and *d*.



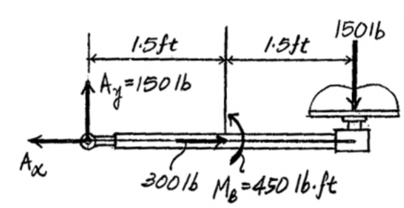
(c)



(d)

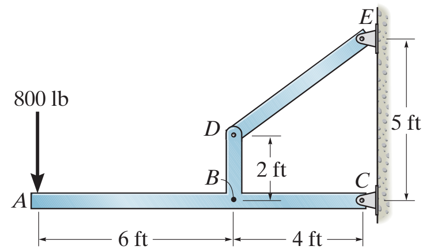


(a)



(b)

6–11. The overhanging beam has been fabricated with a projected arm BD on it. Draw the shear and moment diagrams for the beam ABC if it supports a load of 800 lb. *Hint:* The loading in the supporting strut DE must be replaced by equivalent loads at point B on the axis of the beam.



Support Reactions:

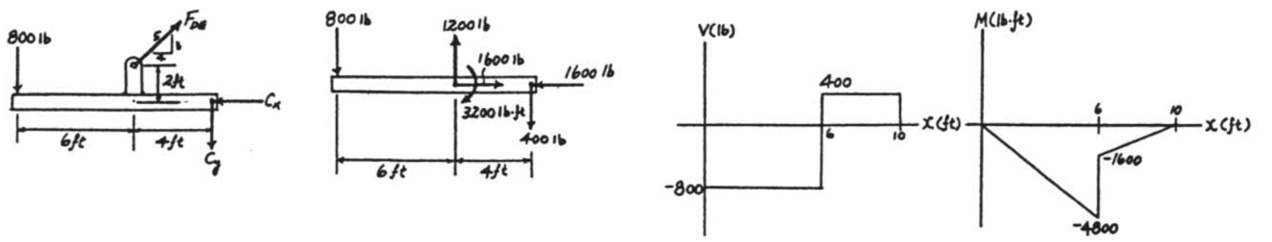
$$\circlearrowleft +\Sigma M_C = 0; \qquad 800(10) - \frac{3}{5}F_{DE}(4) - \frac{4}{5}F_{DE}(2) = 0$$

$$F_{DE} = 2000 \text{ lb}$$

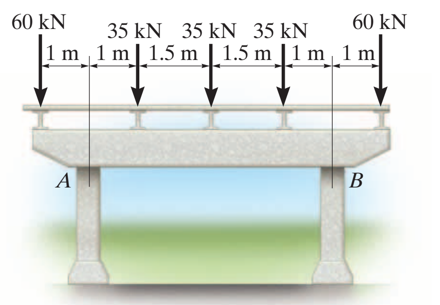
$$+\uparrow \Sigma F_y = 0; \qquad -800 + \frac{3}{5}(2000) - C_y = 0 \qquad C_y = 400 \text{ lb}$$

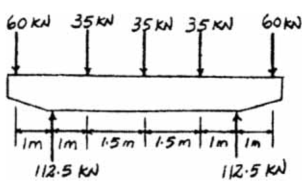
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -C_x + \frac{4}{5}(2000) = 0 \qquad C_x = 1600 \text{ lb}$$

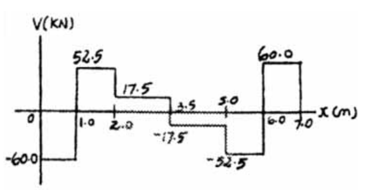
Shear and Moment Diagram:

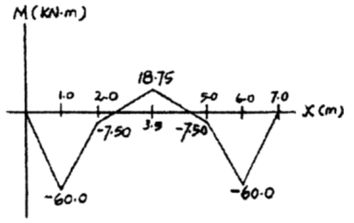


***6–12.** A reinforced concrete pier is used to support the stringers for a bridge deck. Draw the shear and moment diagrams for the pier when it is subjected to the stringer loads shown. Assume the columns at A and B exert only vertical reactions on the pier.

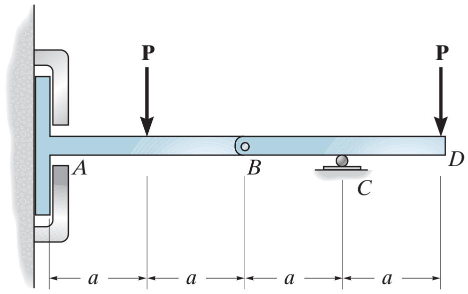








6–13. Draw the shear and moment diagrams for the compound beam. It is supported by a smooth plate at A which slides within the groove and so it cannot support a vertical force, although it can support a moment and axial load.



Support Reactions:

From the FBD of segment BD

$$\zeta + \Sigma M_C = 0; \qquad B_y(a) - P(a) = 0 \qquad B_y = P$$

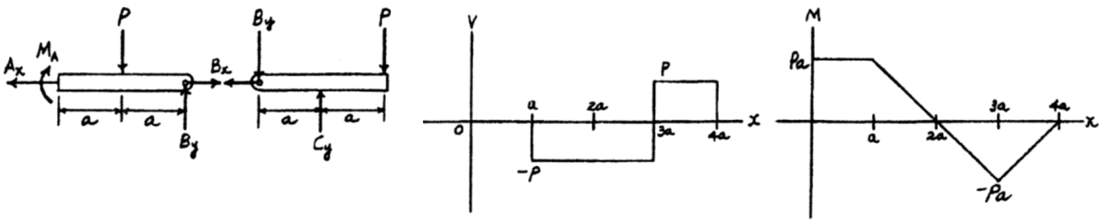
$$+\uparrow \Sigma F_y = 0; \qquad C_y - P - P = 0 \qquad C_y = 2P$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x = 0$$

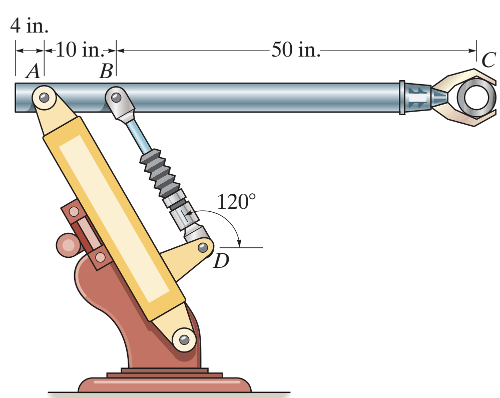
From the FBD of segment AB

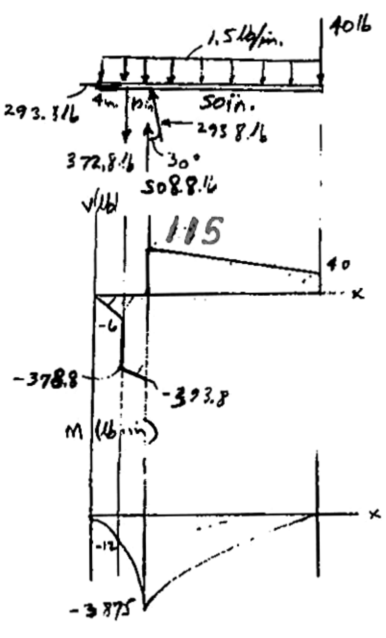
$$\zeta + \Sigma M_A = 0; \qquad P(2a) - P(a) - M_A = 0 \qquad M_A = Pa$$

$$+\uparrow \Sigma F_y = 0; \qquad P - P = 0 \text{ (equilibrium is statisfied!)}$$

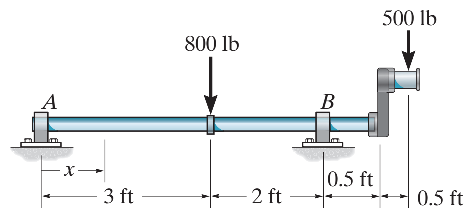


6–14. The industrial robot is held in the stationary position shown. Draw the shear and moment diagrams of the arm ABC if it is pin connected at A and connected to a hydraulic cylinder (two-force member) BD. Assume the arm and grip have a uniform weight of 1.5 lb/in. and support the load of 40 lb at C.





***6–16.** Draw the shear and moment diagrams for the shaft and determine the shear and moment throughout the shaft as a function of x. The bearings at A and B exert only vertical reactions on the shaft.



For $0 < x < 3$ ft

$+\uparrow\Sigma F_y = 0.\qquad 220 - V = 0\qquad V = 220\text{ lb},$ **Ans.**

$\circlearrowleft +\Sigma M_{NA} = 0.\qquad M - 220x = 0$

$M = (220x)\text{ lb ft},$ **Ans.**

For $3\text{ ft} < x < 5$ ft

$+\uparrow\Sigma F_y = 0;\qquad 220 - 800 - V = 0$

$V = -580\text{ lb}$ **Ans.**

$\circlearrowleft +\Sigma M_{NA} = 0;\qquad M + 800(x - 3) - 220x = 0$

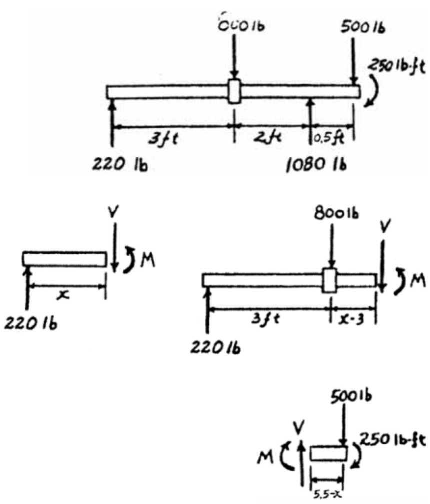
$M = \{-580x + 2400\}\text{ lb ft},$ **Ans.**

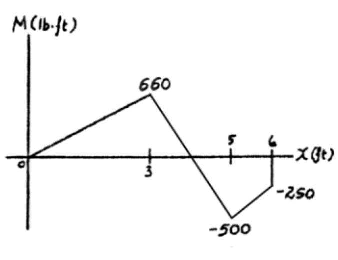
For $5\text{ ft} < x \leq 6$ ft

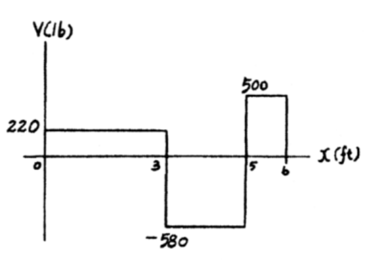
$+\uparrow\Sigma F_y = 0;\qquad V - 500 = 0\qquad V = 500\text{ lb},$ **Ans.**

$\circlearrowleft +\Sigma M_{NA} = 0;\qquad -M - 500(5.5 - x) - 250 = 0$

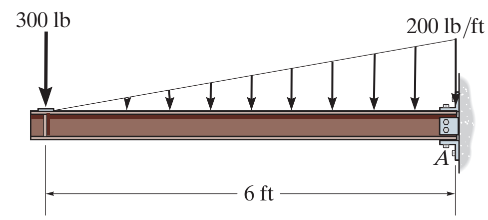
$M = (500x - 3000)\text{ lb ft}$ **Ans.**



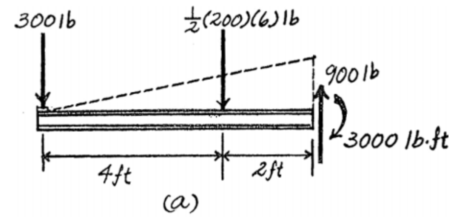




•6–17. Draw the shear and moment diagrams for the cantilevered beam.



The free-body diagram of the beam's left segment sectioned through an arbitrary point shown in Fig. *b* will be used to write the shear and moment equations. The intensity of the triangular distributed load at the point of sectioning is

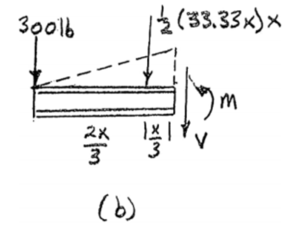


$$w = 200\left(\frac{x}{6}\right) = 33.33x$$

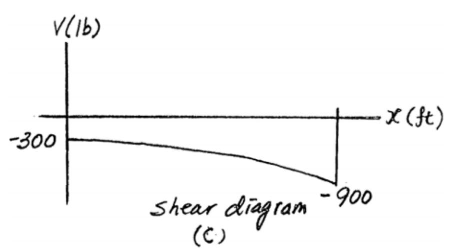
Referring to Fig. *b*,

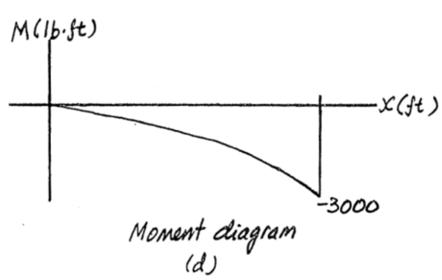
$$+\uparrow \Sigma F_y = 0; \qquad -300 - \frac{1}{2}(33.33x)(x) - V = 0 \qquad V = \{-300 - 16.67x^2\}\text{ lb} \quad (1)$$

$$\circlearrowleft + \Sigma M = 0; \; M + \frac{1}{2}(33.33x)(x)\left(\frac{x}{3}\right) + 300x = 0 \quad M = \{-300x - 5.556x^3\}\text{ lb}\cdot\text{ft} \; (2)$$

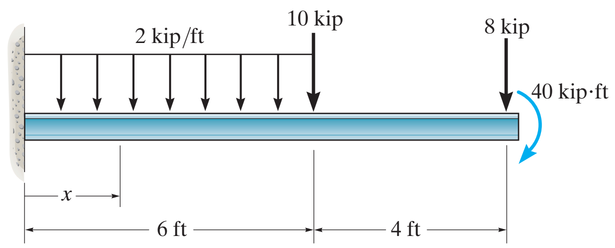


The shear and moment diagrams shown in Figs. *c* and *d* are plotted using Eqs. (1) and (2), respectively.





6–18. Draw the shear and moment diagrams for the beam, and determine the shear and moment throughout the beam as functions of x.



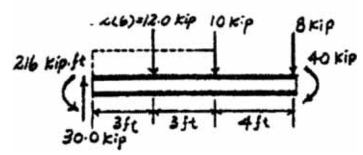
Support Reactions: As shown on FBD.

Shear and Moment Function:

For $0 \leq x < 6$ ft:

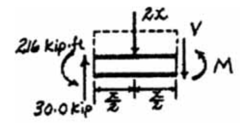
$$+\uparrow \Sigma F_y = 0; \qquad 30.0 - 2x - V = 0$$

$$V = \{30.0 - 2x\} \text{ kip} \qquad \textbf{Ans.}$$



$$\circlearrowleft + \Sigma M_{NA} = 0; \quad M + 216 + 2x\left(\frac{x}{2}\right) - 30.0x = 0$$

$$M = \{-x^2 + 30.0x - 216\} \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

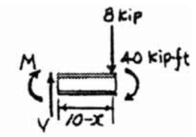


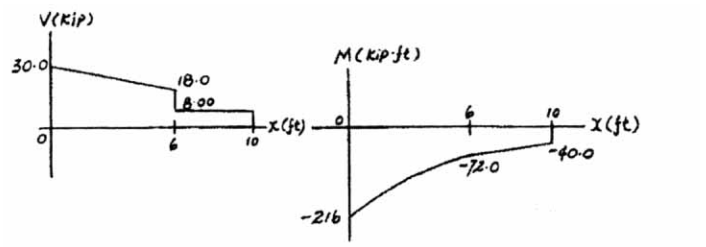
For 6 ft $< x \leq 10$ ft:

$$+\uparrow \Sigma F_y = 0; \qquad V - 8 = 0 \qquad V = 8.00 \text{ kip} \qquad \textbf{Ans.}$$

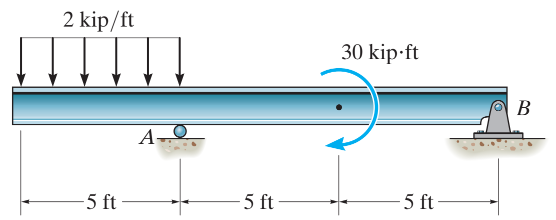
$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad -M - 8(10 - x) - 40 = 0$$

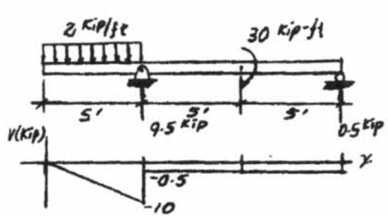
$$M = \{8.00x - 120\} \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

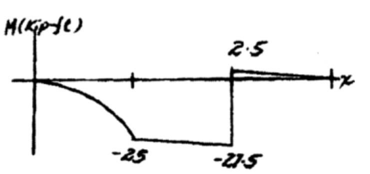




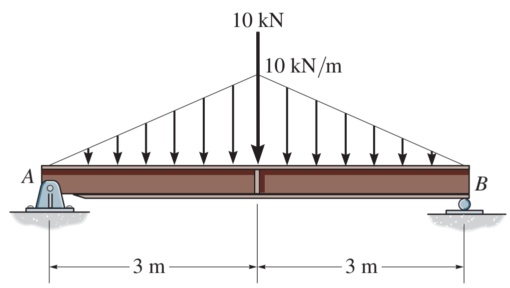
6–19. Draw the shear and moment diagrams for the beam.





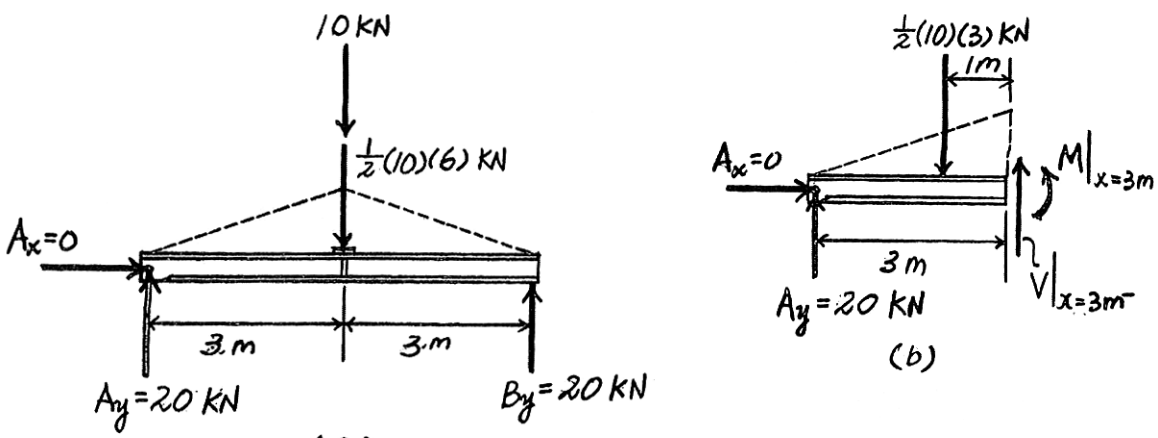


***6–20.** Draw the shear and moment diagrams for the simply supported beam.

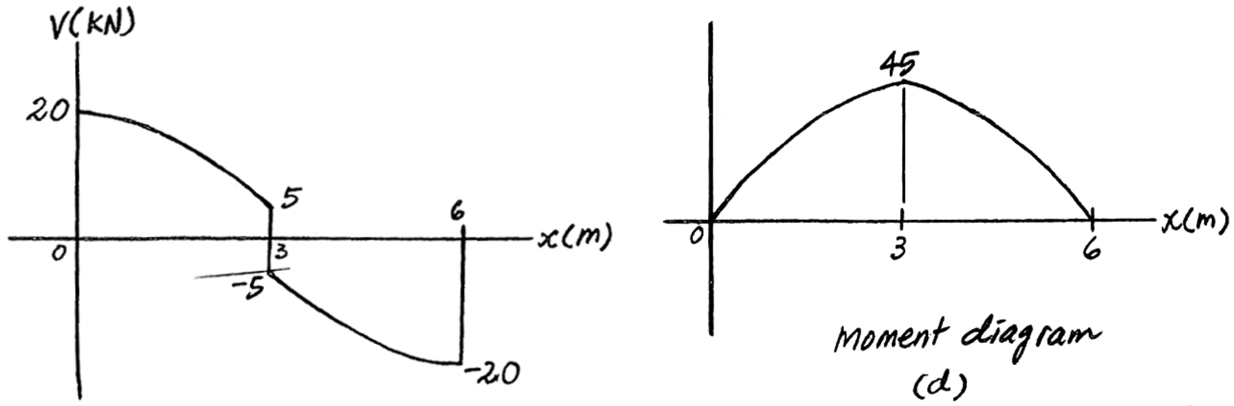


Since the area under the curved shear diagram can not be computed directly, the value of the moment at $x = 3$ m will be computed using the method of sections. By referring to the free-body diagram shown in Fig. *b*,

$\circlearrowleft + \Sigma M = 0;\quad M\big|_{x=3\text{ m}} + \frac{1}{2}(10)(3)(1) - 20(3) = 0 \qquad M\big|_{x=3\text{m}} = 45\text{ kN}\cdot\text{m}$ **Ans.**

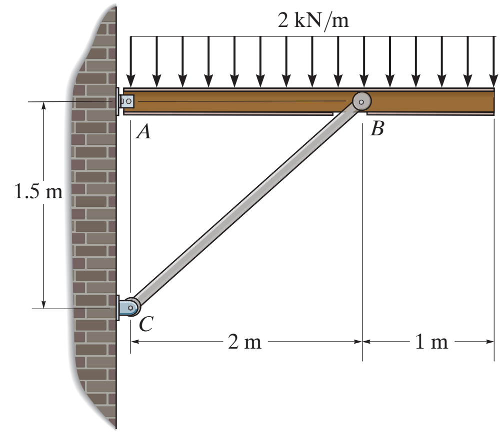


(a) (b)



shear diagram (c) Moment diagram (d)

•6–21. The beam is subjected to the uniform distributed load shown. Draw the shear and moment diagrams for the beam.



Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

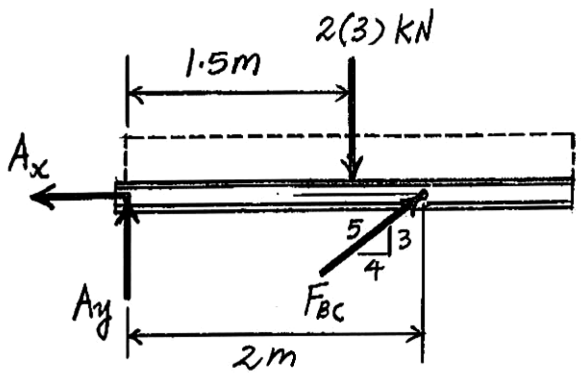
$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{BC}\left(\frac{3}{5}\right)(2) - 2(3)(1.5) = 0$$

$$F_{BC} = 7.5 \text{ kN}$$

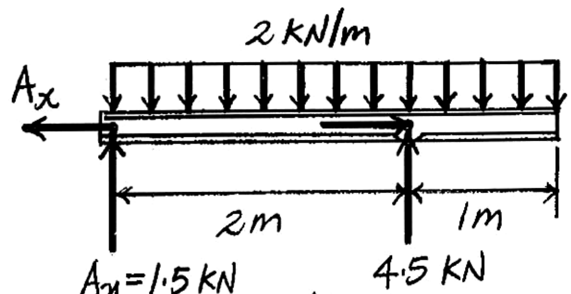
$$+\uparrow \Sigma F_y = 0; \qquad A_y + 7.5\left(\frac{3}{5}\right) - 2(3) = 0$$

$$A_y = 1.5 \text{ kN}$$

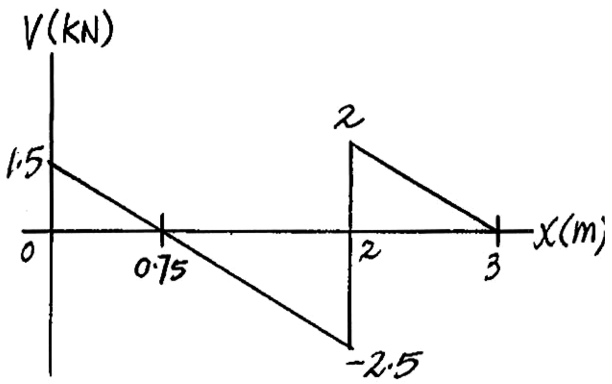
Shear and Moment Diagram: The vertical component of $\mathbf{F}_{BC}$ is $\left(F_{BC}\right)_y = 7.5\left(\frac{3}{5}\right) = 4.5$ kN. The shear and moment diagrams are shown in Figs. *c* and *d*.



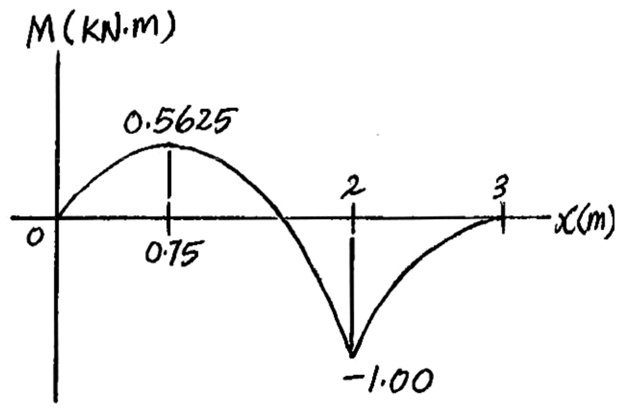
(a)



(b)

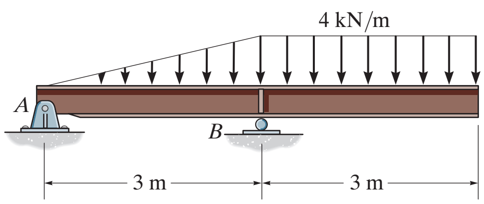


(c)

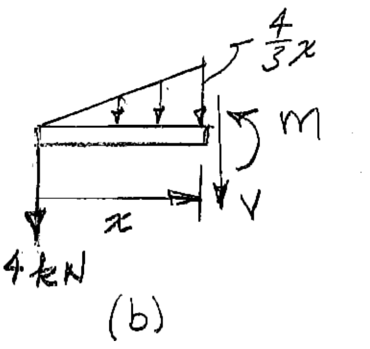


(d)

6–22. Draw the shear and moment diagrams for the overhang beam.



Since the loading is discontinuous at support B, the shear and moment equations must be written for regions $0 \le x < 3$ m and $3 \text{ m} < x \le 6$ m of the beam. The free-body diagram of the beam's segment sectioned through an arbitrary point within these two regions is shown in Figs. *b* and *c*.



Region $0 \le x < 3$ m, Fig. *b*

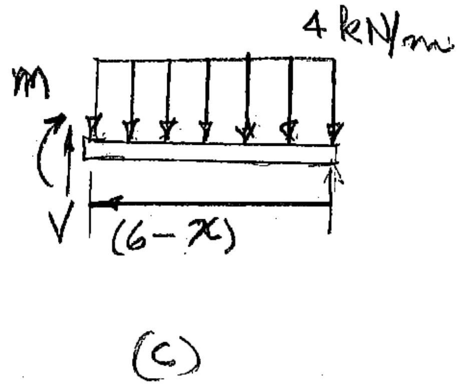
$$+\uparrow \Sigma F_y = 0; \quad -4 - \frac{1}{2}\left(\frac{4}{3}x\right)(x) - V = 0 \qquad V = \left\{-\frac{2}{3}x^2 - 4\right\} \text{kN} \quad (1)$$

$$\circlearrowleft + \Sigma M = 0; \quad M + \frac{1}{2}\left(\frac{4}{3}x\right)(x)\left(\frac{x}{3}\right) + 4x = 0 \qquad M = \left\{-\frac{2}{9}x^3 - 4x\right\} \text{kN}\cdot\text{m} \quad (2)$$

Region $3 \text{ m} < x \le 6$ m, Fig. *c*

$$+\uparrow \Sigma F_y = 0; \quad V - 4(6 - x) = 0 \qquad V = \{24 - 4x\} \text{kN} \quad (3)$$

$$\circlearrowleft + \Sigma M = 0; \quad -M - 4(6 - x)\left[\frac{1}{2}(6 - x)\right] = 0 \qquad M = \{-2(6 - x)^2\}\text{kN}\cdot\text{m} \quad (4)$$



The shear diagram shown in Fig. *d* is plotted using Eqs. (1) and (3). The value of shear just to the left and just to the right of the support is evaluated using Eqs. (1) and (3), respectively.

$$V\big|_{x=3\text{ m}-} = -\frac{2}{3}(3^2) - 4 = -10 \text{ kN}$$

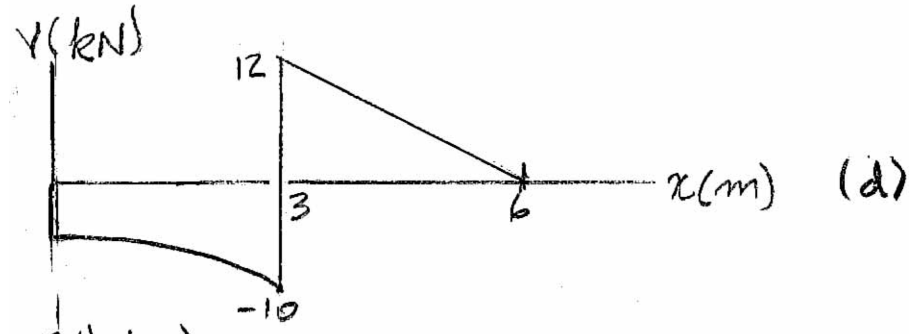
$$V\big|_{x=3\text{ m}+} = 24 - 4(3) = 12 \text{ kN}$$

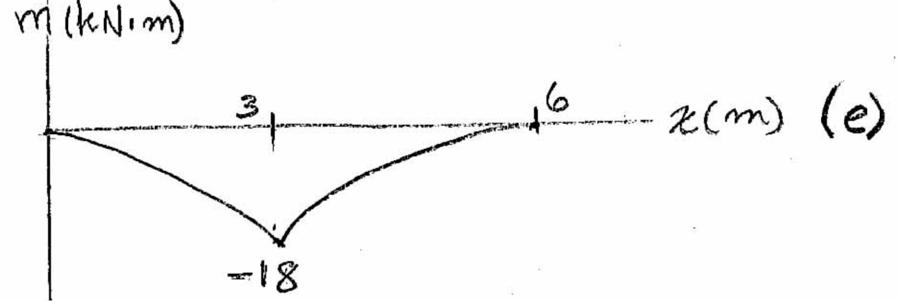
The moment diagram shown in Fig. *e* is plotted using Eqs. (2) and (4). The value of the moment at support B is evaluated using either Eq. (2) or Eq. (4).

$$M\big|_{x=3\text{ m}} = -\frac{2}{9}(3^3) - 4(3) = -18 \text{ kN}\cdot\text{m}$$

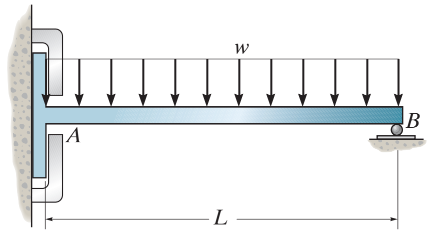
or

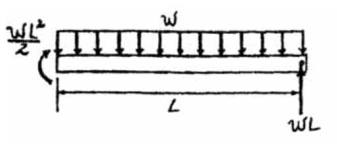
$$M\big|_{x=3\text{ m}} = -2(6 - 3)^2 = -18 \text{ kN}\cdot\text{m}$$

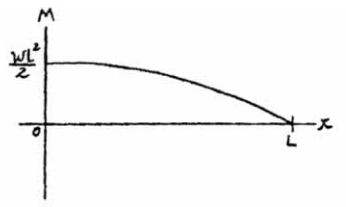


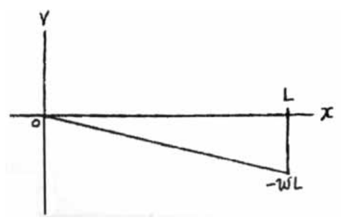


6–23. Draw the shear and moment diagrams for the beam. It is supported by a smooth plate at A which slides within the groove and so it cannot support a vertical force, although it can support a moment and axial load.

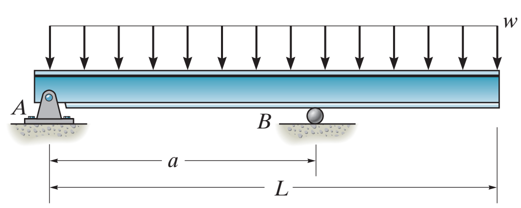








***6–24.** Determine the placement distance a of the roller support so that the largest absolute value of the moment is a minimum. Draw the shear and moment diagrams for this condition.



$$+\uparrow \Sigma F_y = 0; \qquad wL - \frac{wL^2}{2a} - wx = 0$$

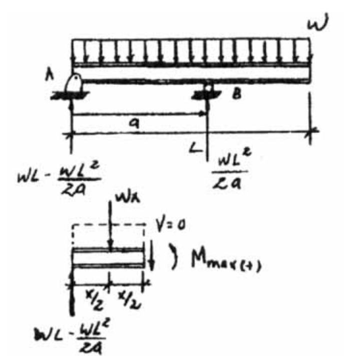
$$x = L - \frac{L^2}{2a}$$

$$\zeta + \Sigma M = 0; \qquad M_{\max(+)} + wx\left(\frac{x}{2}\right) - \left(wL - \frac{wL^2}{2a}\right)x = 0$$

Substitute $x = L - \dfrac{L^2}{2a}$;

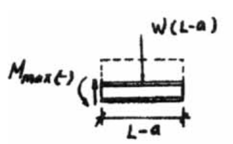
$$M_{\max(+)} = \left(wL - \frac{wL^2}{2a}\right)\left(L - \frac{L^2}{2a}\right) - \frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2$$

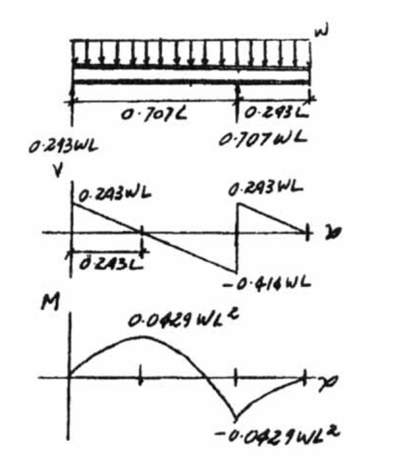
$$= \frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2$$



$$\Sigma M = 0; \qquad M_{\max(-)} - w(L - a)\frac{(L - a)}{2} = 0$$

$$M_{\max(-)} = \frac{w(L - a)^2}{2}$$





To get absolute minimum moment,

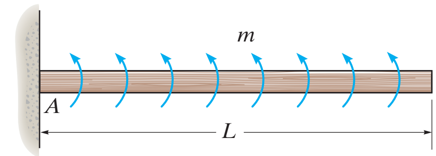
$$M_{\max(+)} = M_{\max(-)}$$

$$\frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2 = \frac{w}{2}(L - a)^2$$

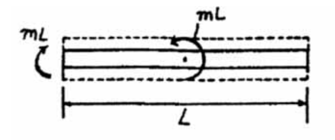
$$L - \frac{L^2}{2a} = L - a$$

$$a = \frac{L}{\sqrt{2}}, \qquad \textbf{Ans.}$$

6–25. The beam is subjected to the uniformly distributed moment m (moment/length). Draw the shear and moment diagrams for the beam.



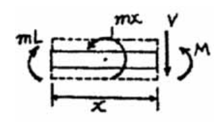
Support Reactions: As shown on FBD.



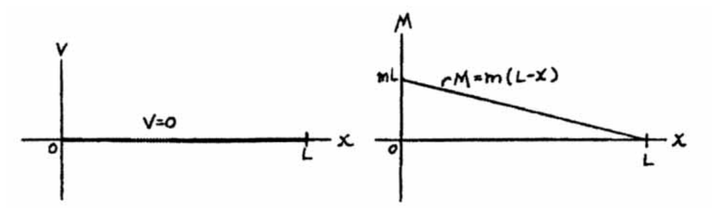
Shear and Moment Function:

$$+\uparrow \Sigma F_y = 0; \qquad V = 0$$

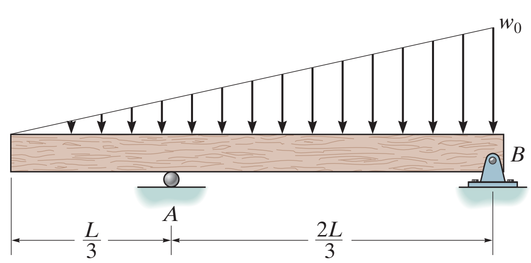
$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad M + mx - mL = 0 \qquad M = m(L - x)$$



Shear and Moment Diagram:



6–27. Draw the shear and moment diagrams for the beam.



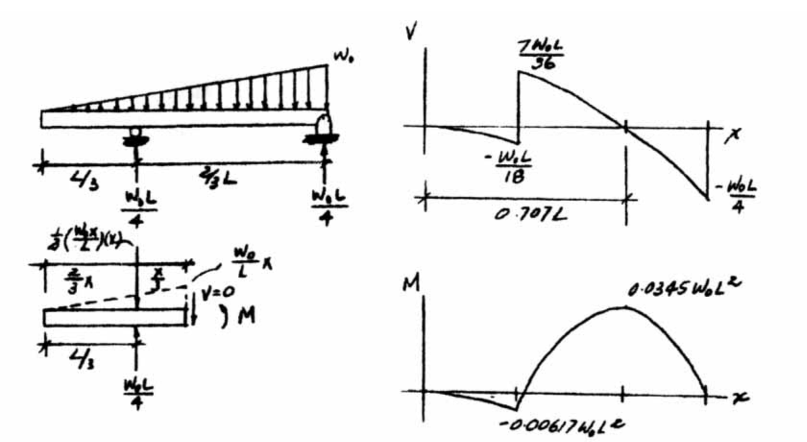
$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{4} - \frac{1}{2}\left(\frac{w_0 x}{L}\right)(x) = 0$$

$$x = 0.7071\, L$$

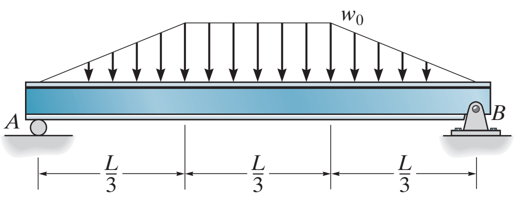
$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad M + \frac{1}{2}\left(\frac{w_0 x}{L}\right)(x)\left(\frac{x}{3}\right) - \frac{w_0 L}{4}\left(x - \frac{L}{3}\right) = 0$$

Substitute $x = 0.7071L$,

$$M = 0.0345\, w_0 L^2$$



***6–28.** Draw the shear and moment diagrams for the beam.



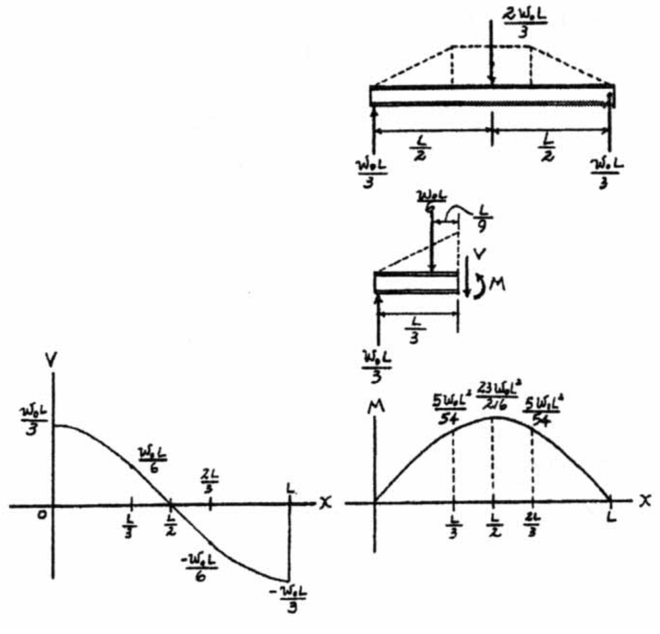
Support Reactions: As shown on FBD.

Shear and Moment Diagram: Shear and moment at $x = L/3$ can be determined using the method of sections.

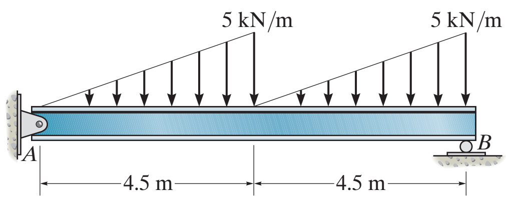
$$+\uparrow \Sigma F_y = 0; \quad \frac{w_0 L}{3} - \frac{w_0 L}{6} - V = 0 \qquad V = \frac{w_0 L}{6}$$

$$\circlearrowleft + \Sigma M_{NA} = 0; \quad M + \frac{w_0 L}{6}\left(\frac{L}{9}\right) - \frac{w_0 L}{3}\left(\frac{L}{3}\right) = 0$$

$$M = \frac{5 w_0 L^2}{54}$$



•6–29. Draw the shear and moment diagrams for the beam.



From FBD(a)

$$+\uparrow \Sigma F_y = 0; \qquad 9.375 - 0.5556x^2 = 0 \qquad x = 4.108 \text{ m}$$

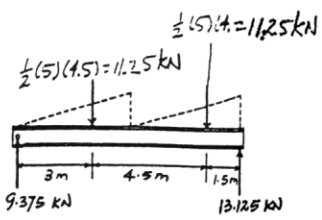
$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad M + (0.5556)\left(4.108^2\right)\left(\frac{4.108}{3}\right) - 9.375(4.108) = 0$$

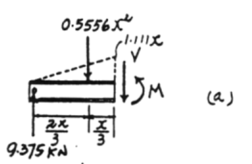
$$M = 25.67 \text{ kN} \cdot \text{m}$$

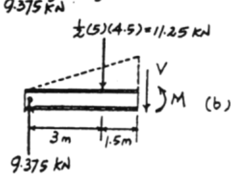
From FBD(b)

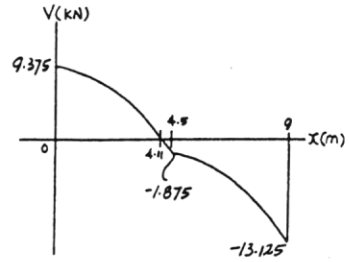
$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad M + 11.25(1.5) - 9.375(4.5) = 0$$

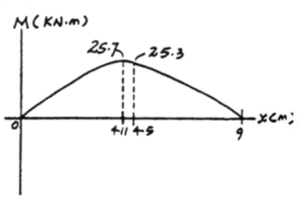
$$M = 25.31 \text{ kN} \cdot \text{m}$$



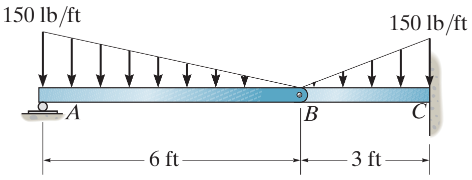








6–30. Draw the shear and moment diagrams for the compound beam.



Support Reactions:

From the FBD of segment AB

$$\circlearrowleft + \Sigma M_B = 0; \qquad 450(4) - A_y(6) = 0 \qquad A_y = 300.0 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y - 450 + 300.0 = 0 \qquad B_y = 150.0 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$$

From the FBD of segment BC

$$\circlearrowleft + \Sigma M_C = 0; \qquad 225(1) + 150.0(3) - M_C = 0$$

$$M_C = 675.0 \text{ lb} \cdot \text{ft}$$

$$+\uparrow \Sigma F_y = 0; \qquad C_y - 150.0 - 225 = 0 \qquad C_y = 375.0 \text{ lb}$$

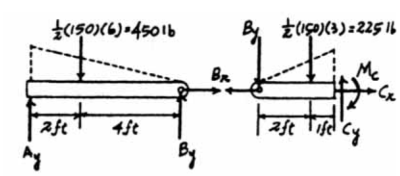
$$\xrightarrow{+} \Sigma F_x = 0; \qquad C_x = 0$$

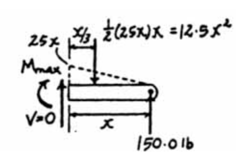
Shear and Moment Diagram: The maximum positive moment occurs when $V = 0$.

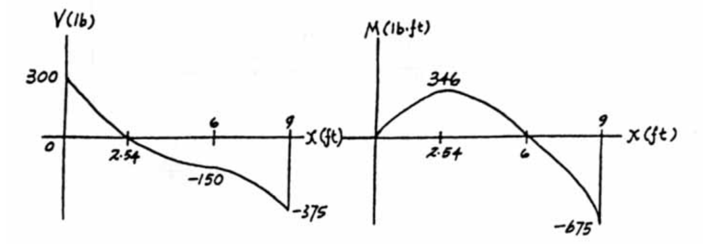
$$+\uparrow \Sigma F_y = 0; \qquad 150.0 - 12.5x^2 = 0 \qquad x = 3.464 \text{ ft}$$

$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad 150(3.464) - 12.5\left(3.464^2\right)\left(\frac{3.464}{3}\right) - M_{max} = 0$$

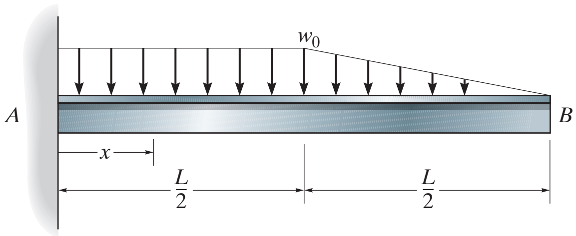
$$M_{max} = 346.4 \text{ lb} \cdot \text{ft}$$







6–31. Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x.



Support Reactions: As shown on FBD.

Shear and Moment Functions:

For $0 \le x < L/2$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{3w_0 L}{4} - w_0 x - V = 0$$

$$V = \frac{w_0}{4}(3L - 4x) \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M_{NA} = 0; \qquad \frac{7w_0 L^2}{24} - \frac{3w_0 L}{4}x + w_0 x\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{w_0}{24}\left(-12x^2 + 18Lx - 7L^2\right) \qquad \textbf{Ans.}$$

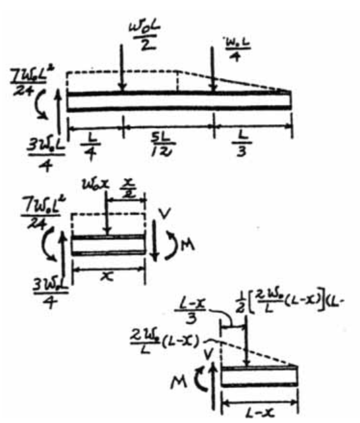
For $L/2 < x \le L$

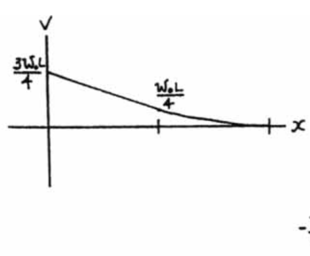
$$+\uparrow \Sigma F_y = 0; \qquad V - \frac{1}{2}\left[\frac{2w_0}{L}(L - x)\right](L - x) = 0$$

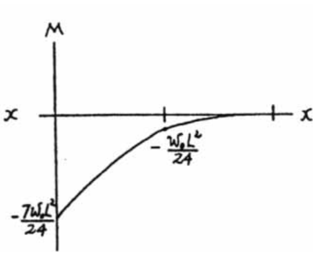
$$V = \frac{w_0}{L}(L - x)^2 \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M_{NA} = 0; \qquad -M - \frac{1}{2}\left[\frac{2w_0}{L}(L - x)\right](L - x)\left(\frac{L - x}{3}\right) = 0$$

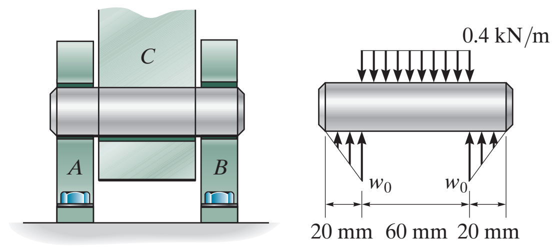
$$M = -\frac{w_0}{3L}(L - x)^3 \qquad \textbf{Ans.}$$





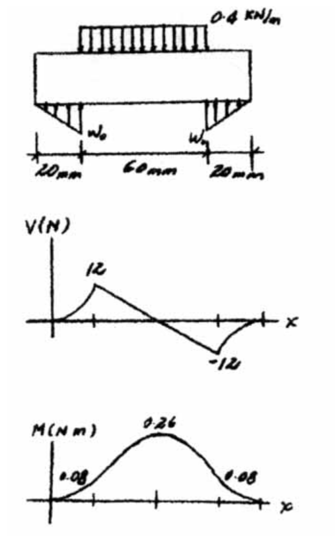


***6–32.** The smooth pin is supported by two leaves A and B and subjected to a compressive load of 0.4 kN/m caused by bar C. Determine the intensity of the distributed load w_0 of the leaves on the pin and draw the shear and moment diagram for the pin.

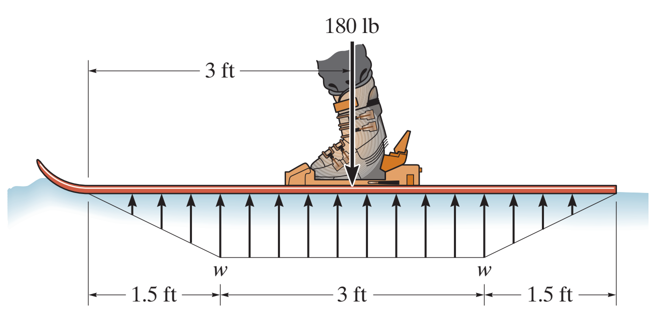


$+\uparrow \Sigma F_y = 0; \qquad 2(w_0)(20)\left(\frac{1}{2}\right) - 60(0.4) = 0$

$w_0 = 1.2 \text{ kN/m}$ **Ans.**



•6–33. The ski supports the 180-lb weight of the man. If the snow loading on its bottom surface is trapezoidal as shown, determine the intensity w, and then draw the shear and moment diagrams for the ski.



Ski:

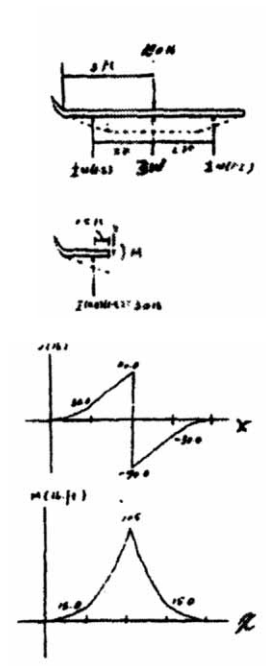
$+\uparrow \Sigma F_y = 0; \qquad \frac{1}{2}w(1.5) + 3w + \frac{1}{2}w(1.5) - 180 = 0$

$w = 40.0 \text{ lb/ft}$ **Ans.**

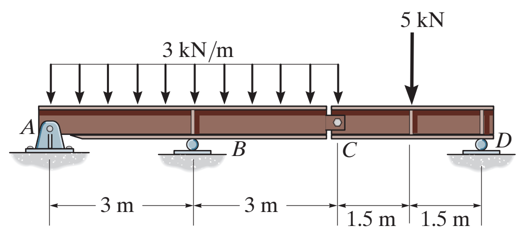
Segment:

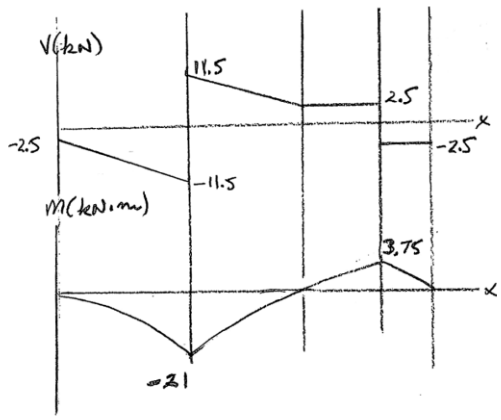
$+\uparrow \Sigma F_y = 0; \qquad 30 - V = 0; \qquad V = 30.0 \text{ lb}$

$\circlearrowleft + \Sigma M = 0; \qquad M - 30(0.5) = 0; \qquad M = 15.0 \text{ lb} \cdot \text{ft}$

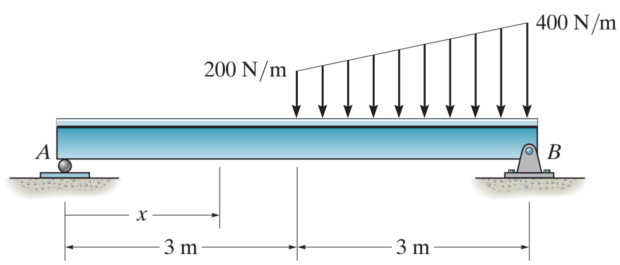


6–34. Draw the shear and moment diagrams for the compound beam.





6–35. Draw the shear and moment diagrams for the beam and determine the shear and moment as functions of x.

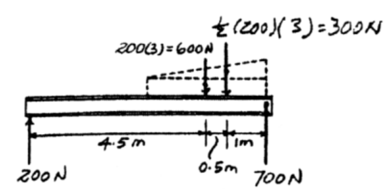


Support Reactions: As shown on FBD.

Shear and Moment Functions:

For $0 \leq x < 3$ m:

$$+\uparrow \Sigma F_y = 0; \qquad 200 - V = 0 \qquad V = 200\text{ N} \qquad \textbf{Ans.}$$

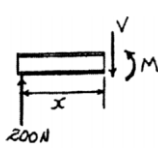


$$\zeta + \Sigma M_{NA} = 0; \qquad M - 200\,x = 0$$

$$M = (200\,x)\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

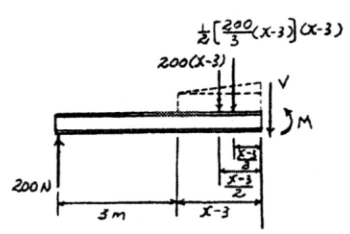
For $3\text{ m} < x \leq 6$ m:

$$+\uparrow \Sigma F_y = 0; \qquad 200 - 200(x-3) - \frac{1}{2}\left[\frac{200}{3}(x-3)\right](x-3) - V = 0$$



$$V = \left\{-\frac{100}{3}x^2 + 500\right\}\text{ N} \qquad \textbf{Ans.}$$

Set $V = 0$, $x = 3.873$ m

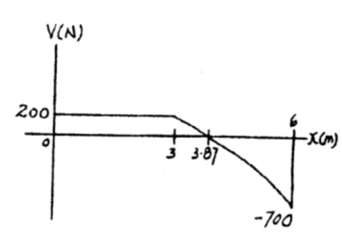


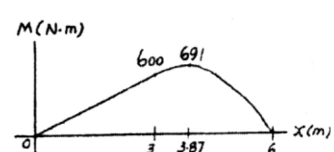
$$\zeta + \Sigma M_{NA} = 0; \qquad M + \frac{1}{2}\left[\frac{200}{3}(x-3)\right](x-3)\left(\frac{x-3}{3}\right)$$

$$+ 200(x-3)\left(\frac{x-3}{2}\right) - 200x = 0$$

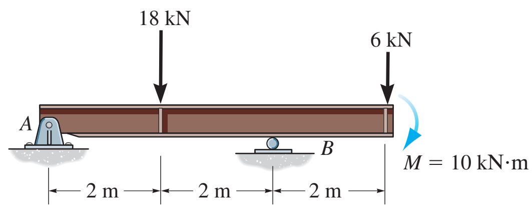
$$M = \left\{-\frac{100}{9}x^3 + 500x - 600\right\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

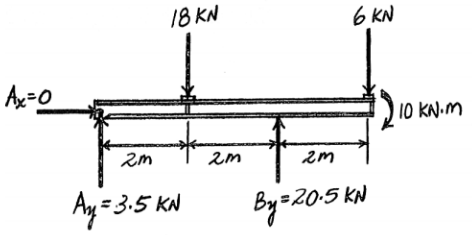
Substitute $x = 3.87$ m, $M = 691$ N·m



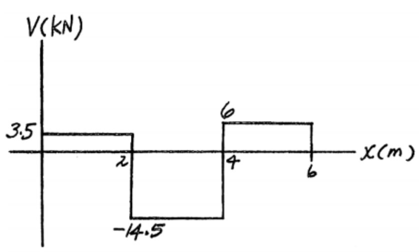


***6–36.** Draw the shear and moment diagrams for the overhang beam.

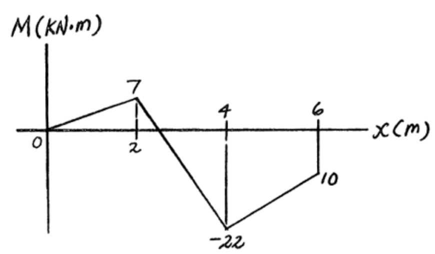




(a)

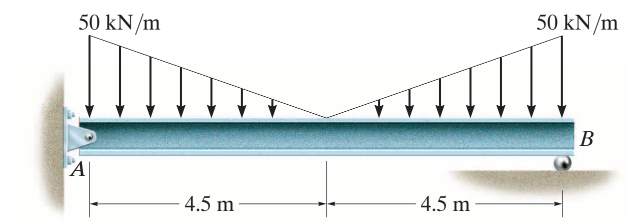


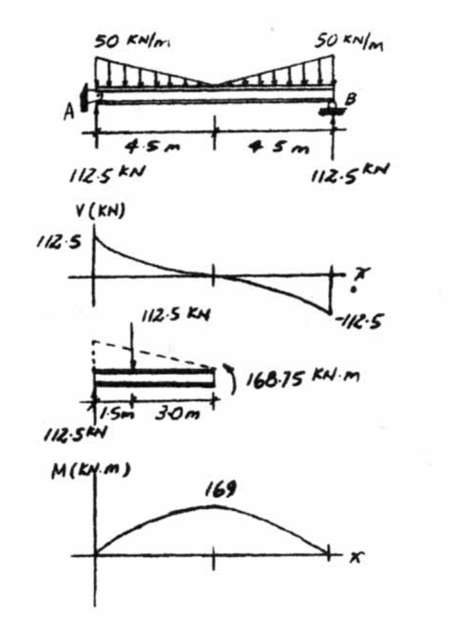
shear diagram
(b)



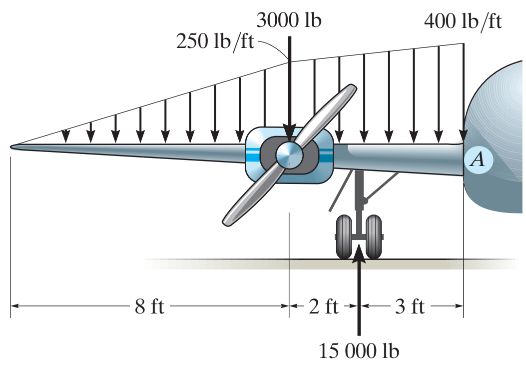
moment diagram
(c)

6–37. Draw the shear and moment diagrams for the beam.





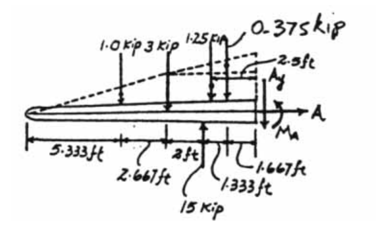
6–38. The dead-weight loading along the centerline of the airplane wing is shown. If the wing is fixed to the fuselage at A, determine the reactions at A, and then draw the shear and moment diagram for the wing.



Support Reactions:

$+\uparrow \Sigma F_y = 0; \qquad -1.00 - 3 + 15 - 1.25 - 0.375 - A_y = 0$

$A_y = 9.375 \text{ kip}$ **Ans.**



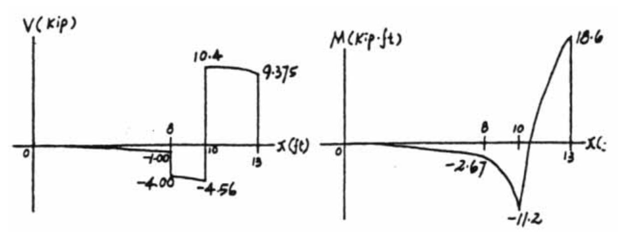
$\zeta + \Sigma M_A = 0; \qquad 1.00(7.667) + 3(5) - 15(3)$

$+ 1.25(2.5) + 0.375(1.667) + M_A = 0$

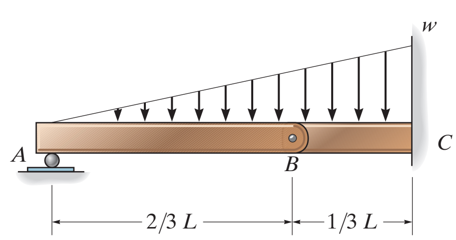
$M_A = 18.583 \text{ kip} \cdot \text{ft} = 18.6 \text{ kip} \cdot \text{ft}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

Shear and Moment Diagram:



6–39. The compound beam consists of two segments that are pinned together at B. Draw the shear and moment diagrams if it supports the distributed loading shown.

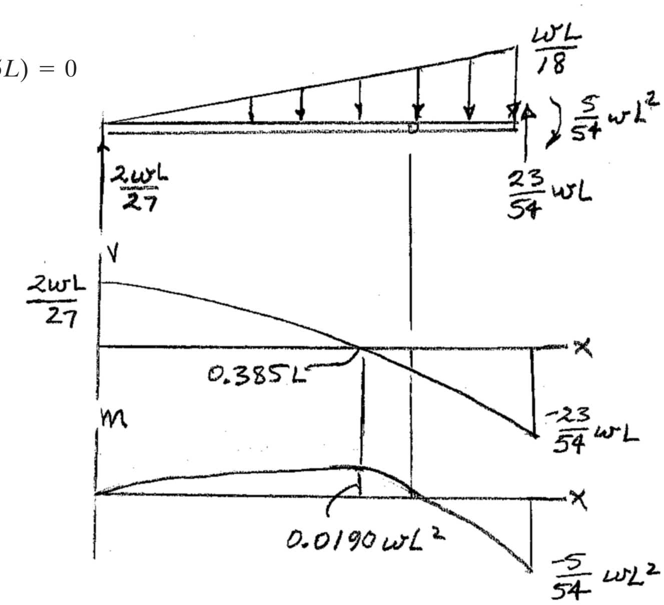


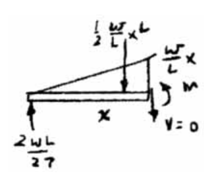
$+\uparrow \Sigma F_y = 0; \qquad \dfrac{2wL}{27} - \dfrac{1}{2}\dfrac{w}{L}x^2 = 0$

$$x = \sqrt{\frac{4}{27}}L = 0.385\,L$$

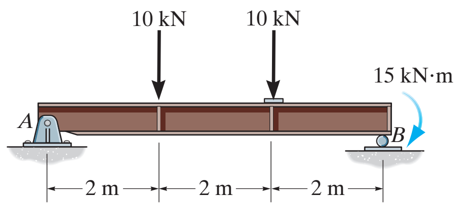
$\zeta + \Sigma M = 0; \qquad M + \dfrac{1}{2}\dfrac{w}{L}(0.385L)^2\left(\dfrac{1}{3}\right)(0.385L) - \dfrac{2wL}{27}(0.385L) = 0$

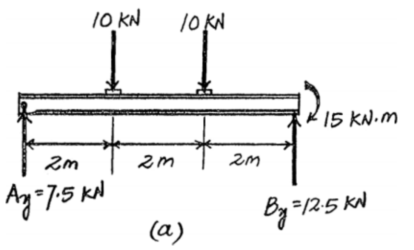
$$M = 0.0190\,wL^2$$

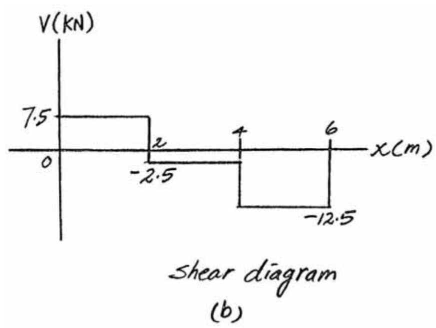


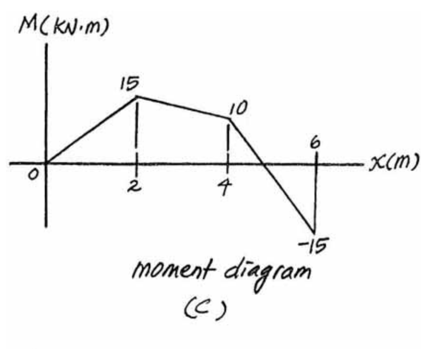


***6–40.** Draw the shear and moment diagrams for the simply supported beam.

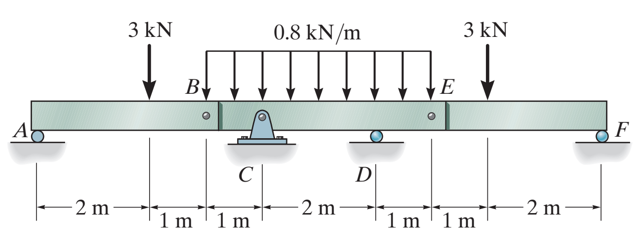


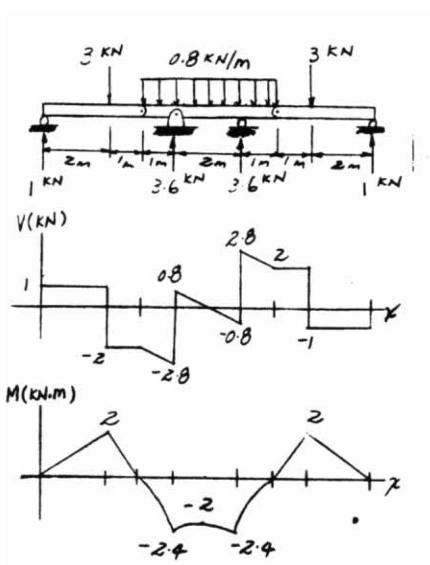




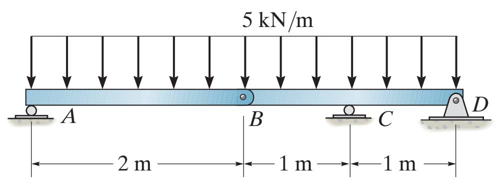


6–41. Draw the shear and moment diagrams for the compound beam. The three segments are connected by pins at B and E.





6–42. Draw the shear and moment diagrams for the compound beam.



Support Reactions:

From the FBD of segment AB

$$\zeta + \Sigma M_A = 0; \qquad B_y(2) - 10.0(1) = 0 \qquad B_y = 5.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 10.0 + 5.00 = 0 \qquad A_y = 5.00 \text{ kN}$$

From the FBD of segment BD

$$\zeta + \Sigma M_C = 0; \qquad 5.00(1) + 10.0(0) - D_y(1) = 0$$

$$D_y = 5.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad C_y - 5.00 - 5.00 - 10.0 = 0$$

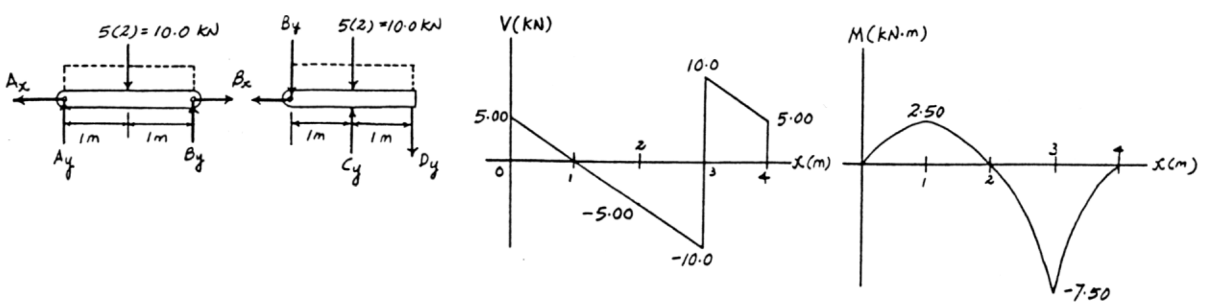
$$C_y = 20.0 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$$

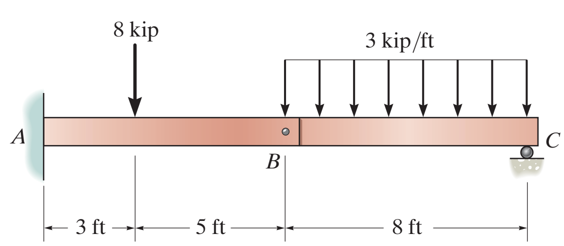
From the FBD of segment AB

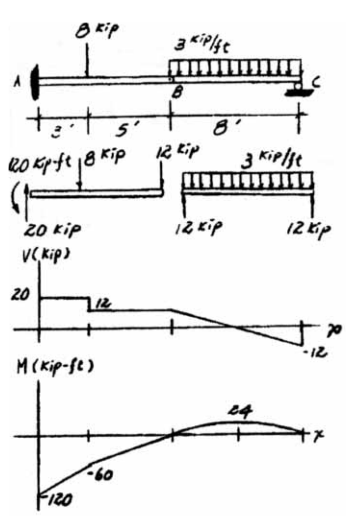
$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$$

Shear and Moment Diagram:

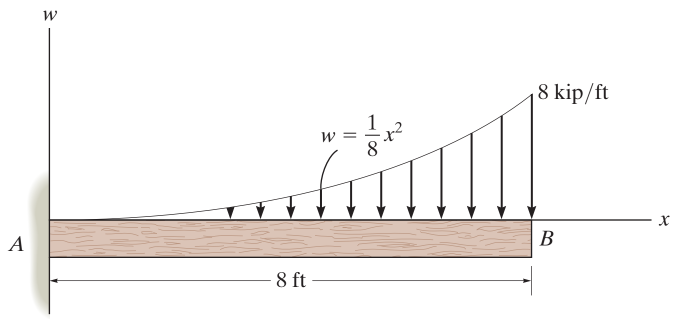


6–43. Draw the shear and moment diagrams for the beam. The two segments are joined together at B.



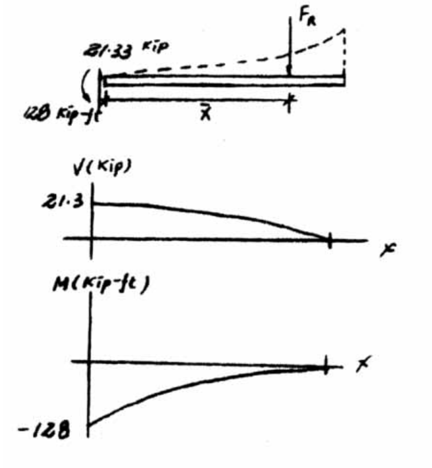


***6–44.** Draw the shear and moment diagrams for the beam.

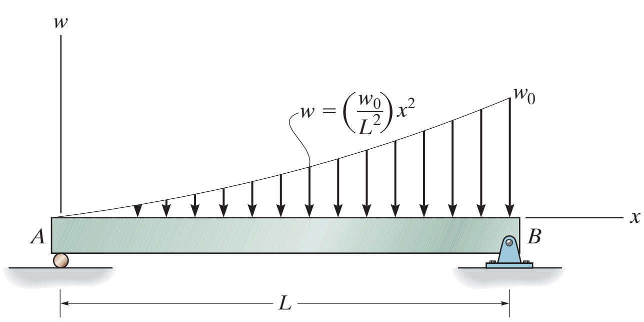


$$F_R = \frac{1}{8}\int_0^8 x^2\,dx = 21.33 \text{ kip}$$

$$\bar{x} = \frac{\frac{1}{8}\int_0^8 x^3\,dx}{21.33} = 6.0 \text{ ft}$$



•6–45. Draw the shear and moment diagrams for the beam.



$$F_R = \int_A dA = \int_0^L w\,dx = \frac{w_0}{L^2}\int_0^L x^2\,dx = \frac{w_0 L}{3}$$

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\frac{w_0}{L^2}\int_0^L x^3 dx}{\frac{w_0 L}{3}} = \frac{3L}{4}$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{12} - \frac{w_0 x^3}{3L^2} = 0$$

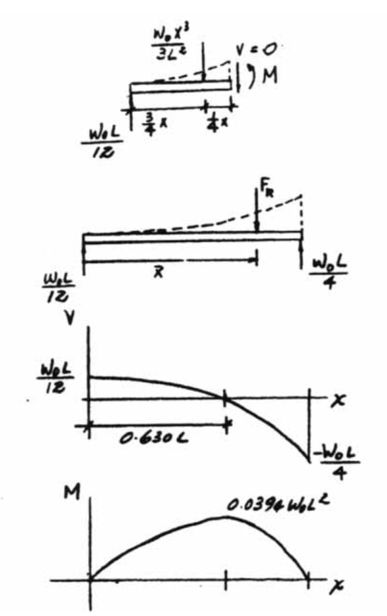
$$x = \left(\frac{1}{4}\right)^{1/3} L = 0.630\,L$$

$$\circlearrowleft +\Sigma M = 0; \qquad \frac{w_0 L}{12}(x) - \frac{w_0 x^3}{3L^2}\left(\frac{1}{4}x\right) - M = 0$$

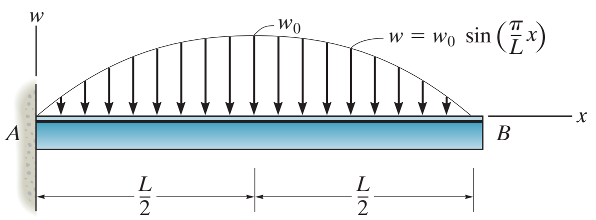
$$M = \frac{w_0 L x}{12} - \frac{w_0 x^4}{12L^2}$$

Substitute $x = 0.630L$

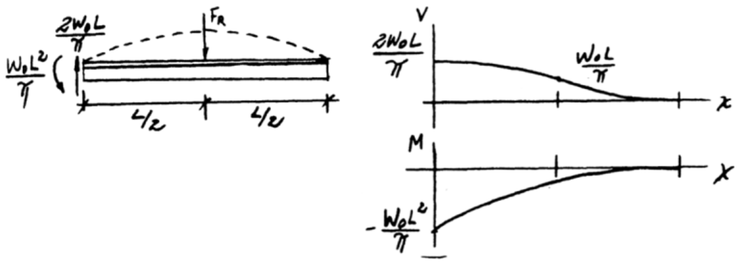
$M = 0.0394\,w_0 L^2$



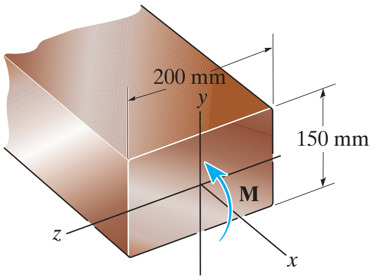
6–46. Draw the shear and moment diagrams for the beam.



$$F_R = \int_A dA = w_0 \int_0^L \sin\left(\frac{\pi}{L}x\right)dx = \frac{2w_0 L}{\pi}$$



6–47. A member having the dimensions shown is used to resist an internal bending moment of $M = 90\text{ kN}\cdot\text{m}$. Determine the maximum stress in the member if the moment is applied (a) about the z axis (as shown) (b) about the y axis. Sketch the stress distribution for each case.



The moment of inertia of the cross-section about z and y axes are

$$I_z = \frac{1}{12}(0.2)(0.15^3) = 56.25(10^{-6})\text{ m}^4$$

$$I_y = \frac{1}{12}(0.15)(0.2^3) = 0.1(10^{-3})\text{ m}^4$$

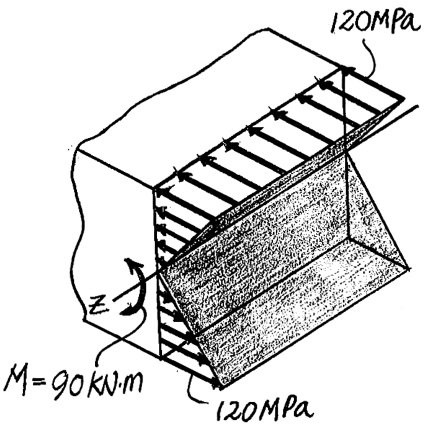
For the bending about z axis, $c = 0.075$ m.

$$\sigma_{max} = \frac{Mc}{I_z} = \frac{90(10^3)\,(0.075)}{56.25\,(10^{-6})} = 120(10^6)\text{Pa} = 120\text{ MPa}$$ **Ans.**

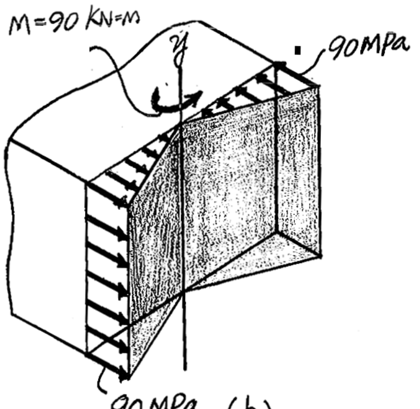
For the bending about y axis, $C = 0.1$ m.

$$\sigma_{max} = \frac{Mc}{I_y} = \frac{90(10^3)\,(0.1)}{0.1\,(10^{-3})} = 90\,(10^6)\text{Pa} = 90\text{ MPa}$$ **Ans.**

The bending stress distribution for bending about z and y axes are shown in Fig. *a* and *b* respectively.

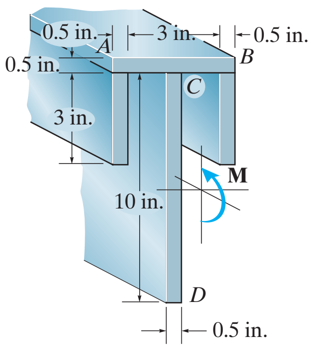


(a)



(b)

***6–48.** Determine the moment M that will produce a maximum stress of 10 ksi on the cross section.



Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.25(4)(0.5) + 2[2(3)(0.5)] + 5.5(10)(0.5)}{4(0.5) + 2[(3)(0.5)] + 10(0.5)} = 3.40 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(4)\left(0.5^3\right) + 4(0.5)(3.40 - 0.25)^2$$

$$+ 2\left[\frac{1}{12}(0.5)(3^3) + 0.5(3)(3.40 - 2)^2\right]$$

$$+ \frac{1}{12}(0.5)\left(10^3\right) + 0.5(10)(5.5 - 3.40)^2$$

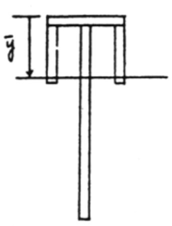
$$= 91.73 \text{ in}^4$$

Maximum Bending Stress: Applying the flexure formula

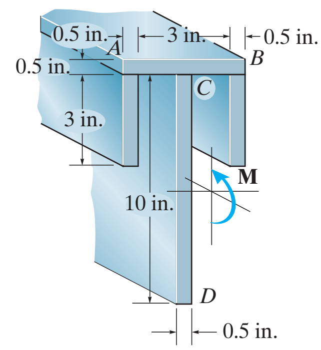
$$\sigma_{\max} = \frac{Mc}{I}$$

$$10 = \frac{M\,(10.5 - 3.4)}{91.73}$$

$$M = 129.2 \text{ kip} \cdot \text{in} = 10.8 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$



•6–49. Determine the maximum tensile and compressive bending stress in the beam if it is subjected to a moment of $M = 4\ \text{kip} \cdot \text{ft}$.



Section Properties:

$$\bar{y} = \frac{\Sigma\,\bar{y}\,A}{\Sigma A}$$

$$= \frac{0.25(4)(0.5) + 2[2(3)(0.5)] + 5.5(10)(0.5)}{4(0.5) + 2[(3)(0.5)] + 10(0.5)} = 3.40\text{ in.}$$

$$I_{NA} = \frac{1}{12}(4)\left(0.5^3\right) + 4(0.5)(3.40 - 0.25)^2$$

$$+ 2\left[\frac{1}{12}(0.5)(3^3) + 0.5(3)(3.40 - 2)^2\right]$$

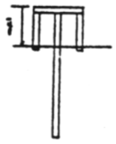
$$+ \frac{1}{12}(0.5)\left(10^3\right) + 0.5(10)(5.5 - 3.40)^2$$

$$= 91.73\text{ in}^4$$

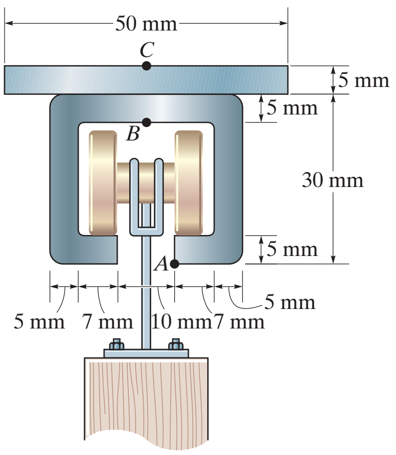
Maximum Bending Stress: Applying the flexure formula $\sigma_{\max} = \dfrac{Mc}{I}$

$$(\sigma_t)_{\max} = \frac{4(10^3)(12)(10.5 - 3.40)}{91.73} = 3715.12\text{ psi} = 3.72\text{ ksi} \qquad \textbf{Ans.}$$

$$(\sigma_c)_{\max} = \frac{4(10^3)(12)(3.40)}{91.73} = 1779.07\text{ psi} = 1.78\text{ ksi} \qquad \textbf{Ans.}$$



6–50. The channel strut is used as a guide rail for a trolley. If the maximum moment in the strut is $M = 30\ \text{N}\cdot\text{m}$, determine the bending stress at points A, B, and C.



$$\bar{y} = \frac{2.5(50)(5) + 7.5(34)(5) + 2[20(5)(20)] + 2[(32.5)(12)(5)]}{50(5) + 34(5) + 2[5(20)] + 2[(12)(5)]}$$

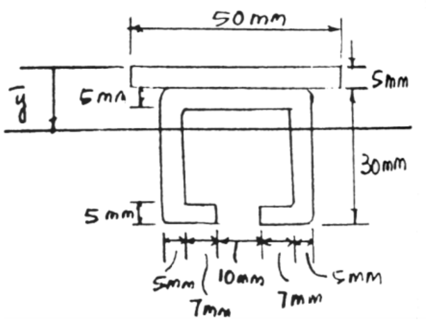
$$= 13.24 \text{ mm}$$

$$I = \left[\frac{1}{12}(50)(5^3) + 50(5)(13.24 - 2.5)^2\right]$$

$$+ \left[\frac{1}{12}(34)(5^3) + 34(5)(13.24 - 7.5)^2\right]$$

$$+ 2\left[\frac{1}{12}(5)(20^3) + 5(20)(20 - 13.24)^2\right] + 2\left[\frac{1}{12}(12)(5^3) + 12(5)(32.5 - 13.24)^2\right]$$

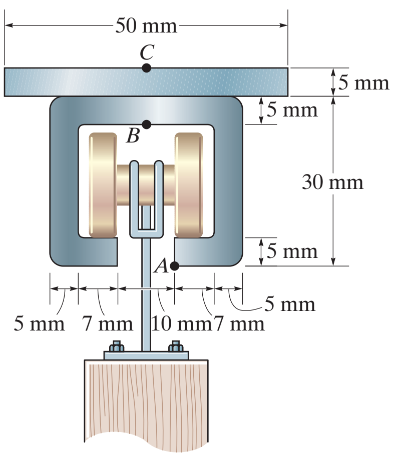
$$= 0.095883(10^{-6}) \text{ m}^4$$



$$\sigma_A = \frac{30(35 - 13.24)(10^{-3})}{0.095883(10^{-6})} = 6.81 \text{ MPa}$$ **Ans.**

$$\sigma_B = \frac{30(13.24 - 10)(10^{-3})}{0.095883(10^{-6})} = 1.01 \text{ MPa}$$ **Ans.**

6–51. The channel strut is used as a guide rail for a trolley. If the allowable bending stress for the material is $\sigma_{\text{allow}} = 175$ MPa, determine the maximum bending moment the strut will resist.



$$\sigma_C = \frac{30(13.24)(10^{-3})}{0.095883(10^{-6})} = 4.14 \text{ MPa}$$ **Ans.**

$$\bar{y} = \frac{\Sigma y^2 A}{\Sigma A} = \frac{2.5(50)(5) + 7.5(34)(5) + 2[20(5)(20)] + 2[(32.5)(12)(5)]}{50(5) + 34(5) + 2[5(20)] + 2[(12)(5)]} = 13.24 \text{ mm}$$

$$I = \left[\frac{1}{12}(50)(5^3) + 50(5)(13.24 - 2.5)^2\right] + \left[\frac{1}{12}(34)(5^3) + 34(5)(13.24 - 7.5)^2\right]$$

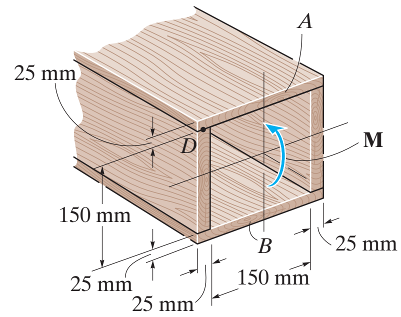
$$+ 2\left[\frac{1}{12}(5)(20^3) + 5(20)(20 - 13.24)^2\right] + 2\left[\frac{1}{12}(12)(5^3) + 12(5)(32.5 - 13.24)^2\right]$$

$$= 0.095883(10^{-6}) \text{ m}^4$$

$$\sigma = \frac{Mc}{I}; \qquad 175(10^6) = \frac{M(35 - 13.24)(10^{-3})}{0.095883(10^{-6})}$$

$$M = 771 \text{ N}\cdot\text{m}$$ **Ans.**

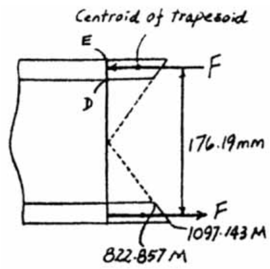
***6–52.** The beam is subjected to a moment **M**. Determine the percentage of this moment that is resisted by the stresses acting on both the top and bottom boards, *A* and *B*, of the beam.



Section Property:

$$I = \frac{1}{12}(0.2)\left(0.2^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right) = 91.14583\left(10^{-6}\right)\ \text{m}^4$$

Bending Stress: Applying the flexure formula



$$\sigma = \frac{My}{I}$$

$$\sigma_E = \frac{M(0.1)}{91.14583(10^{-6})} = 1097.143\ M$$

$$\sigma_D = \frac{M(0.075)}{91.14583(10^{-6})} = 822.857\ M$$

Resultant Force and Moment: For board *A* or *B*

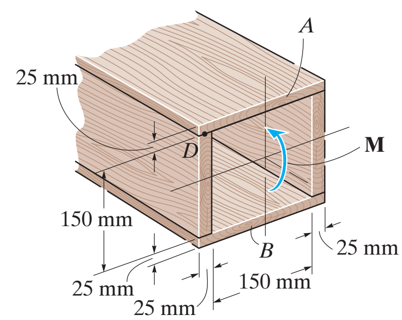
$$F = 822.857M(0.025)(0.2) + \frac{1}{2}(1097.143M - 822.857M)(0.025)(0.2)$$

$$= 4.800\ M$$

$$M' = F(0.17619) = 4.80M(0.17619) = 0.8457\ M$$

$$\sigma_c\left(\frac{M'}{M}\right) = 0.8457(100\%) = 84.6\ \%$$ **Ans.**

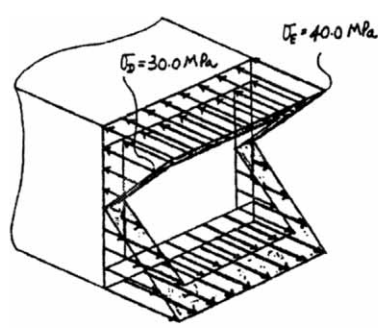
•6–53. Determine the moment **M** that should be applied to the beam in order to create a compressive stress at point *D* of $\sigma_D = 30$ MPa. Also sketch the stress distribution acting over the cross section and compute the maximum stress developed in the beam.



Section Property:

$$I = \frac{1}{12}(0.2)\left(0.2^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right) = 91.14583\left(10^{-6}\right)\ \text{m}^4$$

Bending Stress: Applying the flexure formula



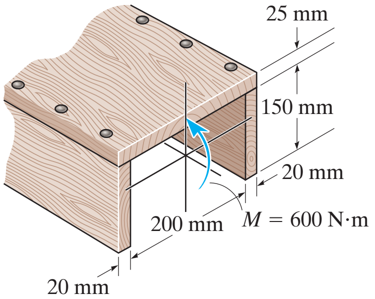
$$\sigma = \frac{My}{I}$$

$$30\left(10^6\right) = \frac{M(0.075)}{91.14583(10^{-6})}$$

$$M = 36458\ \text{N}\cdot\text{m} = 36.5\ \text{kN}\cdot\text{m}$$ **Ans.**

$$\sigma_{\max} = \frac{Mc}{I} = \frac{36458(0.1)}{91.14583(10^{-6})} = 40.0\ \text{MPa}$$ **Ans.**

6–54. The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 600\ \text{N}\cdot\text{m}$, determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution acting over the cross section.

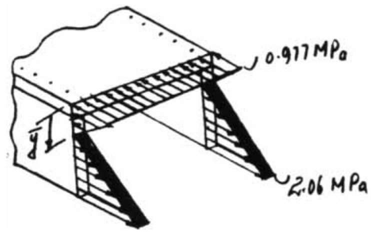


$$\bar{y} = \frac{(0.0125)(0.24)(0.025) + 2\,(0.1)(0.15)(0.2)}{0.24\,(0.025) + 2\,(0.15)(0.02)} = 0.05625 \text{ m}$$

$$I = \frac{1}{12}(0.24)(0.025^3) + (0.24)(0.025)(0.04375^2)$$

$$+\ 2\left(\frac{1}{12}\right)(0.02)(0.15^3) + 2(0.15)(0.02)(0.04375^2)$$

$$= 34.53125\,(10^{-6})\text{ m}^4$$



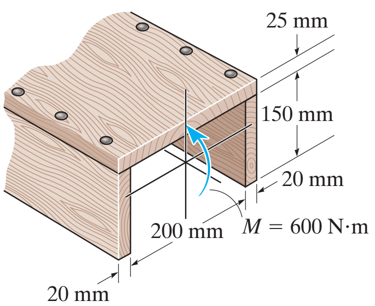
$$\sigma_{max} = \sigma_B = \frac{Mc}{I}$$

$$= \frac{600\,(0.175 - 0.05625)}{34.53125\,(10^{-6})}$$

$$= 2.06 \text{ MPa}$$ **Ans.**

$$\sigma_C = \frac{My}{I} = \frac{600\,(0.05625)}{34.53125\,(10^{-6})} = 0.977 \text{ MPa}$$

6–55. The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 600\ \text{N}\cdot\text{m}$, determine the resultant force the bending stress produces on the top board.

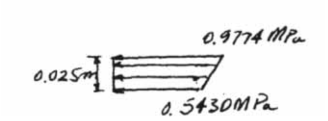


$$\bar{y} = \frac{(0.0125)(0.24)(0.025) + 2\,(0.15)(0.1)(0.02)}{0.24\,(0.025) + 2\,(0.15)(0.02)} = 0.05625 \text{ m}$$

$$I = \frac{1}{12}(0.24)(0.025^3) + (0.24)(0.025)(0.04375^2)$$

$$+\ 2\left(\frac{1}{12}\right)(0.02)(0.15^3) + 2(0.15)(0.02)(0.04375^2)$$

$$= 34.53125\,(10^{-6})\text{ m}^4$$

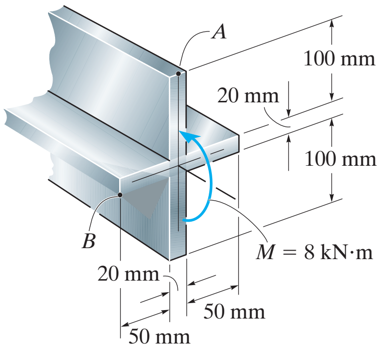


$$\sigma_1 = \frac{My}{I} = \frac{600(0.05625)}{34.53125(10^{-6})} = 0.9774 \text{ MPa}$$

$$\sigma_b = \frac{My}{I} = \frac{600(0.05625 - 0.025)}{34.53125(10^{-6})} = 0.5430 \text{ MPa}$$

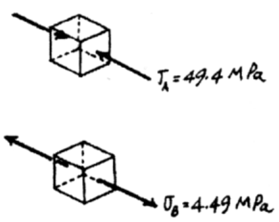
$$F = \frac{1}{2}(0.025)(0.9774 + 0.5430)(10^6)(0.240) = 4.56 \text{ kN}$$ **Ans.**

***6–56.** The aluminum strut has a cross-sectional area in the form of a cross. If it is subjected to the moment $M = 8\ \text{kN}\cdot\text{m}$, determine the bending stress acting at points A and B, and show the results acting on volume elements located at these points.



Section Property:

$$I = \frac{1}{12}(0.02)\left(0.22^3\right) + \frac{1}{12}(0.1)\left(0.02^3\right) = 17.8133\left(10^{-6}\right)\ \text{m}^4$$

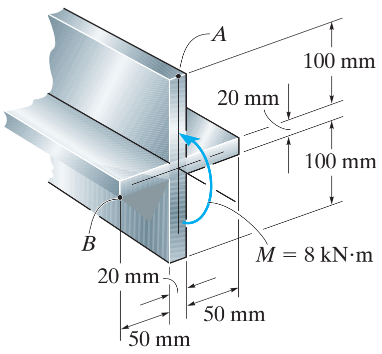


Bending Stress: Applying the flexure formula $\sigma = \dfrac{My}{I}$

$$\sigma_A = \frac{8(10^3)(0.11)}{17.8133(10^{-6})} = 49.4\ \text{MPa (C)}$$ **Ans.**

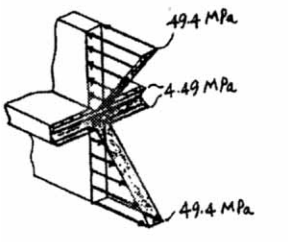
$$\sigma_B = \frac{8(10^3)(0.01)}{17.8133(10^{-6})} = 4.49\ \text{MPa (T)}$$ **Ans.**

•6–57. The aluminum strut has a cross-sectional area in the form of a cross. If it is subjected to the moment $M = 8\ \text{kN}\cdot\text{m}$, determine the maximum bending stress in the beam, and sketch a three-dimensional view of the stress distribution acting over the entire cross-sectional area.



Section Property:

$$I = \frac{1}{12}(0.02)\left(0.22^3\right) + \frac{1}{12}(0.1)\left(0.02^3\right) = 17.8133\left(10^{-6}\right)\ \text{m}^4$$

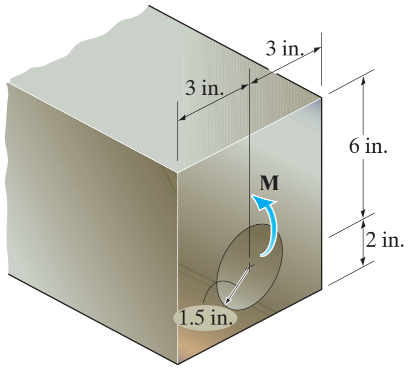


Bending Stress: Applying the flexure formula $\sigma_{max} = \dfrac{Mc}{I}$ and $\sigma = \dfrac{My}{I}$,

$$\sigma_{max} = \frac{8(10^3)(0.11)}{17.8133(10^{-6})} = 49.4\ \text{MPa}$$ **Ans.**

$$\sigma_{y=0.01\text{m}} = \frac{8(10^3)(0.01)}{17.8133(10^{-6})} = 4.49\ \text{MPa}$$

6–58. If the beam is subjected to an internal moment of $M = 100\text{ kip}\cdot\text{ft}$, determine the maximum tensile and compressive bending stress in the beam.



Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. a. The location of C is

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{4(8)(6) - 2\left[\pi\left(1.5^2\right)\right]}{8(6) - \pi\left(1.5^2\right)} = 4.3454 \text{ in.}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2$$

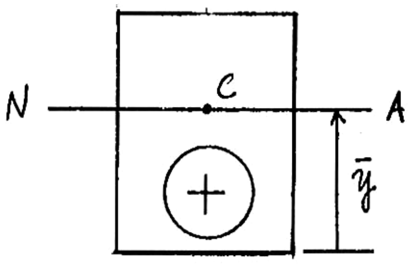
$$= \frac{1}{12}(6)\left(8^3\right) + 6(8)\left(4.3454 - 4\right)^2 - \left[\frac{1}{4}\pi\left(1.5^4\right) + \pi\left(1.5^2\right)\left(4.3454 - 2\right)^2\right]$$

$$= 218.87 \text{ in}^4$$

Maximum Bending Stress: The maximum compressive and tensile bending stress occurs at the top and bottom edges of the cross section.

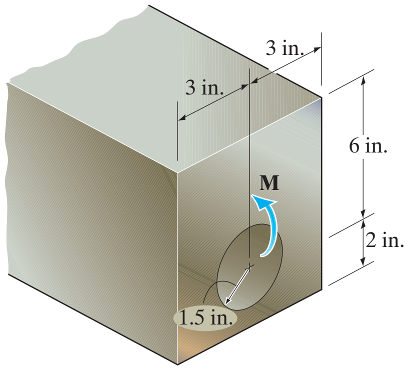
$$\left(\sigma_{\max}\right)_T = \frac{Mc}{I} = \frac{100(12)(4.3454)}{218.87} = 23.8 \text{ ksi (T)} \qquad \textbf{Ans.}$$

$$\left(\sigma_{\max}\right)_C = \frac{My}{I} = \frac{100(12)(8 - 4.3454)}{218.87} = 20.0 \text{ ksi (C)} \qquad \textbf{Ans.}$$



(a)

6–59. If the beam is made of material having an allowable tensile and compressive stress of $(\sigma_{allow})_t = 24$ ksi and $(\sigma_{allow})_c = 22$ ksi, respectively, determine the maximum allowable internal moment **M** that can be applied to the beam.



Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. a. The location of C is

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{4(8)(6) - 2\left[\pi\left(1.5^2\right)\right]}{8(6) - \pi\left(1.5^2\right)} = 4.3454 \text{ in.}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2$$

$$= \frac{1}{12}(6)\left(8^3\right) + 6(8)\left(4.3454 - 4\right)^2 - \left[\frac{1}{4}\pi\left(1.5^4\right) + \pi\left(1.5^2\right)\left(4.3454 - 2\right)^2\right]$$

$$= 218.87 \text{ in}^4$$

Allowable Bending Stress: The maximum compressive and tensile bending stress occurs at the top and bottom edges of the cross section. For the top edge,

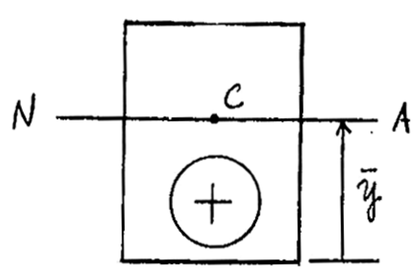
$$(\sigma_{allow})_c = \frac{My}{I}; \qquad 22 = \frac{M(8 - 4.3454)}{218.87}$$

$$M = 1317.53 \text{ kip} \cdot \text{in}\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = 109.79 \text{ kip} \cdot \text{ft}$$

For the bottom edge,

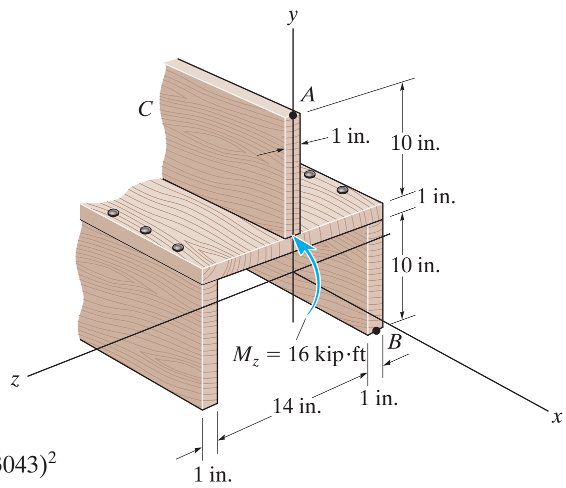
$$\left(\sigma_{max}\right)_t = \frac{Mc}{I}; \qquad 24 = \frac{M(4.3454)}{218.87}$$

$$M = 1208.82 \text{ kip} \cdot \text{in}\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = 101 \text{ kip} \cdot \text{ft (controls)} \qquad \textbf{Ans.}$$



(a)

***6–60.** The beam is constructed from four boards as shown. If it is subjected to a moment of $M_z = 16$ kip·ft, determine the stress at points A and B. Sketch a three-dimensional view of the stress distribution.



$$\bar{y} = \frac{2[5(10)(1)] + 10.5(16)(1) + 16(10)(1)}{2(10)(1) + 16(1) + 10(1)}$$

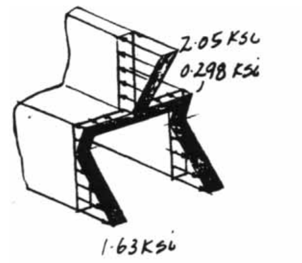
$$= 9.3043 \text{ in.}$$

$$I = 2\left[\frac{1}{12}(1)(10^3) + 1(10)(9.3043 - 5)^2\right] + \frac{1}{12}(16)(1^3) + 16(1)(10.5 - 9.3043)^2$$

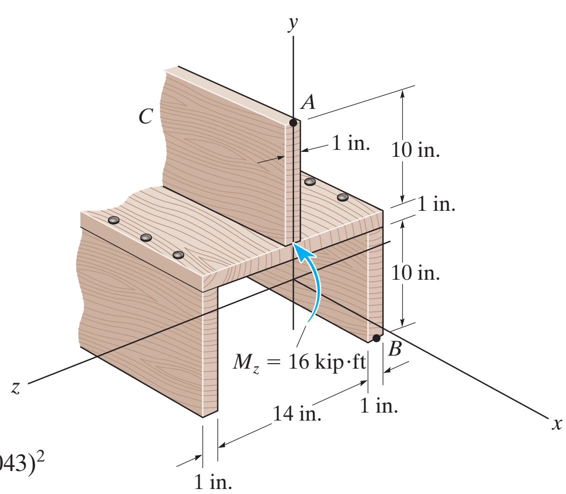
$$+ \frac{1}{12}(1)(10^3) + 1(10)(16 - 9.3043)^2 = 1093.07 \text{ in}^4$$

$$\sigma_A = \frac{Mc}{I} = \frac{16(12)(21 - 9.3043)}{1093.07} = 2.05 \text{ ksi}$$ **Ans.**

$$\sigma_B = \frac{My}{I} = \frac{16(12)(9.3043)}{1093.07} = 1.63 \text{ ksi}$$ **Ans.**



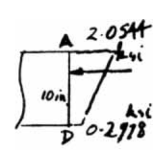
•6–61. The beam is constructed from four boards as shown. If it is subjected to a moment of $M_z = 16$ kip·ft, determine the resultant force the stress produces on the top board C.



$$\bar{y} = \frac{2[5(10)(1)] + 10.5(16)(1) + 16(10)(1)}{2(10)(1) + 16(1) + 10(1)} = 9.3043 \text{ in.}$$

$$I = 2\left[\frac{1}{12}(1)(10^3) + (10)(9.3043 - 5)^2\right] + \frac{1}{12}(16)(1^3) + 16(1)(10.5 - 9.3043)^2$$

$$+ \frac{1}{12}(1)(10^3) + 1(10)(16 - 9.3043)^2 = 1093.07 \text{ in}^4$$

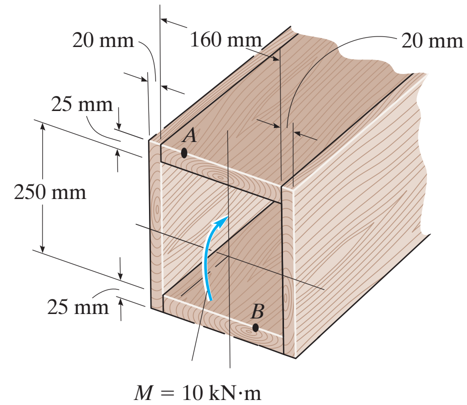


$$\sigma_A = \frac{Mc}{I} = \frac{16(12)(21 - 9.3043)}{1093.07} = 2.0544 \text{ ksi}$$

$$\sigma_D = \frac{My}{I} = \frac{16(12)(11 - 9.3043)}{1093.07} = 0.2978 \text{ ksi}$$

$$(F_R)_C = \frac{1}{2}(2.0544 + 0.2978)(10)(1) = 11.8 \text{ kip}$$ **Ans.**

6–62. A box beam is constructed from four pieces of wood, glued together as shown. If the moment acting on the cross section is 10 kN · m, determine the stress at points A and B and show the results acting on volume elements located at these points.



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.3^3) - \frac{1}{12}(0.16)(0.25^3) = 0.2417(10^{-3})\text{ m}^4.$$

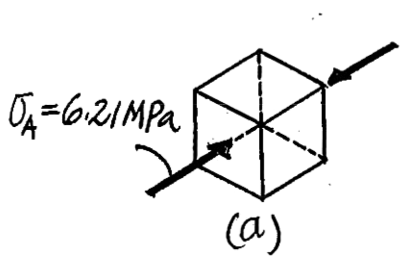
For point A, $y_A = C = 0.15$ m.

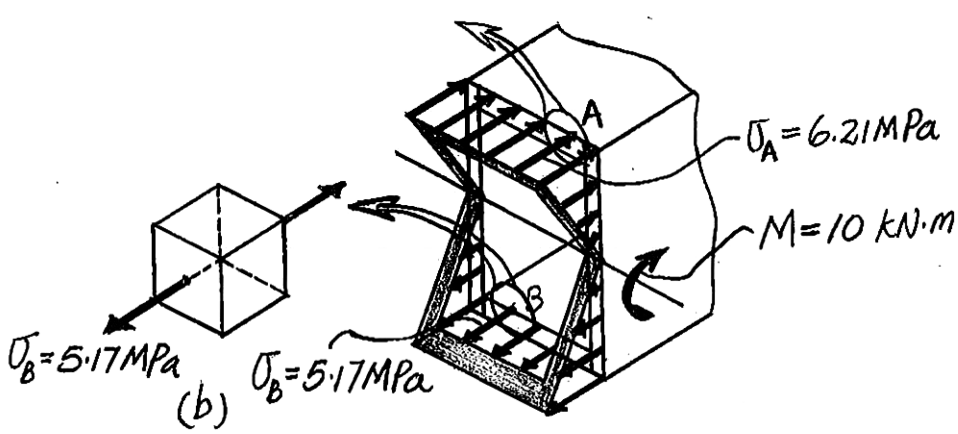
$$\sigma_A = \frac{My_A}{I} = \frac{10(10^3)\,(0.15)}{0.2417(10^{-3})} = 6.207(10^6)\text{Pa} = 6.21\text{ MPa (C)} \qquad \textbf{Ans.}$$

For point B, $y_B = 0.125$ m.

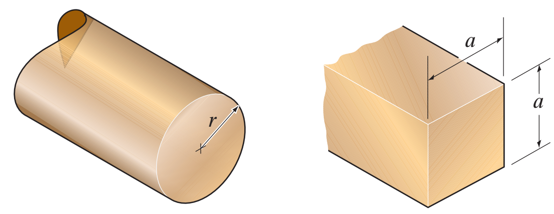
$$\sigma_B = \frac{My_B}{I} = \frac{10(10^3)(0.125)}{0.2417(10^{-3})} = 5.172(10^6)\text{Pa} = 5.17\text{ MPa (T)} \qquad \textbf{Ans.}$$

The state of stress at point A and B are represented by the volume element shown in Figs. a and b respectively.





6–63. Determine the dimension a of a beam having a square cross section in terms of the radius r of a beam with a circular cross section if both beams are subjected to the same internal moment which results in the same maximum bending stress.



Section Properties: The moments of inertia of the square and circular cross sections about the neutral axis are

$$I_S = \frac{1}{12}a\left(a^3\right) = \frac{a^4}{12} \qquad I_C = \frac{1}{4}\pi r^4$$

Maximum Bending Stress: For the square cross section, $c = a/2$.

$$\left(\sigma_{\max}\right)_S = \frac{Mc}{I_S} = \frac{M(a/2)}{a^4/12} = \frac{6M}{a^3}$$

For the circular cross section, $c = r$.

$$\left(\sigma_{\max}\right)_c = \frac{Mc}{I_c} = \frac{Mr}{\frac{1}{4}\pi r^4} - \frac{4M}{\pi r^3}$$

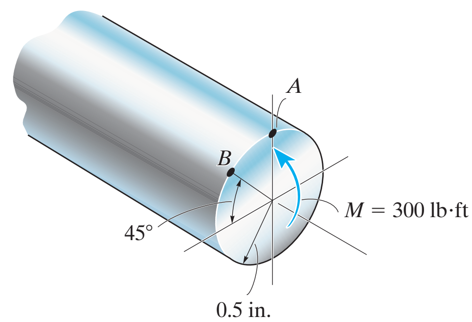
It is required that

$$\left(\sigma_{\max}\right)_S = \left(\sigma_{\max}\right)_C$$

$$\frac{6M}{a^3} = \frac{4M}{\pi r^3}$$

$$a = 1.677r \qquad \textbf{Ans.}$$

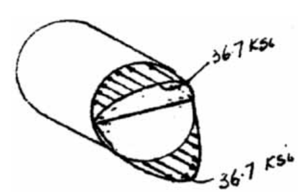
***6–64.** The steel rod having a diameter of 1 in. is subjected to an internal moment of $M = 300\ \text{lb}\cdot\text{ft}$. Determine the stress created at points A and B. Also, sketch a three-dimensional view of the stress distribution acting over the cross section.



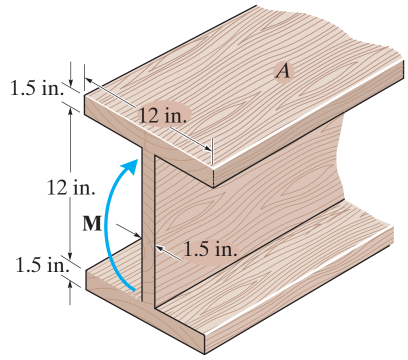
$$I = \frac{\pi}{4}r^4 = \frac{\pi}{4}(0.5^4) = 0.0490874\ \text{in}^4$$

$$\sigma_A = \frac{Mc}{I} = \frac{300(12)(0.5)}{0.0490874} = 36.7\ \text{ksi} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{My}{I} = \frac{300(12)(0.5\sin 45°)}{0.0490874} = 25.9\ \text{ksi} \qquad \textbf{Ans.}$$



•6–65. If the moment acting on the cross section of the beam is $M = 4\ \text{kip}\cdot\text{ft}$, determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution acting over the cross section.



The moment of inertia of the cross-section about the neutral axis is

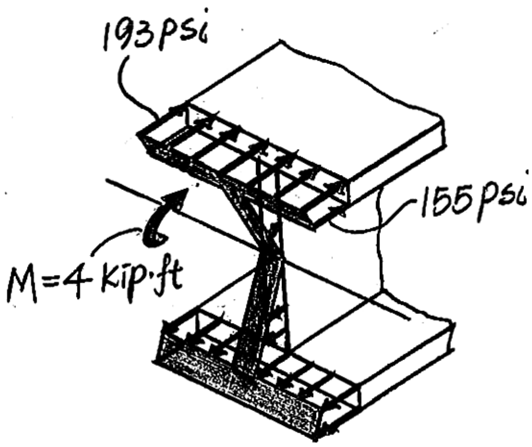
$$I = \frac{1}{12}(12)(15^3) - \frac{1}{12}(10.5)(12^3) = 1863\ \text{in}^4$$

Along the top edge of the flange $y = c = 7.5$ in. Thus

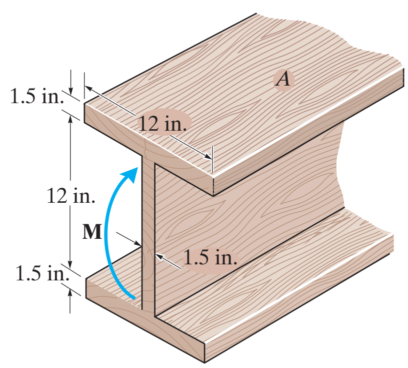
$$\sigma_{\max} = \frac{Mc}{I} = \frac{4(10^3)(12)(7.5)}{1863} = 193\ \text{psi}$$ **Ans.**

Along the bottom edge to the flange, $y = 6$ in. Thus

$$\sigma = \frac{My}{I} = \frac{4(10^3)(12)(6)}{1863} = 155\ \text{psi}$$



6–66. If $M = 4\ \text{kip}\cdot\text{ft}$, determine the resultant force the bending stress produces on the top board A of the beam.



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(12)(15^3) - \frac{1}{12}(10.5)(12^3) = 1863\ \text{in}^4$$

Along the top edge of the flange $y = c = 7.5$ in. Thus

$$\sigma_{\max} = \frac{Mc}{I} = \frac{4(10^3)(12)(7.5)}{1863} = 193.24\ \text{psi}$$

Along the bottom edge of the flange, $y = 6$ in. Thus

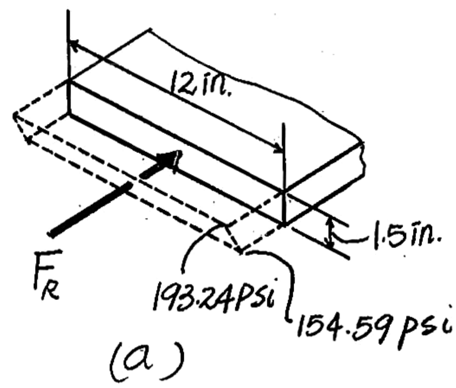
$$\sigma = \frac{My}{I} = \frac{4(10^3)(12)(6)}{1863} = 154.59\ \text{psi}$$

The resultant force acting on board A is equal to the volume of the trapezoidal stress block shown in Fig. *a*.

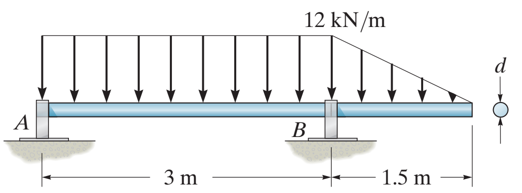
$$F_R = \frac{1}{2}(193.24 + 154.59)(1.5)(12)$$

$$= 3130.43\ \text{lb}$$

$$= 3.13\ \text{kip}$$ **Ans.**



6–67. The rod is supported by smooth journal bearings at A and B that only exert vertical reactions on the shaft. If $d = 90$ mm, determine the absolute maximum bending stress in the beam, and sketch the stress distribution acting over the cross section.

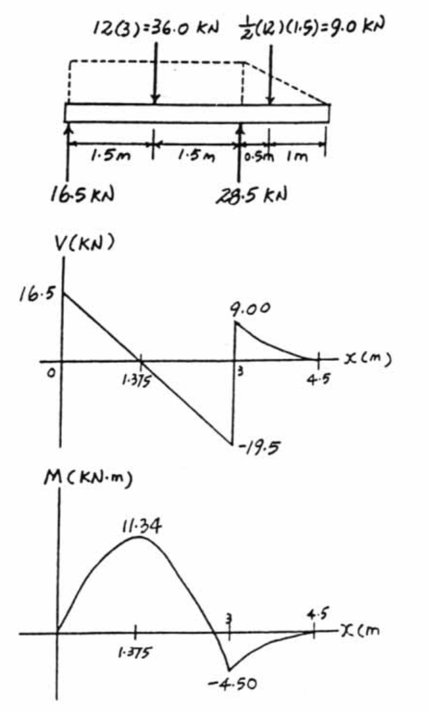


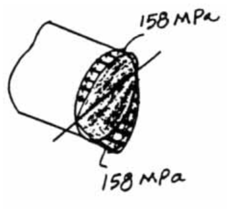
Absolute Maximum Bending Stress: The maximum moment is $M_{max} = 11.34$ kN · m as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{max} = \frac{M_{max}\, c}{I}$$

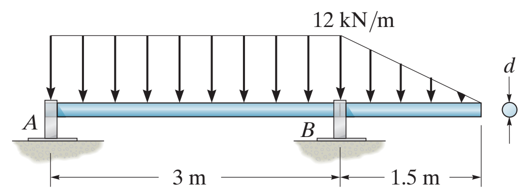
$$= \frac{11.34(10^3)(0.045)}{\frac{\pi}{4}(0.045^4)}$$

$$= 158 \text{ MPa}$$ **Ans.**

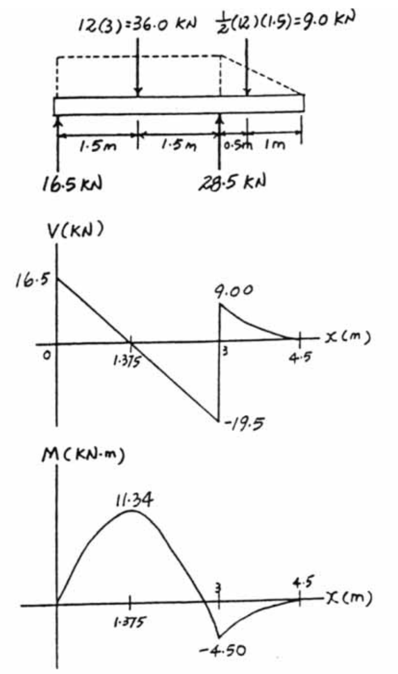




***6–68.** The rod is supported by smooth journal bearings at A and B that only exert vertical reactions on the shaft. Determine its smallest diameter d if the allowable bending stress is $\sigma_{allow} = 180$ MPa.



Allowable Bending Stress: The maximum moment is $M_{max} = 11.34\ \text{kN}\cdot\text{m}$ as indicated on the moment diagram. Applying the flexure formula

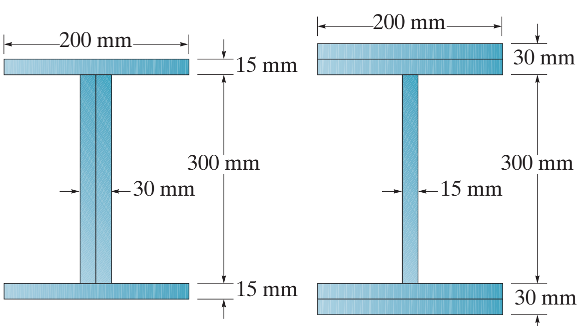


$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$180\left(10^6\right) = \frac{11.34(10^3)\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 0.08626 \text{ m} = 86.3 \text{ mm}$$ **Ans.**

•6–69. Two designs for a beam are to be considered. Determine which one will support a moment of $M = 150\ \text{kN}\cdot\text{m}$ with the least amount of bending stress. What is that stress?



(a) (b)

Section Property:

For section (a)

$$I = \frac{1}{12}(0.2)\left(0.33^3\right) - \frac{1}{12}(0.17)(0.3)^3 = 0.21645(10^{-3})\ \text{m}^4$$

For section (b)

$$I = \frac{1}{12}(0.2)\left(0.36^3\right) - \frac{1}{12}(0.185)\left(0.3^3\right) = 0.36135(10^{-3})\ \text{m}^4$$

Maximum Bending Stress: Applying the flexure formula $\sigma_{max} = \dfrac{Mc}{I}$

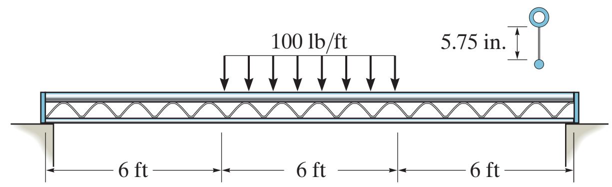
For section (a)

$$\sigma_{max} = \frac{150(10^3)(0.165)}{0.21645(10^{-3})} = 114.3 \text{ MPa}$$

For section (b)

$$\sigma_{max} = \frac{150(10^3)(0.18)}{0.36135(10^{-3})} = 74.72 \text{ MPa} = 74.7 \text{ MPa}$$ **Ans.**

6–70. The simply supported truss is subjected to the central distributed load. Neglect the effect of the diagonal lacing and determine the absolute maximum bending stress in the truss. The top member is a pipe having an outer diameter of 1 in. and thickness of $\frac{3}{16}$ in., and the bottom member is a solid rod having a diameter of $\frac{1}{2}$ in.



$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0 + (6.50)(0.4786)}{0.4786 + 0.19635} = 4.6091 \text{ in.}$$

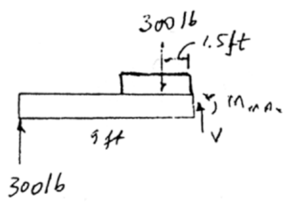
$$I = \left[\frac{1}{4}\pi(0.5)^4 - \frac{1}{4}\pi(0.3125)^4\right] + 0.4786(6.50 - 4.6091)^2 + \frac{1}{4}\pi(0.25)^4$$

$$+ 0.19635(4.6091)^2 = 5.9271 \text{ in}^4$$

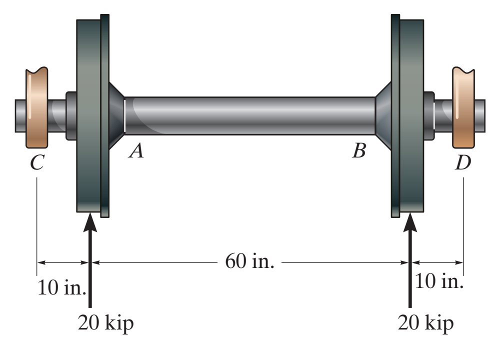
$$M_{max} = 300(9 - 1.5)(12) = 27\,000 \text{ lb} \cdot \text{in.}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{27\,000(4.6091 + 0.25)}{5.9271}$$

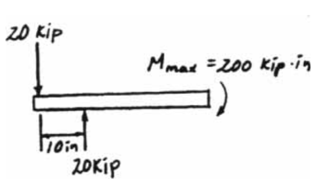
$$= 22.1 \text{ ksi}$$ **Ans.**



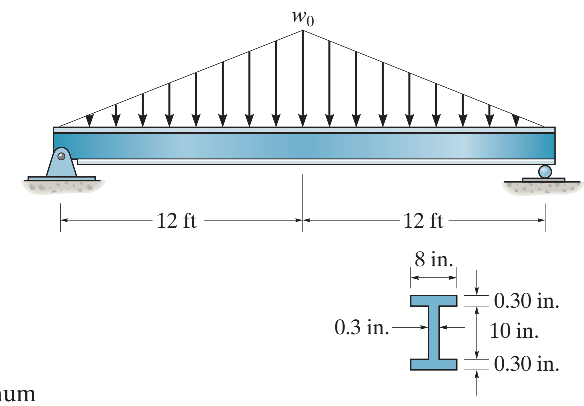
6–71. The axle of the freight car is subjected to wheel loadings of 20 kip. If it is supported by two journal bearings at C and D, determine the maximum bending stress developed at the center of the axle, where the diameter is 5.5 in.



$$\sigma_{max} = \frac{Mc}{I} = \frac{200(2.75)}{\frac{1}{4}\pi(2.75)^4} = 12.2 \text{ ksi}$$ **Ans.**



***6–72.** The steel beam has the cross-sectional area shown. Determine the largest intensity of distributed load w_0 that it can support so that the maximum bending stress in the beam does not exceed $\sigma_{max} = 22$ ksi.



Support Reactions: As shown on FBD.

Internal Moment: The maximum moment occurs at mid span. The maximum moment is determined using the method of sections.

Section Property:

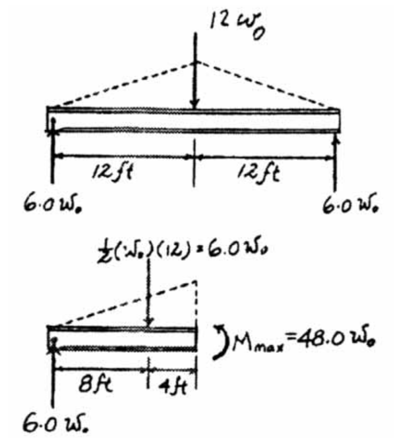
$$I = \frac{1}{12}(8)\left(10.6^3\right) - \frac{1}{12}(7.7)\left(10^3\right) = 152.344 \text{ in}^4$$

Absolute Maximum Bending Stress: The maximum moment is $M_{max} = 48.0w_0$ as indicated on the FBD. Applying the flexure formula

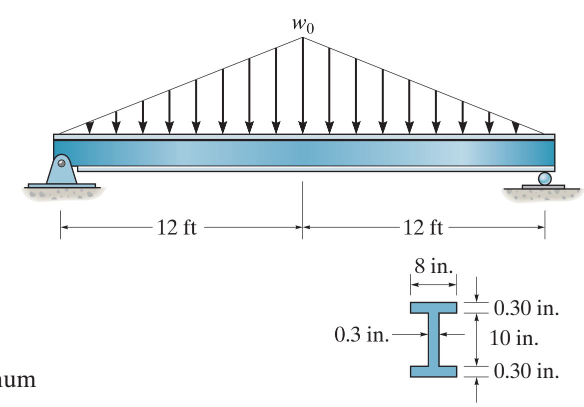
$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$22 = \frac{48.0w_0\,(12)(5.30)}{152.344}$$

$$w_0 = 1.10 \text{ kip/ft} \qquad \textbf{Ans.}$$



•6–73. The steel beam has the cross-sectional area shown. If $w_0 = 0.5$ kip/ft, determine the maximum bending stress in the beam.



Support Reactions: As shown on FBD.

Internal Moment: The maximum moment occurs at mid span. The maximum moment is determined using the method of sections.

Section Property:

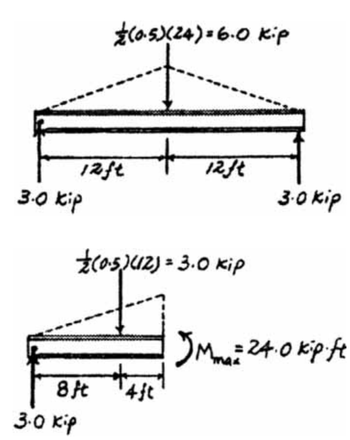
$$I = \frac{1}{12}(8)\left(10.6^3\right) - \frac{1}{12}(7.7)\left(10^3\right) = 152.344 \text{ in}^4$$

Absolute Maximum Bending Stress: The maximum moment is $M_{max} = 24.0 \text{ kip} \cdot \text{ft}$ as indicated on the FBD. Applying the flexure formula

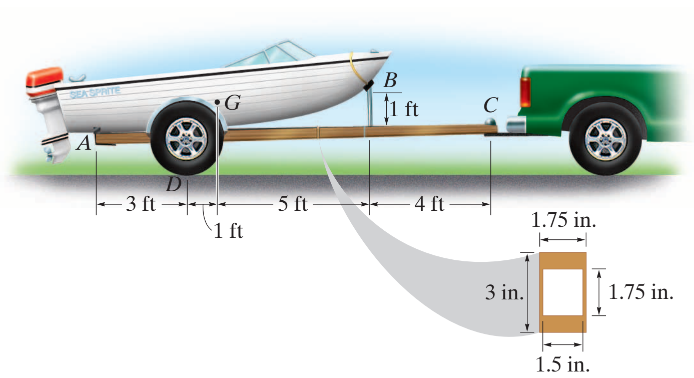
$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$= \frac{24.0(12)(5.30)}{152.344}$$

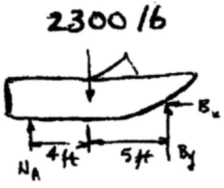
$$= 10.0 \text{ ksi} \qquad \textbf{Ans.}$$



6–74. The boat has a weight of 2300 lb and a center of gravity at G. If it rests on the trailer at the smooth contact A and can be considered pinned at B, determine the absolute maximum bending stress developed in the main strut of the trailer. Consider the strut to be a box-beam having the dimensions shown and pinned at C.



Boat:



$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$

$\circlearrowleft + \Sigma M_B = 0; \qquad -N_A(9) + 2300(5) = 0$

$N_A = 1277.78 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad 1277.78 - 2300 + B_y = 0$

$B_y = 1022.22 \text{ lb}$

Assembly:

$\circlearrowleft + \Sigma M_C = 0; \qquad -N_D(10) + 2300(9) = 0$

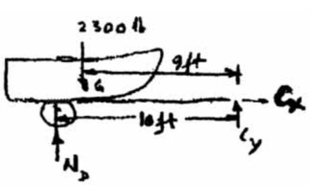
$N_D = 2070 \text{ lb}$

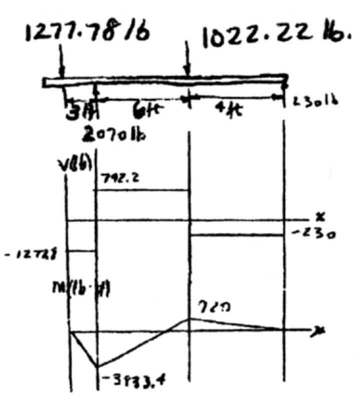
$+\uparrow \Sigma F_y = 0; \qquad C_y + 2070 - 2300 = 0$

$C_y = 230 \text{ lb}$

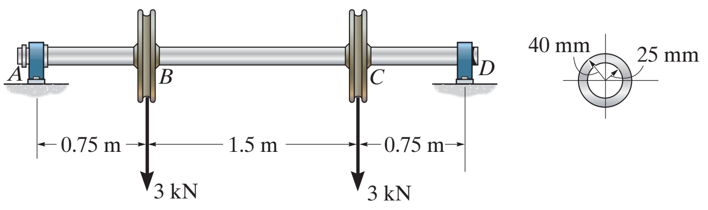
$$I = \frac{1}{12}(1.75)(3)^3 - \frac{1}{12}(1.5)(1.75)^3 = 3.2676 \text{ in}^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{3833.3(12)(1.5)}{3.2676} = 21.1 \text{ ksi}$$ **Ans.**





6–75. The shaft is supported by a smooth thrust bearing at *A* and smooth journal bearing at *D*. If the shaft has the cross section shown, determine the absolute maximum bending stress in the shaft.



Shear and Moment Diagrams: As shown in Fig. *a*.

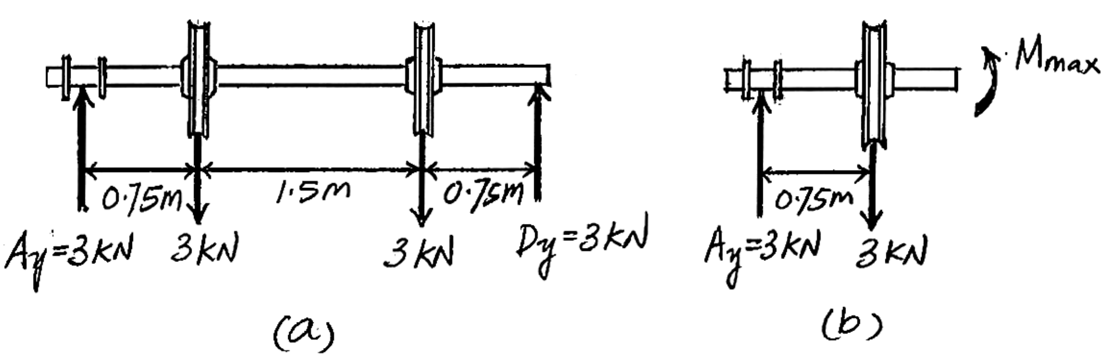
Maximum Moment: Due to symmetry, the maximum moment occurs in region *BC* of the shaft. Referring to the free-body diagram of the segment shown in Fig. *b*.

Section Properties: The moment of inertia of the cross section about the neutral axis is

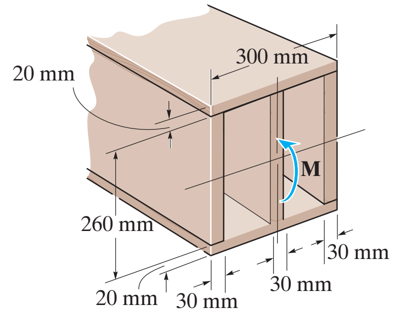
$$I = \frac{\pi}{4}\left(0.04^4 - 0.025^4\right) = 1.7038\left(10^{-6}\right)\text{m}^4$$

Absolute Maximum Bending Stress:

$$\sigma_{\text{allow}} = \frac{M_{\max}c}{I} = \frac{2.25\left(10^3\right)(0.04)}{1.7038\left(10^{-6}\right)} = 52.8 \text{ MPa}$$ **Ans.**



***6–76.** Determine the moment **M** that must be applied to the beam in order to create a maximum stress of 80 MPa. Also sketch the stress distribution acting over the cross section.



The moment of inertia of the cross-section about the neutral axis is

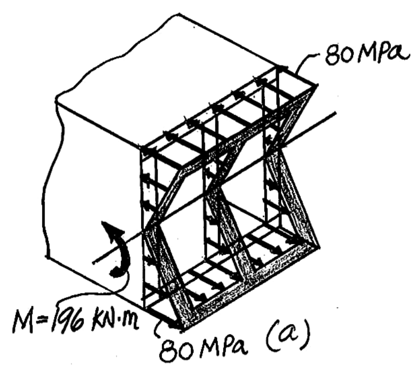
$$I = \frac{1}{12}(0.3)(0.3^3) - \frac{1}{12}(0.21)(0.26^3) = 0.36742(10^{-3})\text{ m}^4$$

Thus,

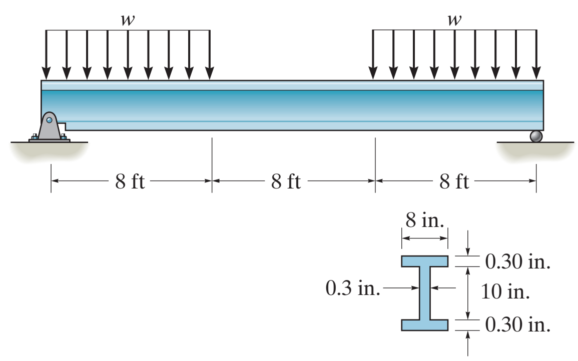
$$\sigma_{\max} = \frac{Mc}{I}; \qquad 80(10^6) = \frac{M(0.15)}{0.36742(10^{-3})}$$

$$M = 195.96\,(10^3)\text{ N}\cdot\text{m} = 196\text{ kN}\cdot\text{m}$$ **Ans.**

The bending stress distribution over the cross-section is shown in Fig. *a*.



•6–77. The steel beam has the cross-sectional area shown. Determine the largest intensity of distributed load w that it can support so that the bending stress does not exceed $\sigma_{max} = 22$ ksi.

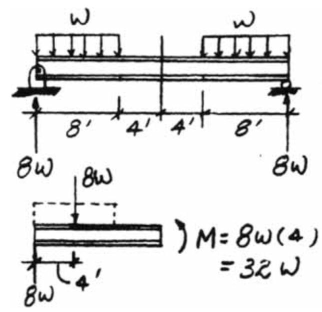


$$I = \frac{1}{12}(8)(10.6)^3 - \frac{1}{12}(7.7)(10^3) = 152.344 \text{ in}^4$$

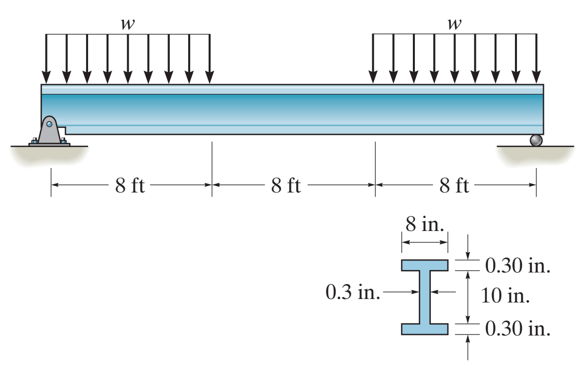
$$\sigma_{max} = \frac{Mc}{I}$$

$$22 = \frac{32w(12)(5.3)}{152.344}$$

$$w = 1.65 \text{ kip/ft}$$ **Ans.**



6–78. The steel beam has the cross-sectional area shown. If w = 5 kip/ft, determine the absolute maximum bending stress in the beam.



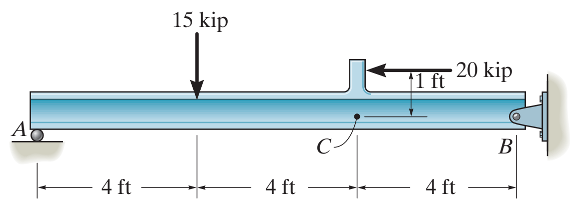
From Prob. 6-78:

$$M = 32w = 32(5)(12) = 1920 \text{ kip} \cdot \text{in.}$$

$$I = 152.344 \text{ in}^4$$

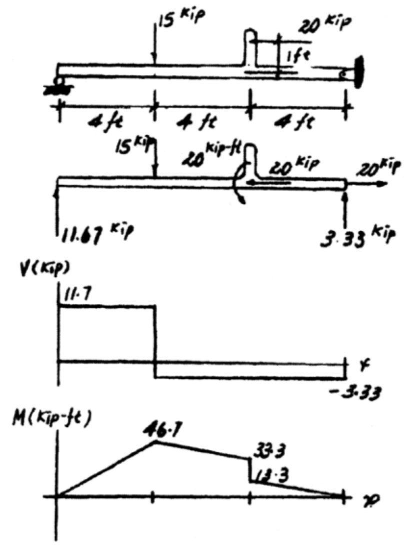
$$\sigma_{max} = \frac{Mc}{I} = \frac{1920(5.3)}{152.344} = 66.8 \text{ ksi}$$ **Ans.**

6–79. If the beam *ACB* in Prob. 6–9 has a square cross section, 6 in. by 6 in., determine the absolute maximum bending stress in the beam.

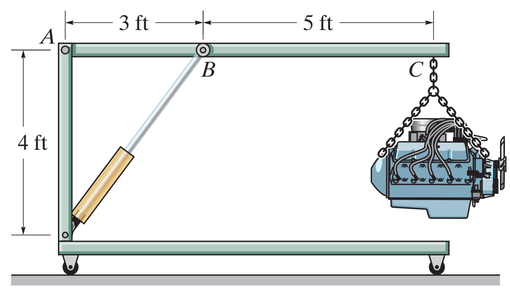


$$M_{max} = 46.7 \text{ kip} \cdot \text{ft}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{46.7(10^3)(12)(3)}{\frac{1}{12}(6)(6^3)} = 15.6 \text{ ksi}$$ **Ans.**



***6–80.** If the crane boom *ABC* in Prob. 6–3 has a rectangular cross section with a base of 2.5 in., determine its required height *h* to the nearest $\frac{1}{4}$ in. if the allowable bending stress is $\sigma_{allow} = 24$ ksi.



$$\zeta + \Sigma M_A = 0; \quad \frac{4}{5}F_B(3) - 1200(8) = 0; \quad F_B = 4000 \text{ lb}$$

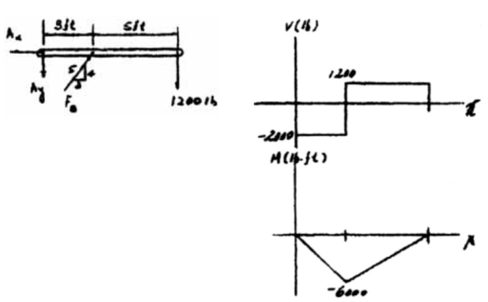
$$+\uparrow \Sigma F_y = 0; \quad -A_y + \frac{4}{5}(4000) - 1200 = 0; \quad A_y = 2000 \text{ lb}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad A_x - \frac{3}{5}(4000) = 0; \quad A_x = 2400 \text{ lb}$$

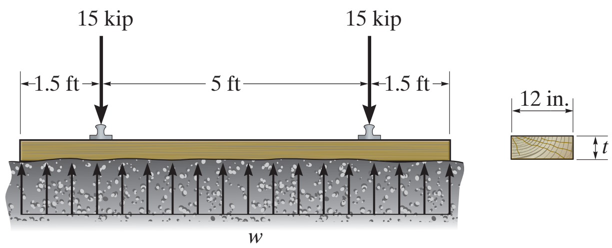
$$\sigma_{max} = \frac{Mc}{I} = \frac{6000(12)\left(\frac{h}{2}\right)}{\frac{1}{12}(2.5)(h^3)} = 24(10)^3$$

$h = 2.68$ in. **Ans.**

Use $h = 2.75$ in. **Ans.**



•6–81. If the reaction of the ballast on the railway tie can be assumed uniformly distributed over its length as shown, determine the maximum bending stress developed in the tie. The tie has the rectangular cross section with thickness $t = 6$ in.



Support Reactions: Referring to the free - body diagram of the tie shown in Fig. *a*, we have

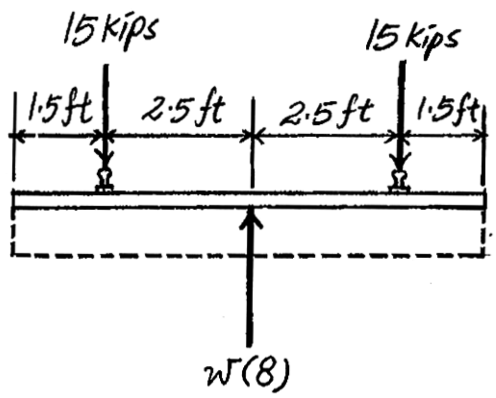
$$+\uparrow \Sigma F_y = 0; \qquad w(8) - 2(15) = 0$$

$$w = 3.75 \text{ kip/ft}$$

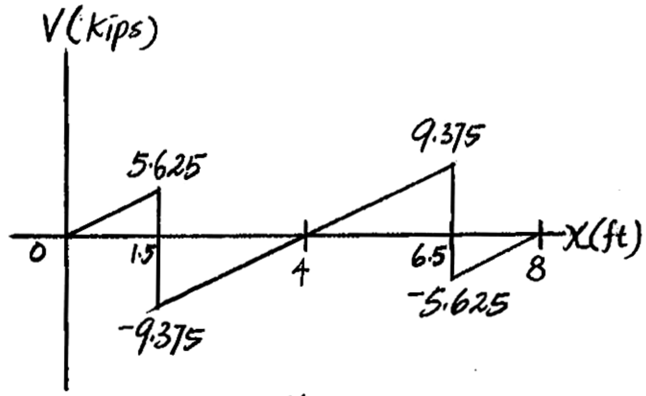
Maximum Moment: The shear and moment diagrams are shown in Figs. *b* and *c*. As indicated on the moment diagram, the maximum moment is $|M_{max}| = 7.5 \text{ kip} \cdot \text{ft}$.

Absolute Maximum Bending Stress:

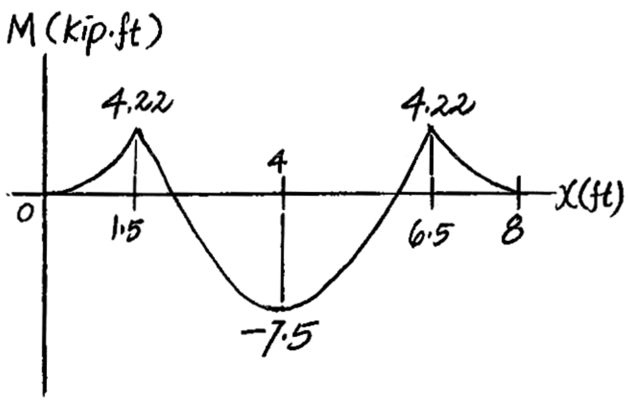
$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{7.5(12)(3)}{\frac{1}{12}(12)(6^3)} = 1.25 \text{ ksi}$$ **Ans.**



(a)

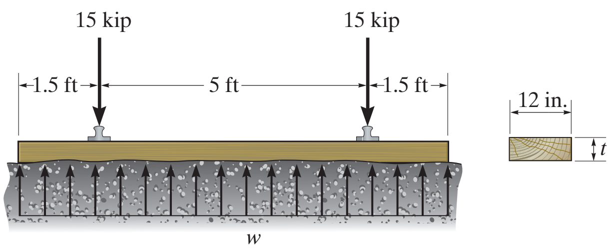


(b)



(c)

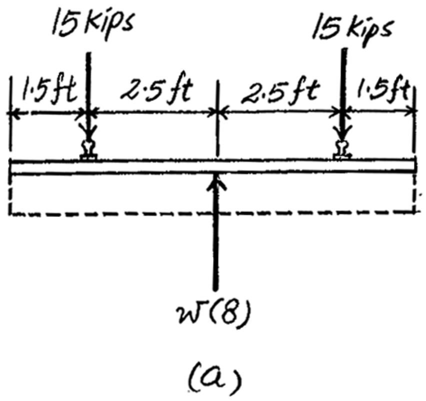
6–82. The reaction of the ballast on the railway tie can be assumed uniformly distributed over its length as shown. If the wood has an allowable bending stress of $\sigma_{allow} = 1.5$ ksi, determine the required minimum thickness t of the rectangular cross sectional area of the tie to the nearest $\frac{1}{8}$ in.



Support Reactions: Referring to the free-body diagram of the tie shown in Fig. *a*, we have

$+\uparrow \Sigma F_y = 0; \qquad w(8) - 2(15) = 0$

$w = 3.75 \text{ kip/ft}$



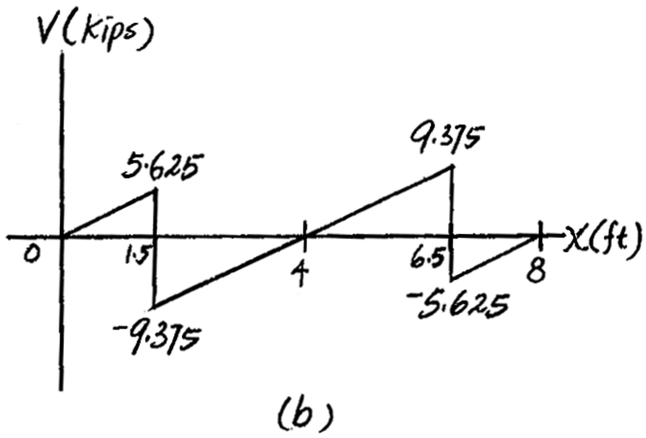
Maximum Moment: The shear and moment diagrams are shown in Figs. *b* and *c*. As indicated on the moment diagram, the maximum moment is $|M_{max}| = 7.5 \text{ kip} \cdot \text{ft}$.

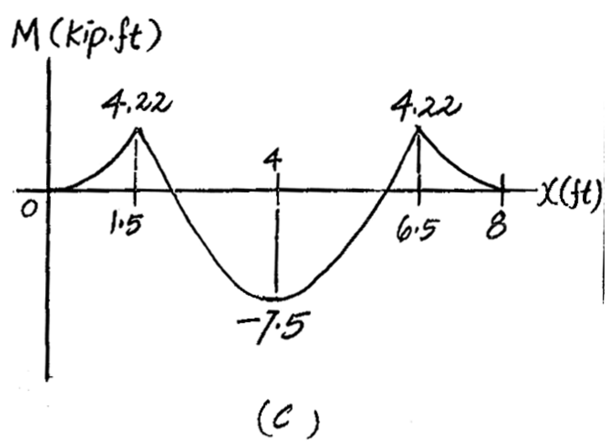
Absolute Maximum Bending Stress:

$$\sigma_{max} = \frac{Mc}{I}; \qquad 1.5 = \frac{7.5(12)\left(\frac{t}{2}\right)}{\frac{1}{12}(12)t^3}$$

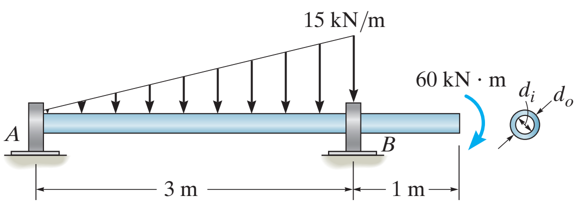
$$t = 5.48 \text{ in.}$$

Use $\qquad t = 5\frac{1}{2}$ in. **Ans.**





6–83. Determine the absolute maximum bending stress in the tubular shaft if $d_i = 160$ mm and $d_o = 200$ mm.



Section Property:

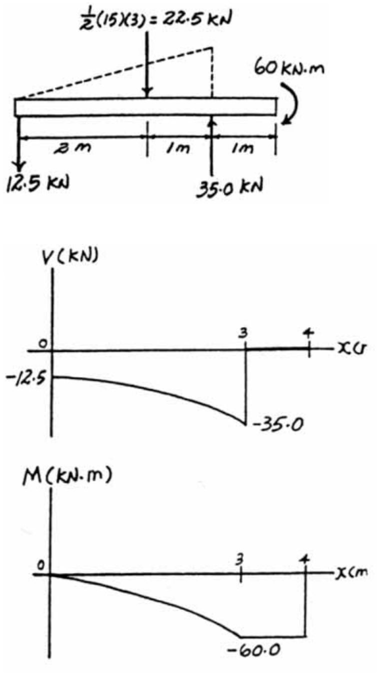
$$I = \frac{\pi}{4}\left(0.1^4 - 0.08^4\right) = 46.370\left(10^{-6}\right)\text{ m}^4$$

Absolute Maximum Bending Stress: The maximum moment is $M_{max} = 60.0\text{ kN} \cdot \text{m}$ as indicated on the moment diagram. Applying the flexure formula

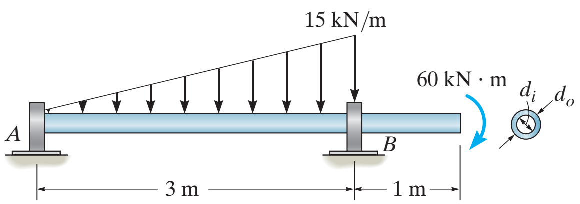
$$\sigma_{max} = \frac{M_{max}c}{I}$$

$$= \frac{60.0(10^3)(0.1)}{46.370(10^{-6})}$$

$$= 129\text{ MPa}$$ **Ans.**



***6–84.** The tubular shaft is to have a cross section such that its inner diameter and outer diameter are related by $d_i = 0.8d_o$. Determine these required dimensions if the allowable bending stress is $\sigma_{allow} = 155$ MPa.



Section Property:

$$I = \frac{\pi}{4}\left[\left(\frac{d_o}{2}\right)^4 - \left(\frac{d_l}{2}\right)^4\right] = \frac{\pi}{4}\left[\frac{d_o{}^4}{16} - \left(\frac{0.8d_o}{2}\right)^4\right] = 0.009225\pi d_o^4$$

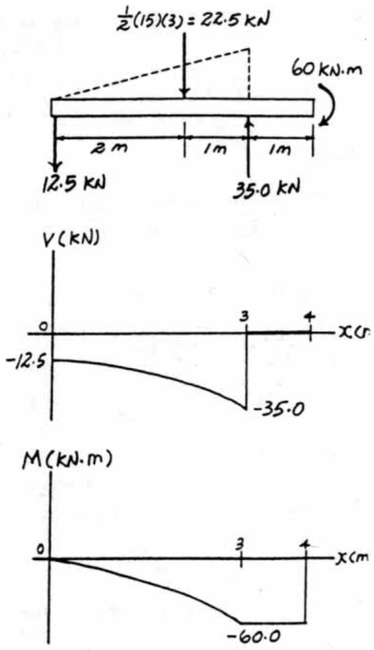
Allowable Bending Stress: The maximum moment is $M_{max} = 60.0\ \text{kN}\cdot\text{m}$ as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\,c}{I}$$

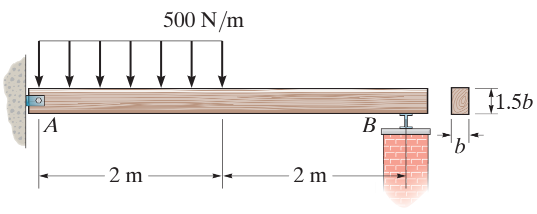
$$155\left(10^6\right) = \frac{60.0(10^3)\left(\frac{d_o}{2}\right)}{0.009225\pi d_o^4}$$

$$d_o = 0.1883\ \text{m} = 188\ \text{mm}$$ **Ans.**

Thus, $$d_l = 0.8d_o = 151\ \text{mm}$$ **Ans.**



6–85. The wood beam has a rectangular cross section in the proportion shown. Determine its required dimension b if the allowable bending stress is $\sigma_{allow} = 10$ MPa.

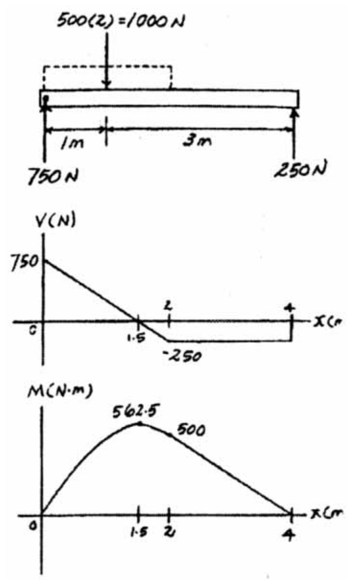


Allowable Bending Stress: The maximum moment is $M_{max} = 562.5$ N · m as indicated on the moment diagram. Applying the flexure formula

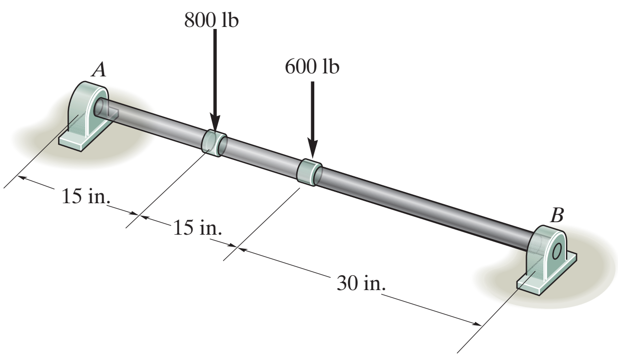
$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$10\left(10^6\right) = \frac{562.5(0.75b)}{\frac{1}{12}(b)(1.5b)^3}$$

$$b = 0.05313 \text{ m} = 53.1 \text{ mm}$$ **Ans.**



6–86. Determine the absolute maximum bending stress in the 2-in.-diameter shaft which is subjected to the concentrated forces. The journal bearings at A and B only support vertical forces.



The FBD of the shaft is shown in Fig. *a*.

The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $M_{max} = 15000 \text{ lb} \cdot \text{in}$.

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{\pi}{4}(1^4) = 0.25\,\pi \text{ in}^4$$

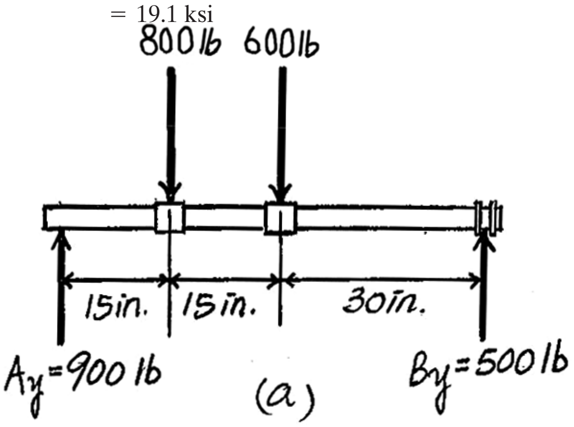
Here, $c = 1$ in. Thus

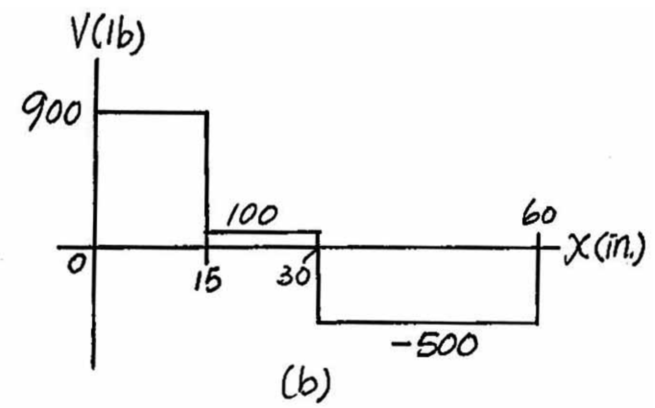
$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

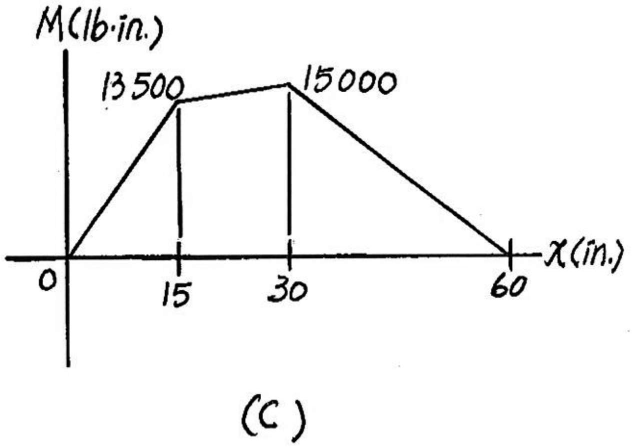
$$= \frac{15000(1)}{0.25\,\pi}$$

$$= 19.10(10^3) \text{ psi}$$

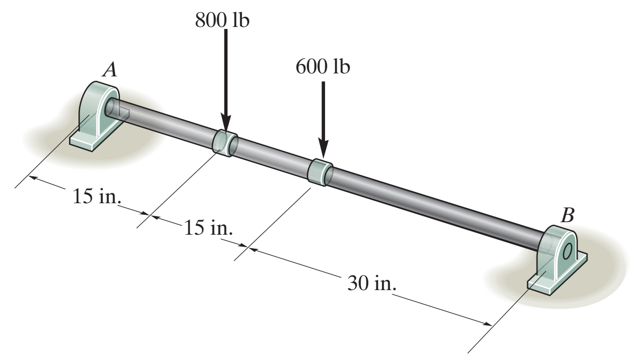
$$= 19.1 \text{ ksi}$$ **Ans.**







6–87. Determine the smallest allowable diameter of the shaft which is subjected to the concentrated forces. The journal bearings at A and B only support vertical forces. The allowable bending stress is $\sigma_{allow} = 22$ ksi.



The FBD of the shaft is shown in Fig. *a*

The shear and moment diagrams are shown in Fig. *b* and *c* respectively. As indicated on the moment diagram, $M_{max} = 15{,}000 \text{ lb} \cdot \text{in}$

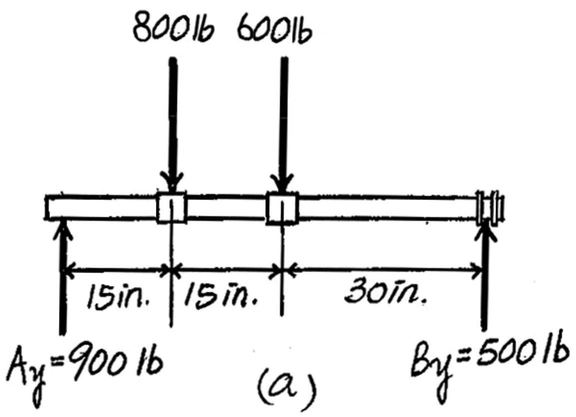
The moment of inertia of the cross-section about the neutral axis is

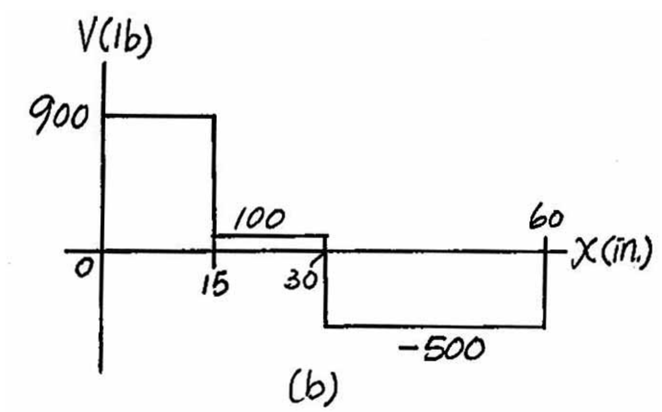
$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64} d^4$$

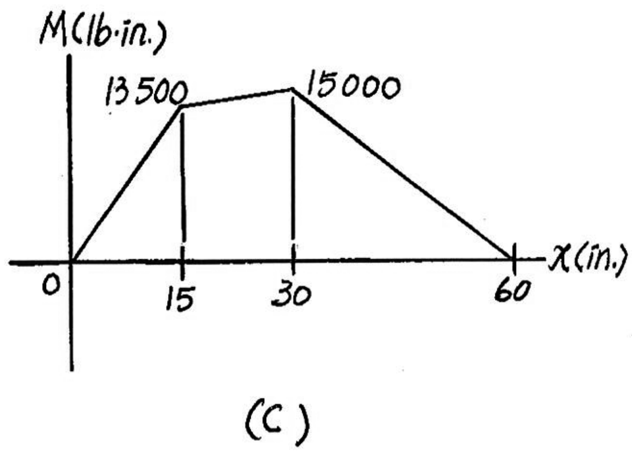
Here, $c = d/2$. Thus

$$\sigma_{allow} = \frac{M_{max}\, c}{I}; \qquad 22(10^3) = \frac{15000(^d/_2)}{\pi d^4/64}$$

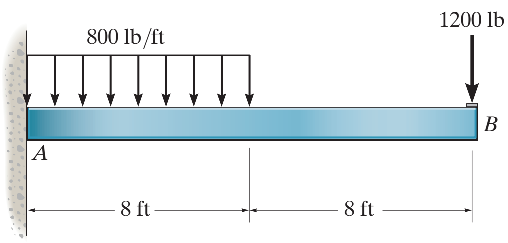
$$d = 1.908 \text{ in} = 2 \text{ in.}$$ **Ans.**





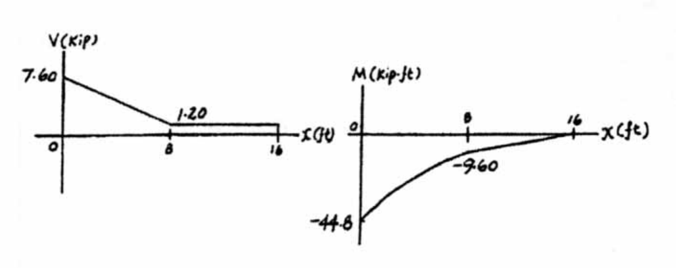


***6–88.** If the beam has a square cross section of 9 in. on each side, determine the absolute maximum bending stress in the beam.



Absolute Maximum Bending Stress: The maximum moment is $M_{max} = 44.8\ \text{kip}\cdot\text{ft}$ as indicated on moment diagram. Applying the flexure formula

$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{44.8(12)(4.5)}{\frac{1}{12}(9)(9)^3} = 4.42\ \text{ksi} \qquad \textbf{Ans.}$$



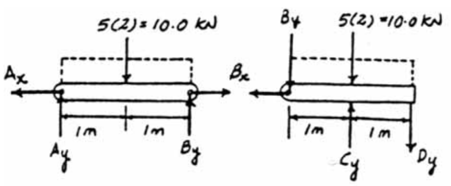
•6–89. If the compound beam in Prob. 6–42 has a square cross section, determine its dimension a if the allowable bending stress is $\sigma_{allow} = 150$ MPa.

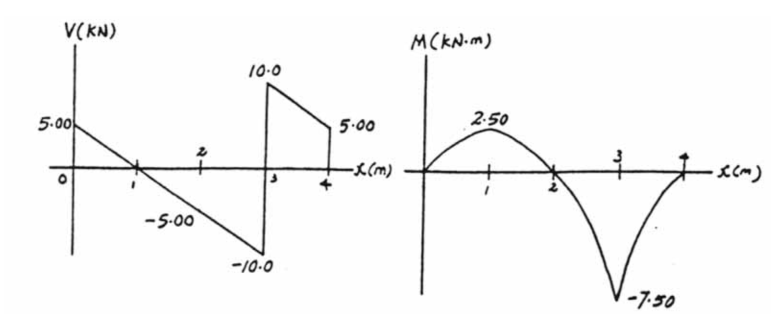
Allowable Bending Stress: The maximum moments is $M_{max} = 7.50\ \text{kN}\cdot\text{m}$ as indicated on moment diagram. Applying the flexure formula

$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$150\left(10^6\right) = \frac{7.50(10^3)\left(\frac{a}{2}\right)}{\frac{1}{12}a^4}$$

$$a = 0.06694\ \text{m} = 66.9\ \text{mm} \qquad \textbf{Ans.}$$

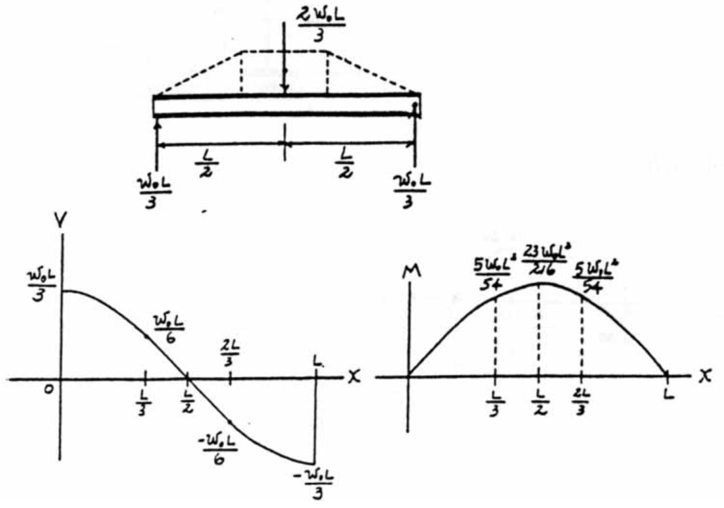




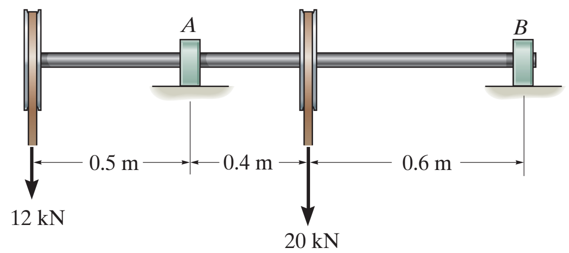
6–90. If the beam in Prob. 6–28 has a rectangular cross section with a width b and a height h, determine the absolute maximum bending stress in the beam.

Absolute Maximum Bending Stress: The maximum moments is $M_{\text{max}} = \dfrac{23w_0 L^2}{216}$ as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{\text{max}} = \frac{M_{\text{max}}\, c}{I} = \frac{\frac{23w_0 L^2}{216}\left(\frac{h}{2}\right)}{\frac{1}{12}\, bh^3} = \frac{23w_0 L^2}{36bh^2} \qquad \textbf{Ans.}$$



6–91. Determine the absolute maximum bending stress in the 80-mm-diameter shaft which is subjected to the concentrated forces. The journal bearings at A and B only support vertical forces.



The FBD of the shaft is shown in Fig. *a*

The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $|M_{max}| = 6\ \text{kN}\cdot\text{m}$.

The moment of inertia of the cross-section about the neutral axis is

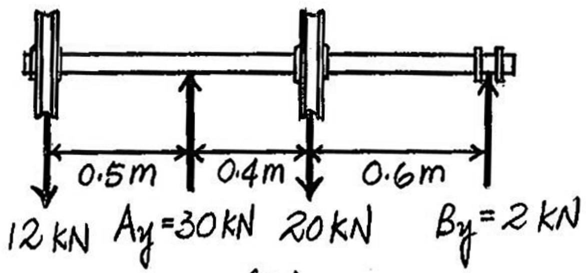
$$I = \frac{\pi}{4}(0.04^4) = 0.64(10^{-6})\pi\ \text{m}^4$$

Here, $c = 0.04$ m. Thus

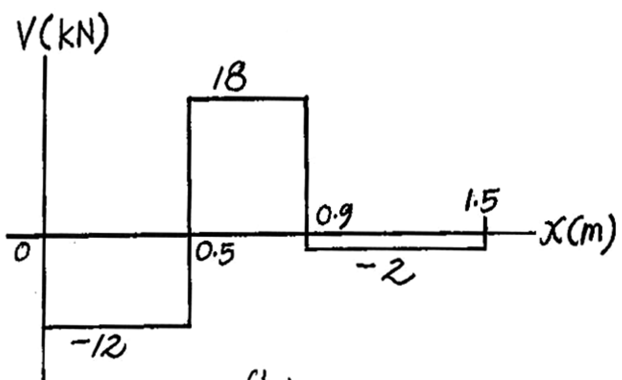
$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{6(10^3)(0.04)}{0.64(10^{-6})\pi}$$

$$= 119.37(10^6)\ \text{Pa}$$

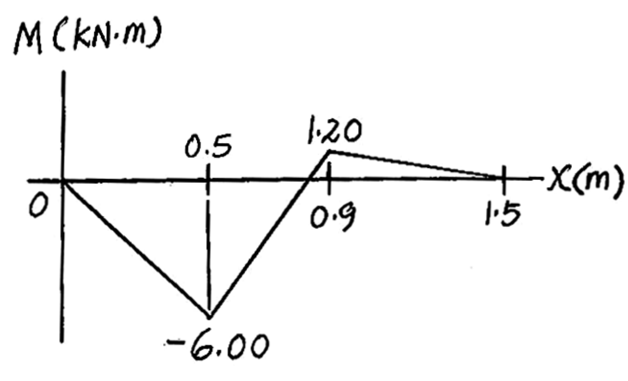
$$= 119\ \text{MPa}$$ **Ans.**



(a)

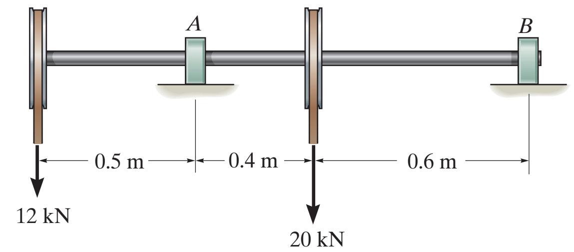


(b)



(C)

***6–92.** Determine the smallest allowable diameter of the shaft which is subjected to the concentrated forces. The journal bearings at *A* and *B* only support vertical forces. The allowable bending stress is $\sigma_{allow} = 150$ MPa.



The FBD of the shaft is shown in Fig. *a*.

The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $\left|M_{max}\right| = 6\text{ kN}\cdot\text{m}$.

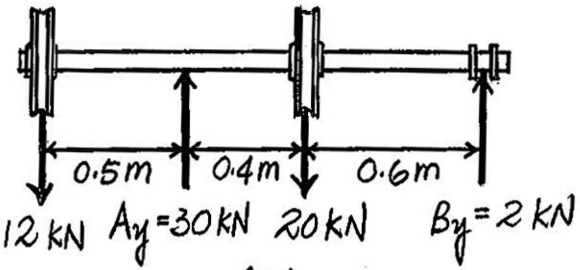
The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

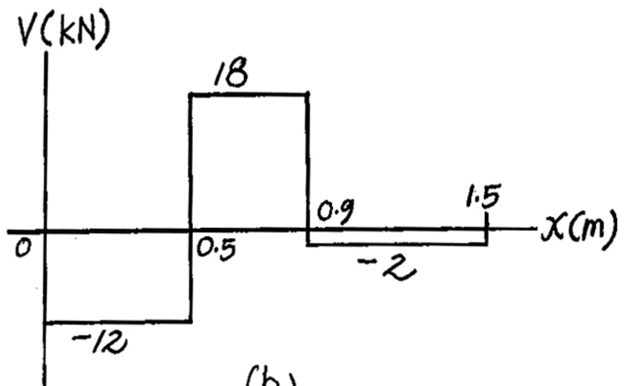
Here, $c = d/2$. Thus

$$\sigma_{allow} = \frac{M_{max}\,c}{I}; \qquad 150(10^6) = \frac{6(10^3)(d/2)}{\pi d^4/64}$$

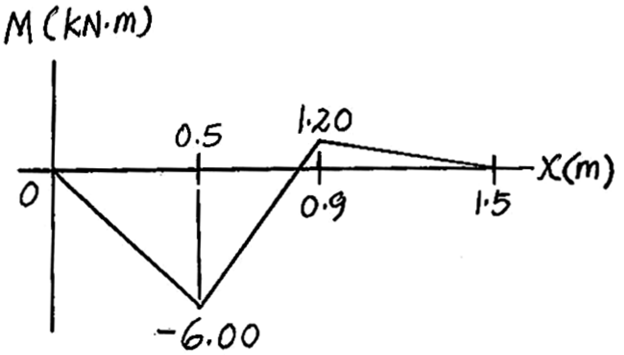
$$d = 0.07413\text{ m} = 74.13\text{ mm} = 75\text{ mm} \qquad \textbf{Ans.}$$



(a)

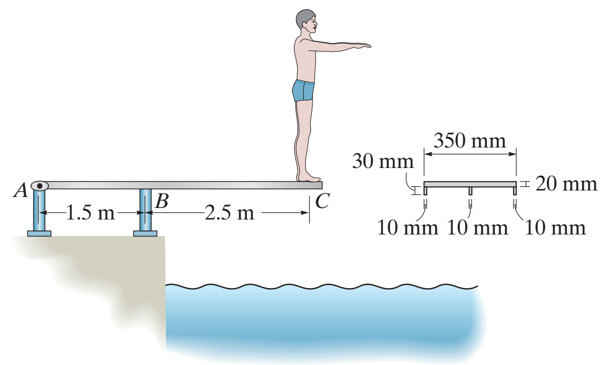


(b)

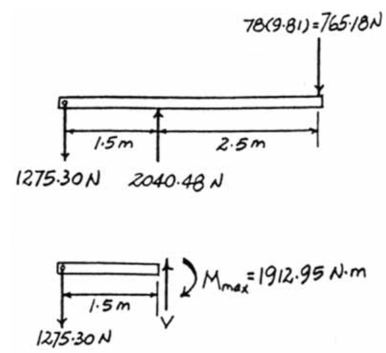


(c)

•6–93. The man has a mass of 78 kg and stands motionless at the end of the diving board. If the board has the cross section shown, determine the maximum normal strain developed in the board. The modulus of elasticity for the material is $E = 125$ GPa. Assume A is a pin and B is a roller.



Internal Moment: The maximum moment occurs at support B. The maximum moment is determined using the method of sections.



Section Property:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.01(0.35)(0.02) + 0.035(0.03)(0.03)}{0.35(0.02) + 0.03(0.03)} = 0.012848 \text{ m}$$

$$I = \frac{1}{12}(0.35)\left(0.02^3\right) + 0.35(0.02)(0.012848 - 0.01)^2$$

$$+ \frac{1}{12}(0.03)\left(0.03^3\right) + 0.03(0.03)(0.035 - 0.012848)^2$$

$$= 0.79925\left(10^{-6}\right) \text{ m}^4$$

Absolute Maximum Bending Stress: The maximum moment is $M_{max} = 1912.95 \text{ N} \cdot \text{m}$ as indicated on the FBD. Applying the flexure formula

$$\sigma_{max} = \frac{M_{max}\, c}{I}$$

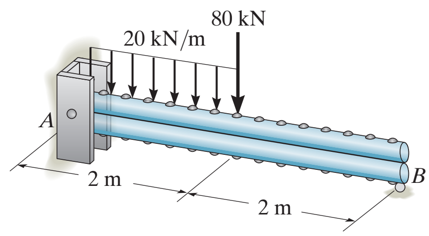
$$= \frac{1912.95(0.05 - 0.012848)}{0.79925(10^{-6})}$$

$$= 88.92 \text{ MPa}$$

Absolute Maximum Normal Strain: Applying Hooke's law, we have

$$\varepsilon_{max} = \frac{\sigma_{max}}{E} = \frac{88.92(10^6)}{125(10^9)} = 0.711\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans.}$$

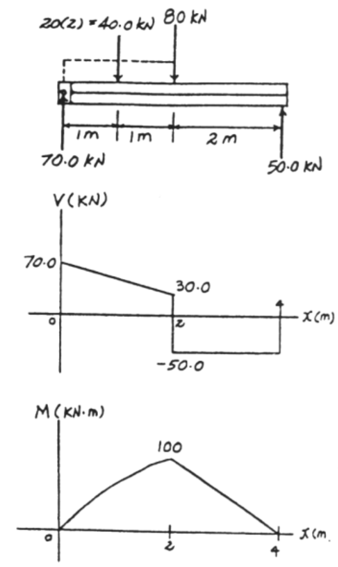
6–94. The two solid steel rods are bolted together along their length and support the loading shown. Assume the support at A is a pin and B is a roller. Determine the required diameter d of each of the rods if the allowable bending stress is $\sigma_{allow} = 130$ MPa.



Section Property:

$$I = 2\left[\frac{\pi}{4}\left(\frac{d}{2}\right)^4 + \frac{\pi}{4}d^2\left(\frac{d}{2}\right)^2\right] = \frac{5\pi}{32}d^4$$

Allowable Bending Stress: The maximum moment is $M_{max} = 100\ \text{kN}\cdot\text{m}$ as indicated on moment diagram. Applying the flexure formula

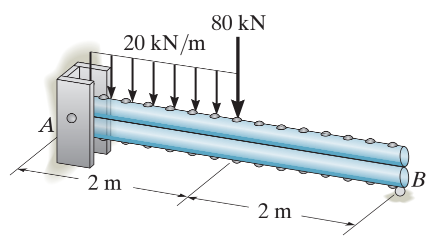


$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$130\left(10^6\right) = \frac{100(10^3)(d)}{\frac{5\pi}{32}d^4}$$

$$d = 0.1162\ \text{m} = 116\ \text{mm}$$ **Ans.**

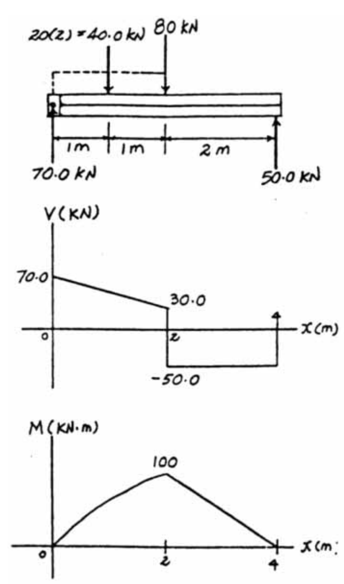
6–95. Solve Prob. 6–94 if the rods are rotated 90° so that both rods rest on the supports at A (pin) and B (roller).



Section Property:

$$I = 2\left[\frac{\pi}{4}\left(\frac{d}{2}\right)^4\right] = \frac{\pi}{32}d^4$$

Allowable Bending Stress: The maximum moment is $M_{max} = 100\ \text{kN}\cdot\text{m}$ as indicated on the moment diagram. Applying the flexure formula

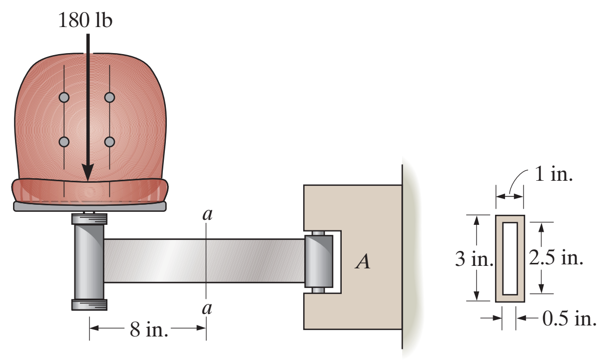


$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$130\left(10^6\right) = \frac{100(10^3)(d)}{\frac{\pi}{32}d^4}$$

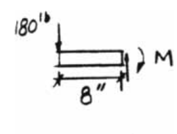
$$d = 0.1986\ \text{m} = 199\ \text{mm}$$ **Ans.**

***6–96.** The chair is supported by an arm that is hinged so it rotates about the vertical axis at A. If the load on the chair is 180 lb and the arm is a hollow tube section having the dimensions shown, determine the maximum bending stress at section a–a.



$$\circlearrowleft + \Sigma M = 0; \qquad M - 180(8) = 0$$

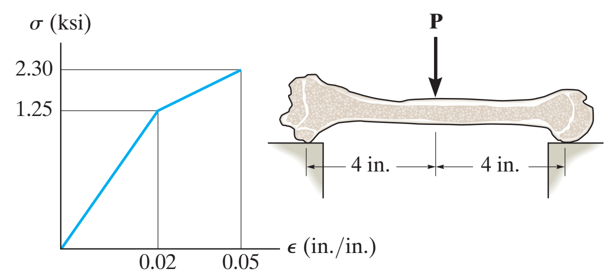
$$M = 1440 \text{ lb} \cdot \text{in.}$$



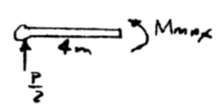
$$I_x = \frac{1}{12}(1)(3^3) - \frac{1}{12}(0.5)(2.5^3) = 1.59896 \text{ in}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{1440\,(1.5)}{1.59896} = 1.35 \text{ ksi}$$ **Ans.**

•6–97. A portion of the femur can be modeled as a tube having an inner diameter of 0.375 in. and an outer diameter of 1.25 in. Determine the maximum elastic static force P that can be applied to its center. Assume the bone to be roller supported at its ends. The $\sigma-\epsilon$ diagram for the bone mass is shown and is the same in tension as in compression.



$$I = \frac{1}{4}\pi\left[\left(\frac{1.25}{2}\right)^4 - \left(\frac{0.375}{2}\right)^4\right] = 0.11887 \text{ in}^4$$



$$M_{\max} = \frac{P}{2}(4) = 2P$$

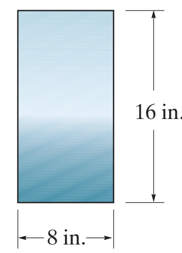
Require $\sigma_{\max} = 1.25$ ksi

$$\sigma_{\max} = \frac{Mc}{I}$$

$$1.25 = \frac{2P(1.25/2)}{0.11887}$$

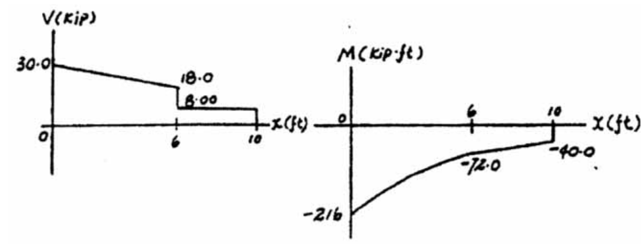
$$P = 0.119 \text{ kip} = 119 \text{ lb}$$ **Ans.**

6–98. If the beam in Prob. 6–18 has a rectangular cross section with a width of 8 in. and a height of 16 in., determine the absolute maximum bending stress in the beam.

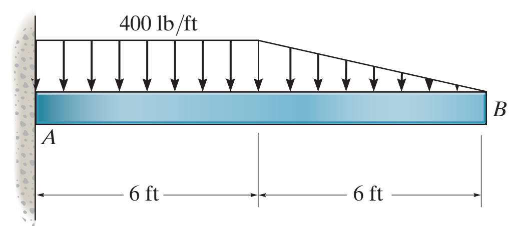


Absolute Maximum Bending Stress: The maximum moment is $M_{max} = 216$ kip · ft as indicated on moment diagram. Applying the flexure formula

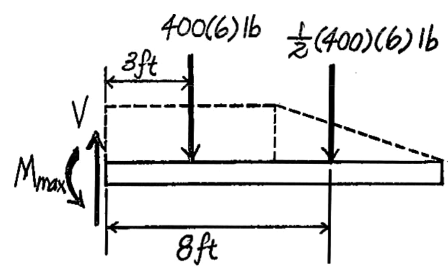
$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{216(12)(8)}{\frac{1}{12}(8)(16^3)} = 7.59 \text{ ksi} \qquad \textbf{Ans.}$$



6–99. If the beam has a square cross section of 6 in. on each side, determine the absolute maximum bending stress in the beam.



The maximum moment occurs at the fixed support A. Referring to the FBD shown in Fig. a,



(a)

$$\circlearrowleft +\Sigma M_A = 0; \qquad M_{max} - 400(6)(3) - \frac{1}{2}(400)(6)(8) = 0$$

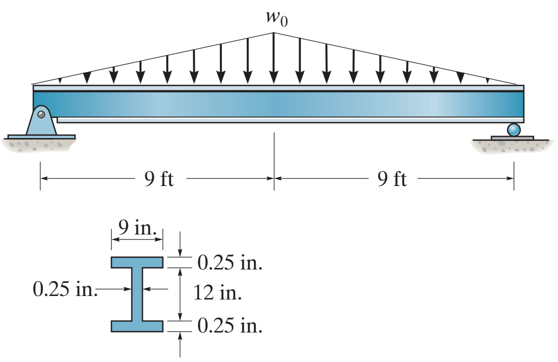
$$M_{max} = 16800 \text{ lb} \cdot \text{ft}$$

The moment of inertia of the about the neutral axis is $I = \frac{1}{12}(6)(6^3) = 108 \text{ in}^4$. Thus,

$$\sigma_{max} = \frac{Mc}{I} = \frac{16800(12)(3)}{108}$$

$$= 5600 \text{ psi} = 5.60 \text{ ksi} \qquad \textbf{Ans.}$$

***6–100.** The steel beam has the cross-sectional area shown. Determine the largest intensity of the distributed load w_0 that it can support so that the maximum bending stress in the beam does not exceed $\sigma_{allow} = 22$ ksi.



Support Reactions. The FBD of the beam is shown in Fig. *a*.

The shear and moment diagrams are shown in Fig. *a* and *b*, respectively. As indicated on the moment diagram, $M_{max} = 27w_o$.

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(9)(12.5^3) - \frac{1}{12}(8.75)(12^3)$$

$$= 204.84375 \text{ in}^4$$

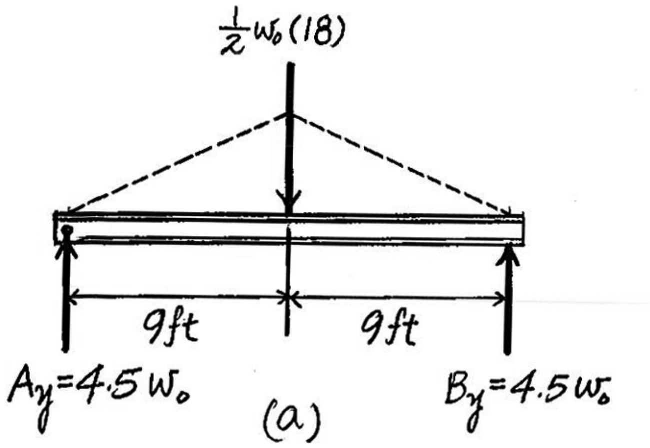
Here, $c = 6.25$ in. Thus,

$$\sigma_{allow} = \frac{M_{max}\,c}{I};$$

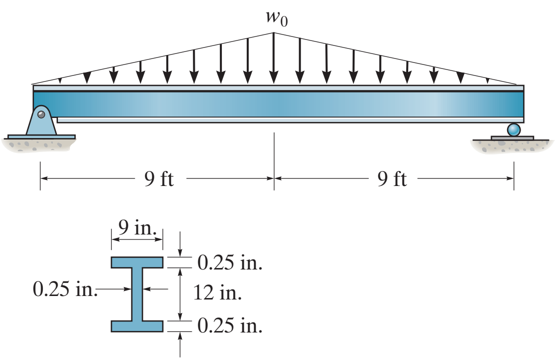
$$22(10^3) = \frac{(27w_o)(12)(6.25)}{204.84375}$$

$$w_o = 2\,225.46 \text{ lb/ft}$$

$$= 2.23 \text{ kip/ft} \qquad \textbf{Ans.}$$



•6–101. The steel beam has the cross-sectional area shown. If $w_0 = 2$ kip/ft, determine the maximum bending stress in the beam.



The FBD of the beam is shown in Fig. *a*

The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $M_{max} = 54$ kip · ft.

The moment of inertia of the I cross-section about the bending axis is

$$I = \frac{1}{12}(9)\left(12.5^3\right) - \frac{1}{12}(8.75)\left(12^3\right)$$

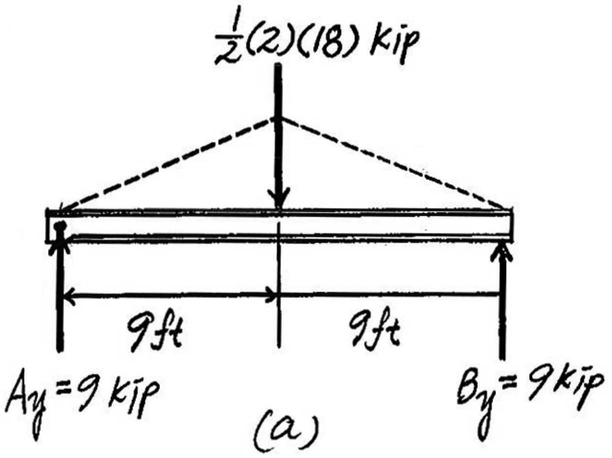
$$= 204.84375 \text{ in}^4$$

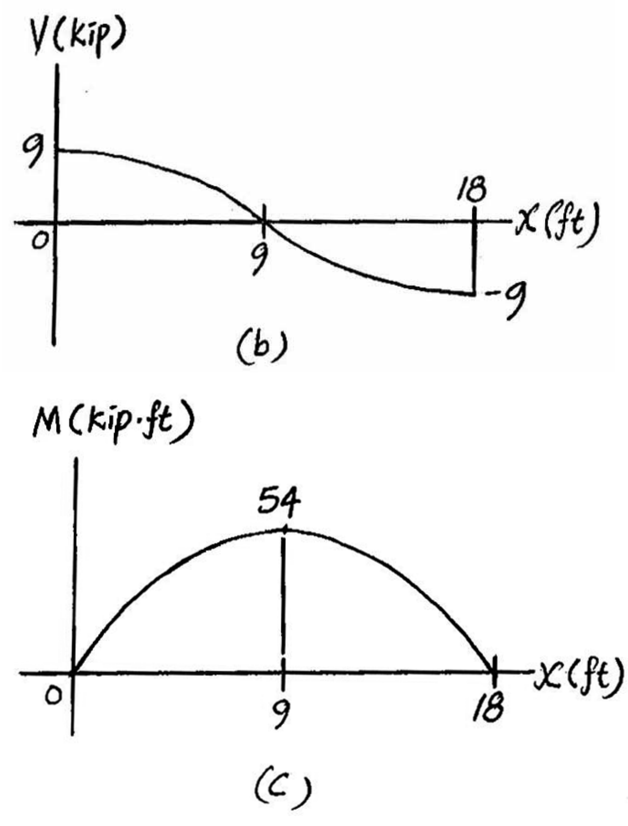
Here, $c = 6.25$ in. Thus

$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

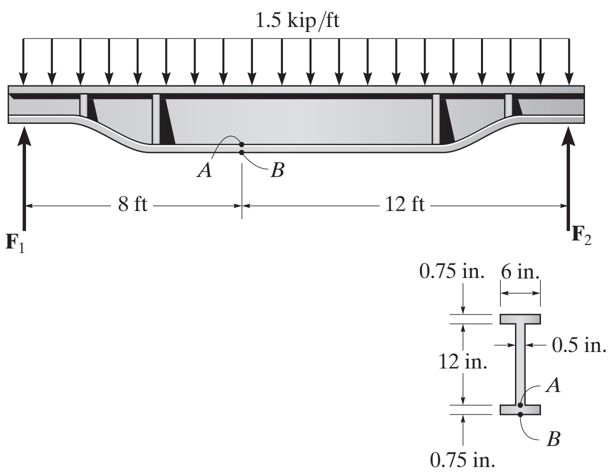
$$= \frac{54\,(12)(6.25)}{204.84375}$$

$$= 19.77 \text{ ksi} = 19.8 \text{ ksi}$$ **Ans.**





6–102. The bolster or main supporting girder of a truck body is subjected to the uniform distributed load. Determine the bending stress at points A and B.



Support Reactions: As shown on FBD.

Internal Moment: Using the method of sections.

$$+\Sigma M_{NA} = 0; \qquad M + 12.0(4) - 15.0(8) = 0$$

$$M = 72.0 \text{ kip} \cdot \text{ft}$$

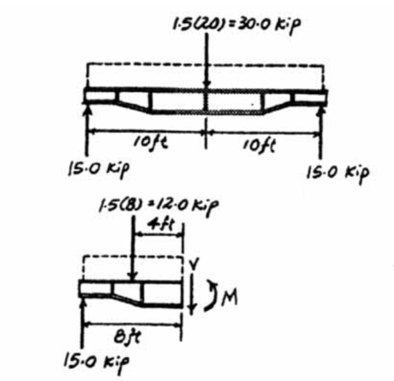
Section Property:

$$I = \frac{1}{12}(6)\left(13.5^3\right) - \frac{1}{12}(5.5)\left(12^3\right) = 438.1875 \text{ in}^4$$

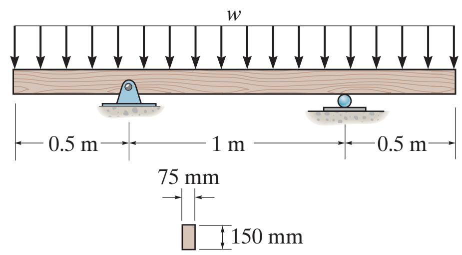
Bending Stress: Applying the flexure formula $\sigma = \dfrac{My}{I}$

$$\sigma_B = \frac{72.0(12)(6.75)}{438.1875} = 13.3 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_A = \frac{72.0(12)(6)}{438.1875} = 11.8 \text{ ksi} \qquad \textbf{Ans.}$$



6–103. Determine the largest uniform distributed load w that can be supported so that the bending stress in the beam does not exceed $\sigma_{\text{allow}} = 5\text{ MPa}$.



The FBD of the beam is shown in Fig. *a*

The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $|M_{\max}| = 0.125\,w$.

The moment of inertia of the cross-section is,

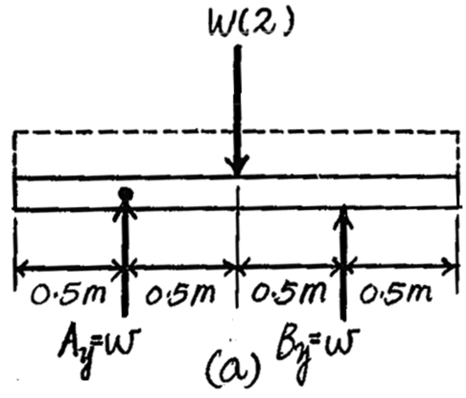
$$I = \frac{1}{12}(0.075)\left(0.15^3\right) = 21.09375\left(10^{-6}\right)\text{ m}^4$$

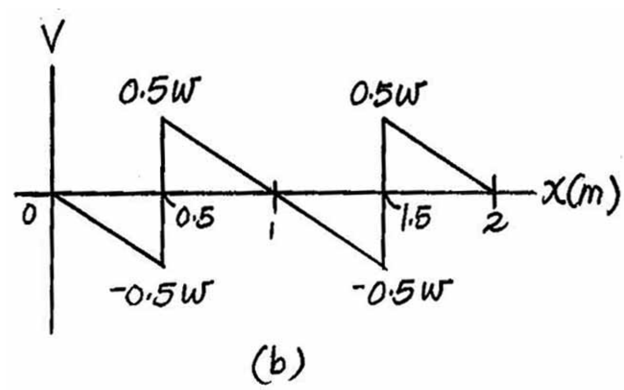
Here, $c = 0.075\ w$. Thus,

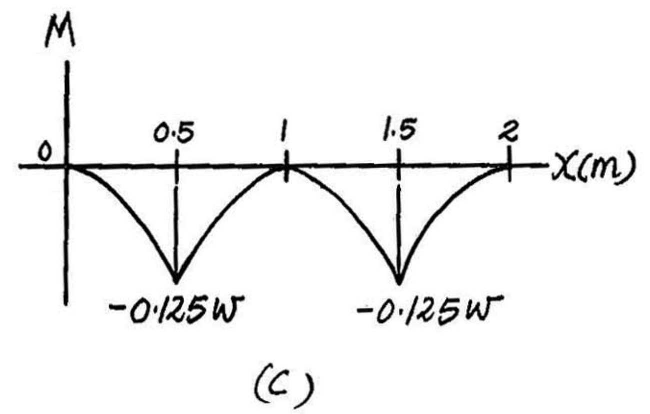
$$\sigma_{\text{allow}} = \frac{M_{\max}\,c}{I};$$

$$5\left(10^6\right) = \frac{0.125w(0.075)}{21.09375\left(10^{-6}\right)}$$

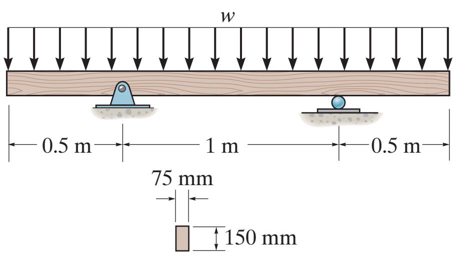
$$w = 11250\text{ N/m} = 11.25\text{ kN/m}$$ **Ans.**



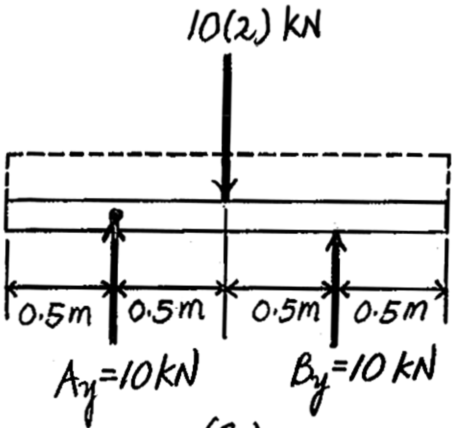




***6–104.** If $w = 10$ kN/m, determine the maximum bending stress in the beam. Sketch the stress distribution acting over the cross section.



Support Reactions. The FBD of the beam is shown in Fig. *a*



(a)

The shear and moment diagrams are shown in Figs. *b* and *c*, respectively. As indicated on the moment diagram, $|M_{max}| = 1.25$ kN · m.

The moment of inertia of the cross-section is

$$I = \frac{1}{12}(0.075)\left(0.15^3\right) = 21.09375\left(10^{-6}\right)\ \text{m}^4$$

Here, $c = 0.075$ m. Thus

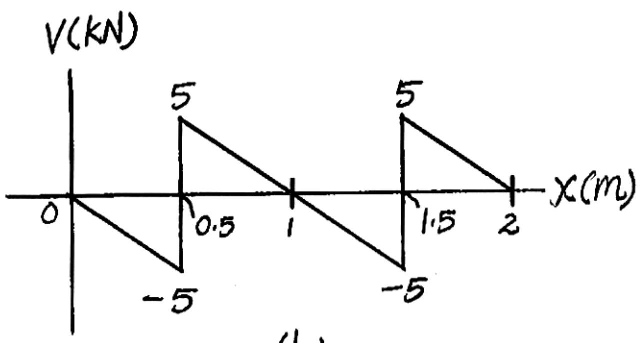
$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$= \frac{1.25\left(10^3\right)(0.075)}{21.09375\left(10^{-6}\right)}$$

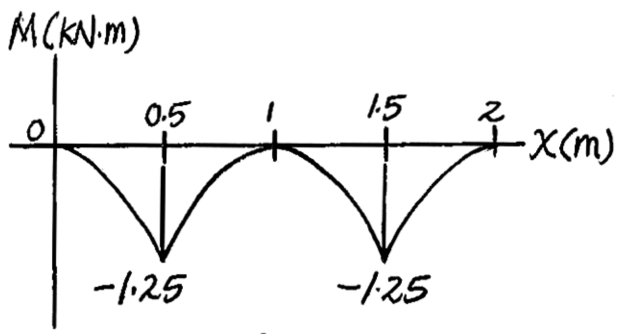
$$= 4.444\left(10^6\right)\ \text{Pa}$$

$$= 4.44\ \text{MPa}$$ **Ans.**

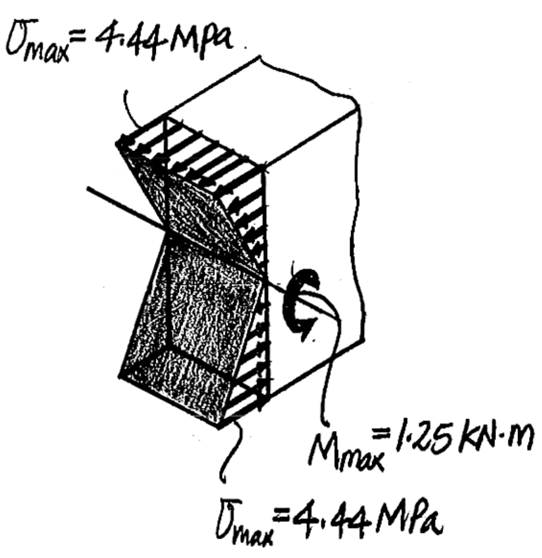
The bending stress distribution over the cross section is shown in Fig. *d*



(b)

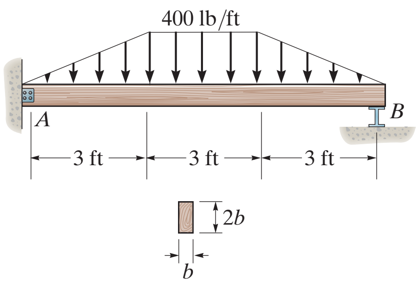


(c)



(d)

•6–105. If the allowable bending stress for the wood beam is $\sigma_{allow} = 150$ psi, determine the required dimension b to the nearest $\frac{1}{4}$ in. of its cross section. Assume the support at A is a pin and B is a roller.



The FBD of the beam is shown in Fig. *a*

The shear and moment diagrams are shown in Figs. *b* and *c*, respectively. As indicated on the moment diagram, $M_{max} = 3450 \text{ lb} \cdot \text{ft}$.

The moment of inertia of the cross section is

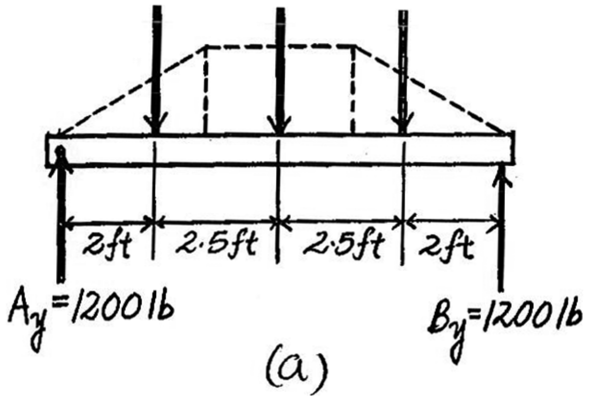
$$I = \frac{1}{12}(b)(2b)^3 = \frac{2}{3}b^4$$

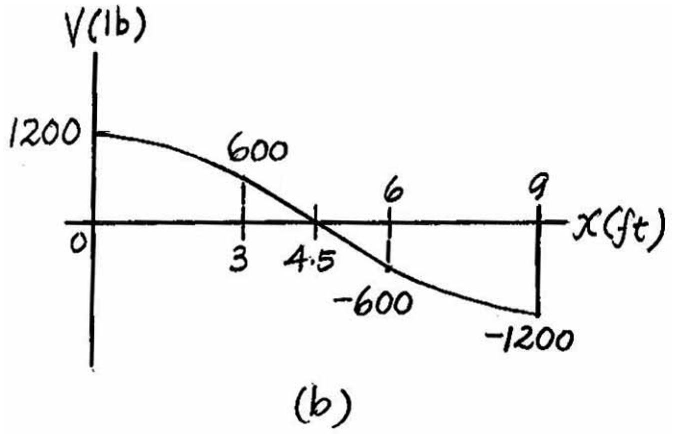
Here, $c = 2b/2 = b$. Thus,

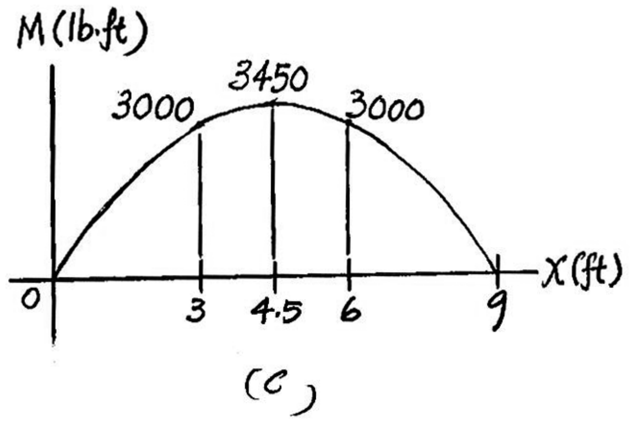
$$\sigma_{allow} = \frac{M_{max}\,c}{I};$$

$$150 = \frac{3450(12)(b)}{{}^2/_3\, b^4}$$

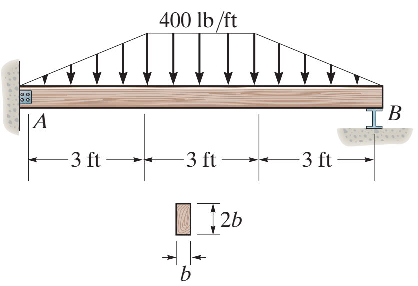
$$b = 7.453 \text{ in} = 7\frac{1}{2} \text{ in.}$$ **Ans.**







6–106. The wood beam has a rectangular cross section in the proportion shown. If $b = 7.5$ in., determine the absolute maximum bending stress in the beam.



The FBD of the beam is shown in Fig. *a*.

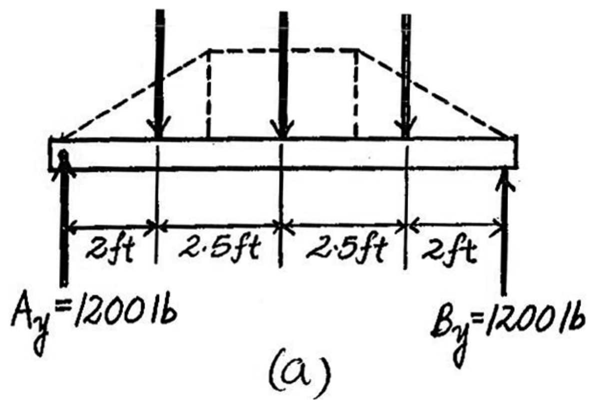
The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $M_{max} = 3450$ lb · ft.

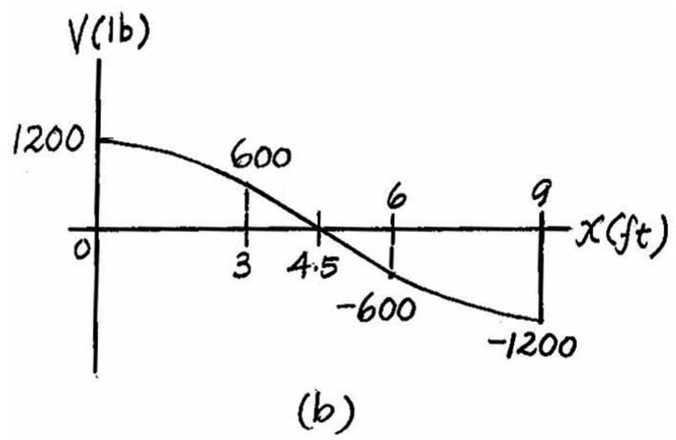
The moment of inertia of the cross-section is

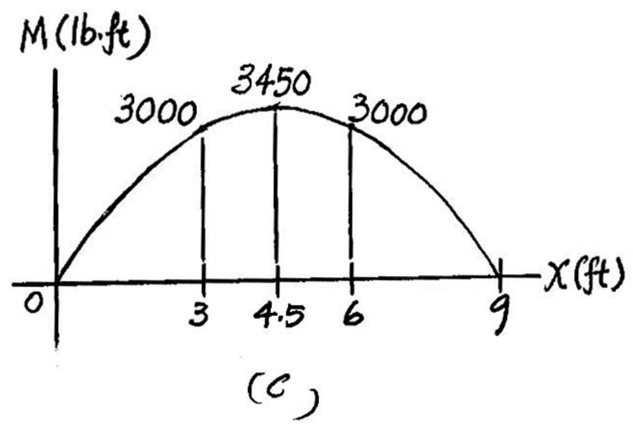
$$I = \frac{1}{12}(7.5)\left(15^3\right) = 2109.375 \text{ in}^4$$

Here, $c = \dfrac{15}{2} = 7.5$ in. Thus

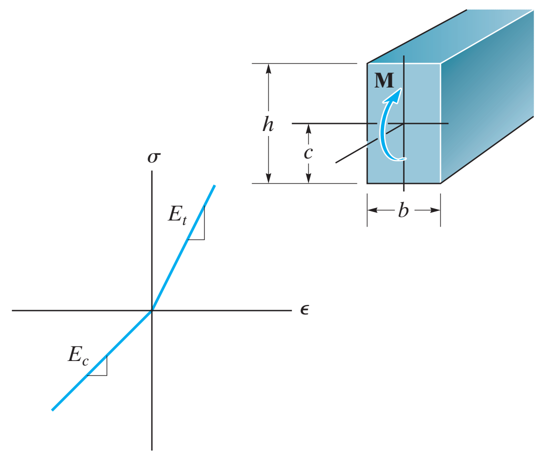
$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{3450(12)(7.5)}{2109.375} = 147 \text{ psi}$$ **Ans.**







6–107. A beam is made of a material that has a modulus of elasticity in compression different from that given for tension. Determine the location c of the neutral axis, and derive an expression for the maximum tensile stress in the beam having the dimensions shown if it is subjected to the bending moment **M**.



$$(\varepsilon_{max})_c = \frac{(\varepsilon_{max})_t\,(h-c)}{c}$$

$$(\sigma_{max})_c = E_c(\varepsilon_{max})_c = \frac{E_c(\varepsilon_{max})_t\,(h-c)}{c}$$

Location of neutral axis:

$$\overset{+}{\rightarrow}\Sigma F = 0; \qquad -\frac{1}{2}(h-c)(\sigma_{max})_c\,(b) + \frac{1}{2}(c)(\sigma_{max})_t\,(b) = 0$$

$$(h-c)(\sigma_{max})_c = c(\sigma_{max})_t \qquad [1]$$

$$(h-c)E_c\,(\varepsilon_{max})_t\,\frac{(h-c)}{c} = cE_t\,(\varepsilon_{max})_t\,; \qquad E_c\,(h-c)^2 = E_t c^2$$

Taking positive root:

$$\frac{c}{h-c} = \sqrt{\frac{E_c}{E_t}}$$

$$c = \frac{h\sqrt{\frac{E_c}{E_t}}}{1+\sqrt{\frac{E_c}{E_t}}} = \frac{h\sqrt{E_c}}{\sqrt{E_t}+\sqrt{E_c}} \qquad [2]\ \textbf{Ans.}$$

$$\Sigma M_{NA} = 0;$$

$$M = \left[\frac{1}{2}(h-c)(\sigma_{max})_c\,(b)\right]\left(\frac{2}{3}\right)(h-c) + \left[\frac{1}{2}(c)(\sigma_{max})_t(b)\right]\left(\frac{2}{3}\right)(c)$$

$$M = \frac{1}{3}(h-c)^2\,(b)(\sigma_{max})_c + \frac{1}{3}c^2 b(\sigma_{max})_t$$

From Eq. [1]. $(\sigma_{max})_c = \dfrac{c}{h-c}(\sigma_{max})_t$

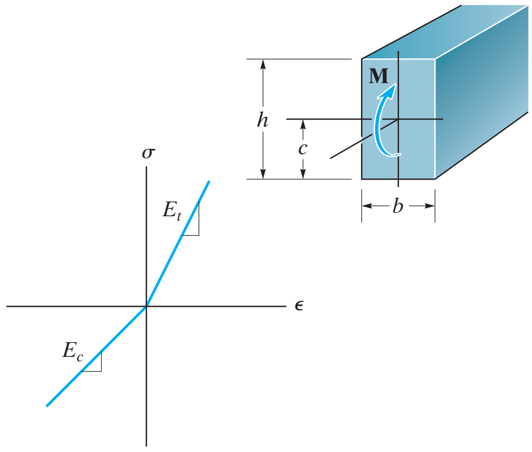
$$M = \frac{1}{3}(h-c)^2\,(b)\left(\frac{c}{h-c}\right)(\sigma_{max})_t + \frac{1}{3}c^2 b(\sigma_{max})_t$$

$$M = \frac{1}{3}bc(\sigma_{max})_t\,(h-c+c)\,; \qquad (\sigma_{max})_t = \frac{3M}{bhc}$$

From Eq. [2]

$$(\sigma_{max})_t = \frac{3M}{b\,h^2}\left(\frac{\sqrt{E_t}+\sqrt{E_c}}{\sqrt{E_c}}\right) \qquad \textbf{Ans.}$$

***6–108.** The beam has a rectangular cross section and is subjected to a bending moment **M**. If the material from which it is made has a different modulus of elasticity for tension and compression as shown, determine the location c of the neutral axis and the maximum compressive stress in the beam.



See the solution to Prob. 6–107

$$c = \frac{h\sqrt{E_c}}{\sqrt{E_t} + \sqrt{E_c}}$$ **Ans.**

Since

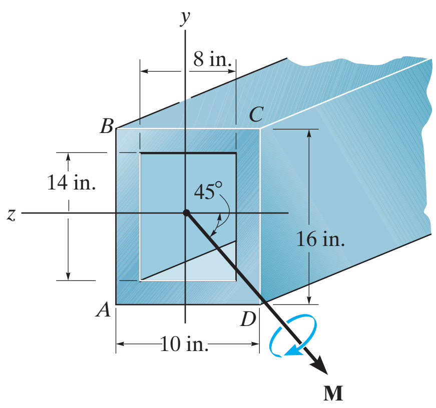
$$(\sigma_{\max})_c = \frac{c}{h - c}(\sigma_{\max})_t = \frac{h\sqrt{E_c}}{(\sqrt{E_t} + \sqrt{E_c})\left[h - \left(\frac{h\sqrt{E_c}}{\sqrt{E_t} + \sqrt{E_c}}\right)\right]}(\sigma_{\max})_t$$

$$(\sigma_{\max})_c = \frac{\sqrt{E_c}}{\sqrt{E_t}}(\sigma_{\max})_t$$

$$(\sigma_{\max})_c = \frac{\sqrt{E_c}}{\sqrt{E_t}}\left(\frac{3M}{bh^2}\right)\left(\frac{\sqrt{E_t} + \sqrt{E_c}}{\sqrt{E_c}}\right)$$

$$(\sigma_{\max})_c = \frac{3M}{bh^2}\left(\frac{\sqrt{E_t} + \sqrt{E_c}}{\sqrt{E_t}}\right)$$ **Ans.**

•6–109. The beam is subjected to a bending moment of $M = 20$ kip · ft directed as shown. Determine the maximum bending stress in the beam and the orientation of the neutral axis.



The y and z components of M are negative, Fig. a. Thus,

$$M_y = -20 \sin 45° = -14.14 \text{ kip} \cdot \text{ft}$$

$$M_z = -20 \cos 45° = -14.14 \text{ kip} \cdot \text{ft}.$$

The moments of inertia of the cross-section about the principal centroidal y and z axes are

$$I_y = \frac{1}{12}(16)\left(10^3\right) - \frac{1}{12}(14)\left(8^3\right) = 736 \text{ in}^4$$

$$I_z = \frac{1}{12}(10)\left(16^3\right) - \frac{1}{12}(8)\left(14^3\right) = 1584 \text{ in}^4$$

By inspection, the bending stress occurs at corners A and C are

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_{max} = \sigma_C = -\frac{-14.14(12)(8)}{1584} + \frac{-14.14(12)(-5)}{736}$$

$$= 2.01 \text{ ksi} \quad \text{(T)}$$ **Ans.**

$$\sigma_{max} = \sigma_A = -\frac{-14.14(12)(-8)}{1584} + \frac{-14.14(12)(5)}{736}$$

$$= -2.01 \text{ ksi} = 2.01 \text{ ksi (C)}$$ **Ans.**

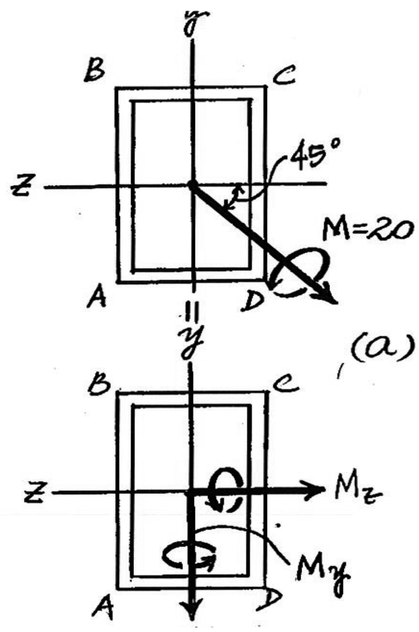
Here, $\theta = 180° + 45° = 225°$

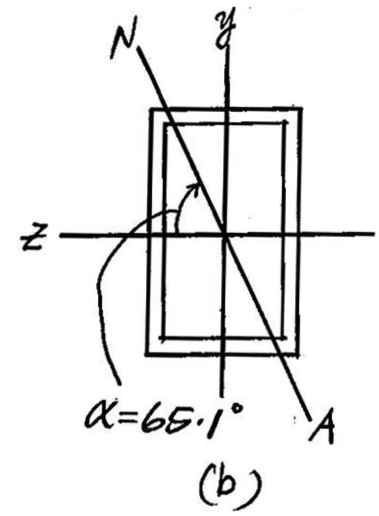
$$\tan \alpha = \frac{I_z}{I_y} \tan \theta$$

$$\tan \alpha = \frac{1584}{736} \tan 225°$$

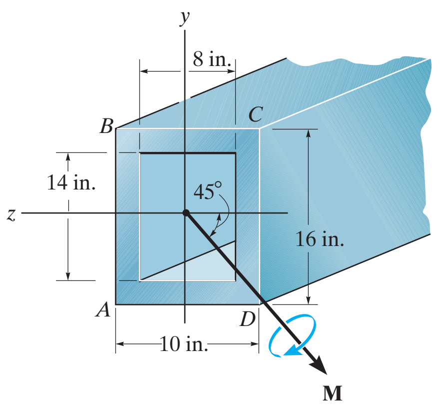
$$\alpha = 65.1°$$ **Ans.**

The orientation of neutral axis is shown in Fig. b.





6–110. Determine the maximum magnitude of the bending moment **M** that can be applied to the beam so that the bending stress in the member does not exceed 12 ksi.



The y and z components of M are negative, Fig. a. Thus,

$$M_y = -M \sin 45° = -0.7071\, M$$

$$M_z = -M \cos 45° = -0.7071\, M$$

The moments of inertia of the cross-section about principal centroidal y and z axes are

$$I_y = \frac{1}{12}(16)\left(10^3\right) - \frac{1}{12}(14)\left(8^3\right) = 736 \text{ in}^4$$

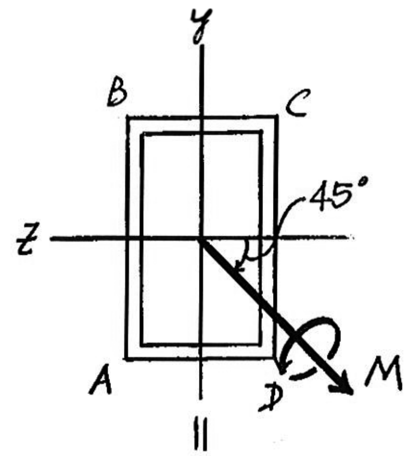
$$I_z = \frac{1}{12}(10)\left(16^3\right) - \frac{1}{12}(8)\left(14^3\right) = 1584 \text{ in}^4$$

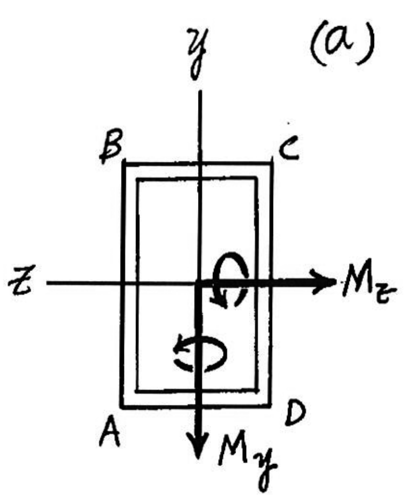
By inspection, the maximum bending stress occurs at corners A and C. Here, we will consider corner C.

$$\sigma_C = \sigma_{\text{allow}} = -\frac{M_z\, y_c}{I_z} + \frac{M_y z_c}{I_y}$$

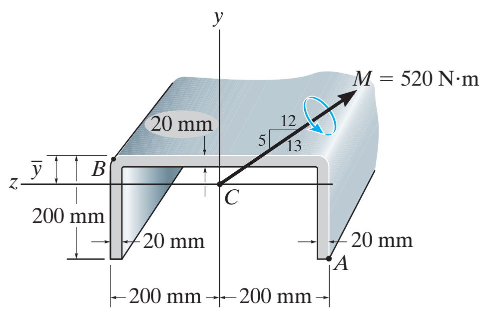
$$12 = -\frac{-0.7071\, M\,(12)(8)}{1584} + \frac{-0.7071\, M(12)(-5)}{736}$$

$$M = 119.40 \text{ kip} \cdot \text{ft} = 119 \text{ kip} \cdot \text{ft}$$ **Ans.**



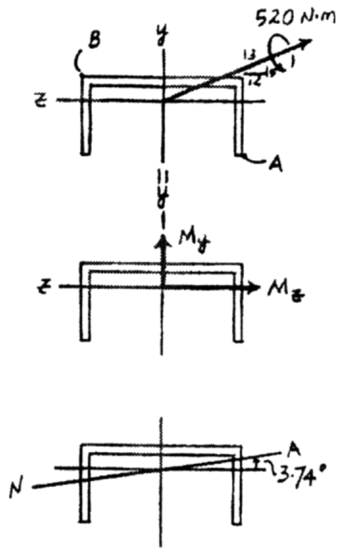


6–111. If the resultant internal moment acting on the cross section of the aluminum strut has a magnitude of $M = 520\ \text{N}\cdot\text{m}$ and is directed as shown, determine the bending stress at points A and B. The location $\bar{y}$ of the centroid C of the strut's cross-sectional area must be determined. Also, specify the orientation of the neutral axis.



Internal Moment Components:

$$M_z = -\frac{12}{13}(520) = -480\ \text{N}\cdot\text{m} \qquad M_y = \frac{5}{13}(520) = 200\ \text{N}\cdot\text{m}$$



Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.01(0.4)(0.02) + 2[(0.110)(0.18)(0.02)]}{0.4(0.02) + 2(0.18)(0.02)}$$

$$= 0.057368\ \text{m} = 57.4\ \text{mm} \qquad \textbf{Ans.}$$

$$I_z = \frac{1}{12}(0.4)\left(0.02^3\right) + (0.4)(0.02)(0.057368 - 0.01)^2$$

$$+ \frac{1}{12}(0.04)\left(0.18^3\right) + 0.04(0.18)(0.110 - 0.057368)^2$$

$$= 57.6014\left(10^{-6}\right)\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.2)\left(0.4^3\right) - \frac{1}{12}(0.18)\left(0.36^3\right) = 0.366827\left(10^{-3}\right)\ \text{m}^4$$

Maximum Bending Stress: Applying the flexure formula for biaxial at points A and B

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-480(-0.142632)}{57.6014(10^{-6})} + \frac{200(-0.2)}{0.366827(10^{-3})}$$

$$= -1.298\ \text{MPa} = 1.30\ \text{MPa}\ (\textbf{C}) \qquad \textbf{Ans.}$$

$$\sigma_B = -\frac{-480(0.057368)}{57.6014(10^{-6})} + \frac{200(0.2)}{0.366827(10^{-3})}$$

$$= 0.587\ \text{MPa}\ (\textbf{T}) \qquad \textbf{Ans.}$$

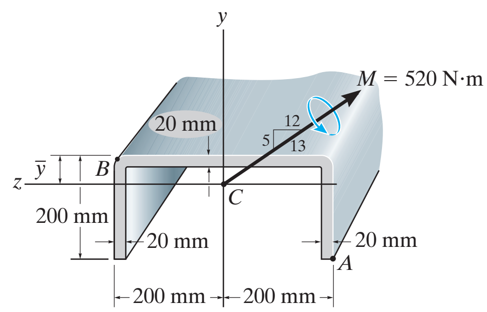
Orientation of Neutral Axis:

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\tan\alpha = \frac{57.6014(10^{-6})}{0.366827(10^{-3})}\tan(-22.62^\circ)$$

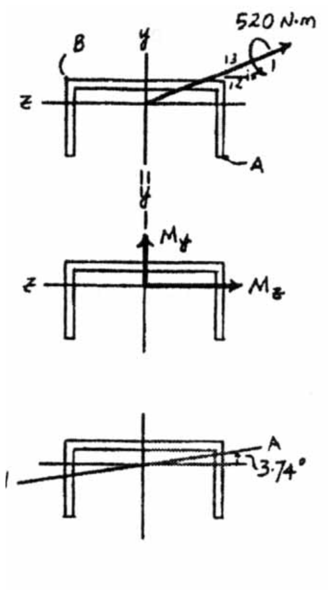
$$\alpha = -3.74^\circ \qquad \textbf{Ans.}$$

***6–112.** The resultant internal moment acting on the cross section of the aluminum strut has a magnitude of $M = 520\ \text{N}\cdot\text{m}$ and is directed as shown. Determine maximum bending stress in the strut. The location $\bar{y}$ of the centroid C of the strut's cross-sectional area must be determined. Also, specify the orientation of the neutral axis.



Internal Moment Components:

$$M_z = -\frac{12}{13}(520) = -480\ \text{N}\cdot\text{m} \qquad M_y = \frac{5}{13}(520) = 200\ \text{N}\cdot\text{m}$$



Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.01(0.4)(0.02) + 2[(0.110)(0.18)(0.02)]}{0.4(0.02) + 2(0.18)(0.02)}$$

$$= 0.057368\ \text{m} = 57.4\ \text{mm} \qquad \textbf{Ans.}$$

$$I_z = \frac{1}{12}(0.4)\left(0.02^3\right) + (0.4)(0.02)(0.057368 - 0.01)^2$$

$$+ \frac{1}{12}(0.04)\left(0.18^3\right) + 0.04(0.18)(0.110 - 0.057368)^2$$

$$= 57.6014\left(10^{-6}\right)\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.2)\left(0.4^3\right) - \frac{1}{12}(0.18)\left(0.36^3\right) = 0.366827\left(10^{-3}\right)\ \text{m}^4$$

Maximum Bending Stress: By inspection, the maximum bending stress can occur at either point A or B. Applying the flexure formula for biaxial bending at points A and B

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-480(-0.142632)}{57.6014(10^{-6})} + \frac{200(-0.2)}{0.366827(10^{-3})}$$

$$= -1.298\ \text{MPa} = 1.30\ \text{MPa (C) (Max)} \qquad \textbf{Ans.}$$

$$\sigma_B = -\frac{-480(0.057368)}{57.6014(10^{-6})} + \frac{200(0.2)}{0.366827(10^{-3})}$$

$$= 0.587\ \text{MPa (T)}$$

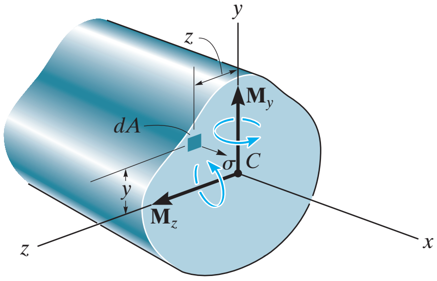
Orientation of Neutral Axis:

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\tan\alpha = \frac{57.6014(10^{-6})}{0.366827(10^{-3})}\tan(-22.62^\circ)$$

$$\alpha = -3.74^\circ \qquad \textbf{Ans.}$$

6–113. Consider the general case of a prismatic beam subjected to bending-moment components $\mathbf{M}_y$ and $\mathbf{M}_z$, as shown, when the x, y, z axes pass through the centroid of the cross section. If the material is linear-elastic, the normal stress in the beam is a linear function of position such that $\sigma = a + by + cz$. Using the equilibrium conditions $0 = \int_A \sigma\, dA$, $M_y = \int_A z\sigma\, dA$, $M_z = \int_A -y\sigma\, dA$, determine the constants a, b, and c, and show that the normal stress can be determined from the equation $\sigma = [-(M_z I_y + M_y I_{yz})y + (M_y I_z + M_z I_{yz})z]/(I_y I_z - I_{yz}^2)$, where the moments and products of inertia are defined in Appendix A.



Equilibrium Condition: $\sigma_x = a + by + cz$

$$0 = \int_A \sigma_x\, dA$$

$$0 = \int_A (a + by + cz)\, dA$$

$$0 = a\int_A dA + b\int_A y\, dA + c\int_A z\, dA \qquad \textbf{[1]}$$

$$M_y = \int_A z\,\sigma_x\, dA$$

$$= \int_A z(a + by + cz)\, dA$$

$$= a\int_A z\, dA + b\int_A yz\, dA + c\int_A z^2\, dA \qquad \textbf{[2]}$$

$$M_z = \int_A -y\,\sigma_x\, dA$$

$$= \int_A -y(a + by + cz)\, dA$$

$$= -a\int_A y dA - b\int_A y^2\, dA - c\int_A yz\, dA \qquad \textbf{[3]}$$

Section Properties: The integrals are defined in Appendix A. Note that $\int_A y\, dA = \int_A z\, dA = 0$. Thus,

From Eq. [1] $\quad Aa = 0$

From Eq. [2] $\quad M_y = bI_{yz} + cI_y$

From Eq. [3] $\quad M_z = -bI_z - cI_{yz}$

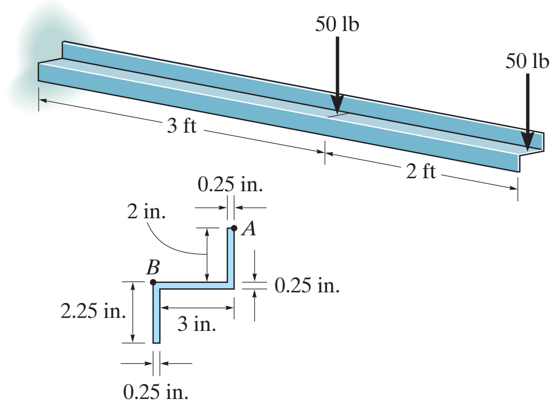
Solving for *a, b, c*:

$$a = 0 \text{ (Since } A \neq 0)$$

$$b = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right) \qquad c = \frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2}$$

Thus, $$\sigma_x = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right)y + \left(\frac{M_y I_y + M_z I_{yz}}{I_y I_z - I_{yz}^2}\right)z \qquad \textbf{(Q.E.D.)}$$

6–114. The cantilevered beam is made from the *Z*-section having the cross-section shown. If it supports the two loadings, determine the bending stress at the wall in the beam at point *A*. Use the result of Prob. 6–113.



$$(M_y)_{max} = 50(3) + 50(5) = 400 \text{ lb} \cdot \text{ft} = 4.80(10^3) \text{lb} \cdot \text{in.}$$

$$I_y = \frac{1}{12}(3.25)(0.25)^3 + 2\left[\frac{1}{12}(0.25)(2)^3 + (0.25)(2)(1.125)^2\right] = 1.60319 \text{ in}^4$$

$$I_z = \frac{1}{12}(0.25)(3.25)^3 + 2\left[\frac{1}{12}(2)(0.25)^3 + (0.25)(2)(1.5)^2\right] = 2.970378 \text{ in}^4$$

$$I_{yz} = 2[1.5(1.125)(2)(0.25)] = 1.6875 \text{ in}^4$$

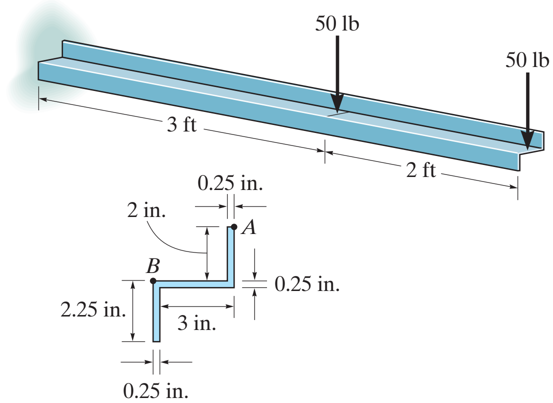
Using the equation developed in Prob. 6-113.

$$\sigma = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right)y + \left(\frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2}\right)z$$

$$\sigma_A = \frac{\{-[0 + (4.80)(10^3)(1.6875)](1.625) + [(4.80)(10^3)(2.970378) + 0](2.125)\}}{[1.60319(2.970378) - (1.6875)^2]}$$

$$= 8.95 \text{ ksi}$$ **Ans.**

6–115. The cantilevered beam is made from the *Z*-section having the cross-section shown. If it supports the two loadings, determine the bending stress at the wall in the beam at point *B*. Use the result of Prob. 6–113.



$$(M_y)_{max} = 50(3) + 50(5) = 400 \text{ lb} \cdot \text{ft} = 4.80(10^3) \text{lb} \cdot \text{in.}$$

$$I_y = \frac{1}{12}(3.25)(0.25)^3 + 2\left[\frac{1}{12}(0.25)(2)^3 + (0.25)(2)(1.125)^2\right] = 1.60319 \text{ in}^4$$

$$I_z = \frac{1}{12}(0.25)(3.25)^3 + 2\left[\frac{1}{12}(2)(0.25)^3 + (0.25)(2)(1.5)^2\right] = 2.970378 \text{ in}^4$$

$$I_{yz} = 2[1.5(1.125)(2)(0.25)] = 1.6875 \text{ in}^4$$

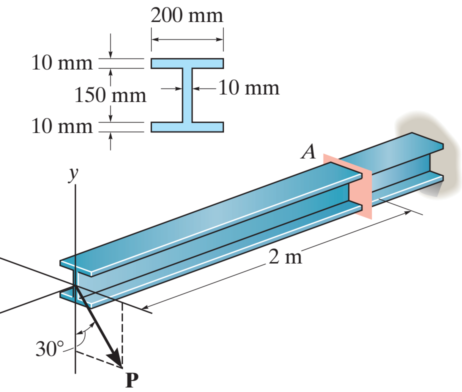
Using the equation developed in Prob. 6-113.

$$\sigma = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right)y + \left(\frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2}\right)z$$

$$\sigma_B = \frac{-[0 + (4.80)(10^3)(1.6875)](-1.625) + [(4.80)(10^3)(2.976378) + 0](0.125)}{[(1.60319)(2.970378) - (1.6875)^2]}$$

$$= 7.81 \text{ ksi}$$ **Ans.**

***6–116.** The cantilevered wide-flange steel beam is subjected to the concentrated force **P** at its end. Determine the largest magnitude of this force so that the bending stress developed at A does not exceed $\sigma_{\text{allow}} = 180$ MPa.



Internal Moment Components: Using method of section

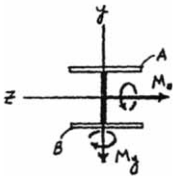
$$\Sigma M_z = 0; \qquad M_z + P\cos 30°(2) = 0 \qquad M_z = -1.732P$$

$$\Sigma M_y = 0; \qquad M_y + P\sin 30°(2) = 0 \qquad M_y = -1.00P$$

Section Properties:

$$I_z = \frac{1}{12}(0.2)\left(0.17^3\right) - \frac{1}{12}(0.19)\left(0.15^3\right) = 28.44583(10^{-6})\ \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)\left(0.2^3\right)\right] + \frac{1}{12}(0.15)\left(0.01^3\right) = 13.34583(10^{-6})\ \text{m}^4$$

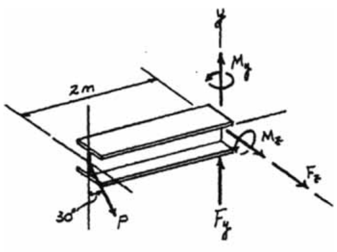


Allowable Bending Stress: By inspection, maximum bending stress occurs at points A and B. Applying the flexure formula for biaxial bending at point A.

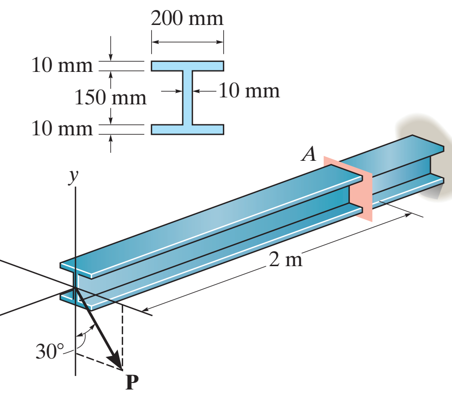
$$\sigma_A = \sigma_{\text{allow}} = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$180\left(10^6\right) = -\frac{(-1.732P)(0.085)}{28.44583(10^{-6})} + \frac{-1.00P(-0.1)}{13.34583(10^{-6})}$$

$$P = 14208\ \text{N} = 14.2\ \text{kN}$$ **Ans.**



•6–117. The cantilevered wide-flange steel beam is subjected to the concentrated force of $P = 600$ N at its end. Determine the maximum bending stress developed in the beam at section A.



Internal Moment Components: Using method of sections

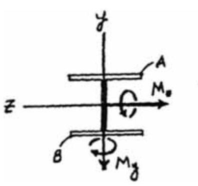
$$\Sigma M_z = 0; \qquad M_z + 600\cos 30°(2) = 0 \qquad M_z = -1039.23\ \text{N}\cdot\text{m}$$

$$\Sigma M_y = 0; \qquad M_y + 600\sin 30°(2) = 0; \qquad M_y = -600.0\ \text{N}\cdot\text{m}$$

Section Properties:

$$I_z = \frac{1}{12}(0.2)\left(0.17^3\right) - \frac{1}{12}(0.19)\left(0.15^3\right) = 28.44583(10^{-6})\ \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)\left(0.2^3\right)\right] + \frac{1}{12}(0.15)\left(0.01^3\right) = 13.34583(10^{-6})\ \text{m}^4$$

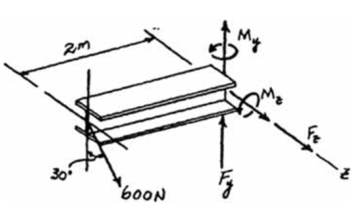


Maximum Bending Stress: By inspection, maximum bending stress occurs at A and B. Applying the flexure formula for biaxial bending at point A

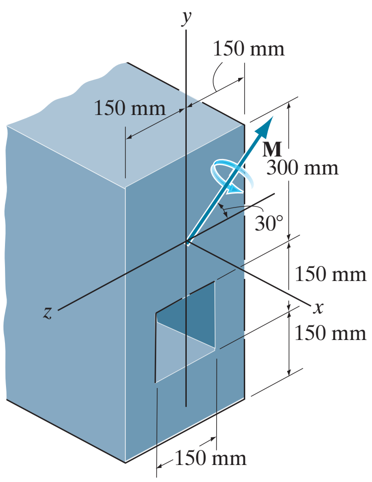
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-1039.32(0.085)}{28.44583(10^{-6})} + \frac{-600.0(-0.1)}{13.34583(10^{-6})}$$

$$= 7.60\ \text{MPa (T)} \qquad \textbf{(Max)}$$ **Ans.**



6–118. If the beam is subjected to the internal moment of $M = 1200\ \text{kN}\cdot\text{m}$, determine the maximum bending stress acting on the beam and the orientation of the neutral axis.



Internal Moment Components: The y component of M is positive since it is directed towards the positive sense of the y axis, whereas the z component of **M**, which is directed towards the negative sense of the z axis, is negative, Fig. *a*. Thus,

$$M_y = 1200 \sin 30° = 600\ \text{kN}\cdot\text{m}$$

$$M_z = -1200 \cos 30° = -1039.23\ \text{kN}\cdot\text{m}$$

Section Properties: The location of the centroid of the cross-section is given by

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.3(0.6)(0.3) - 0.375(0.15)(0.15)}{0.6(0.3) - 0.15(0.15)} = 0.2893\ \text{m}$$

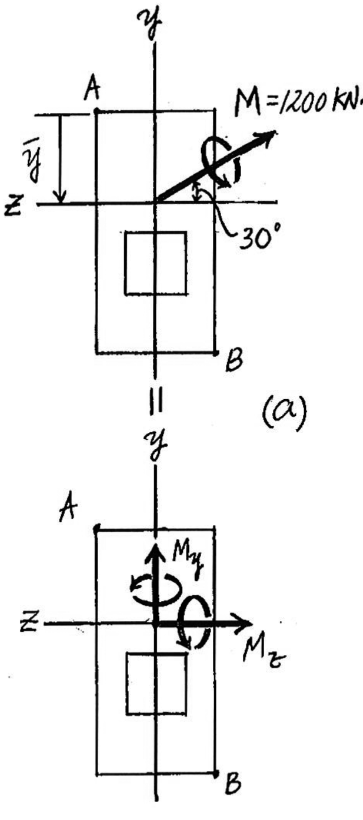
The moments of inertia of the cross section about the principal centroidal y and z axes are

$$I_y = \frac{1}{12}(0.6)\left(0.3^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right) = 1.3078\left(10^{-3}\right)\ \text{m}^4$$

$$I_z = \frac{1}{12}(0.3)\left(0.6^3\right) + 0.3(0.6)(0.3 - 0.2893)^2$$

$$- \left[\frac{1}{12}(0.15)\left(0.15^3\right) + 0.15(0.15)(0.375 - 0.2893)^2\right]$$

$$= 5.2132\left(10^{-3}\right)\text{m}^4$$



Bending Stress: By inspection, the maximum bending stress occurs at either corner A or B.

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{\left[-1039.23\left(10^3\right)\right](0.2893)}{5.2132\left(10^{-3}\right)} + \frac{600\left(10^3\right)(0.15)}{1.3078\left(10^{-3}\right)}$$

$$= 126\ \text{MPa (T)}$$

$$\sigma_B = -\frac{\left[-1039.23\left(10^3\right)\right](-0.3107)}{5.2132\left(10^{-3}\right)} + \frac{600\left(10^3\right)(-0.15)}{1.3078\left(10^{-3}\right)}$$

$$= -131\ \text{MPa} = 131\ \text{MPa (C)(Max.)}$$ **Ans.**

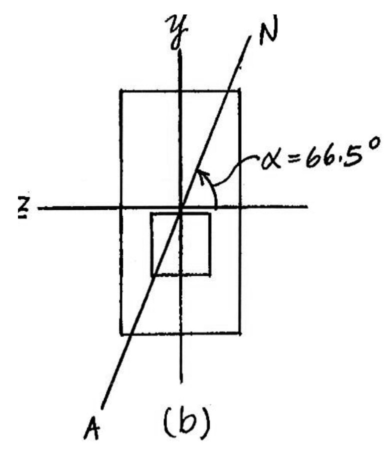
Orientation of Neutral Axis: Here, $\theta = -30°$.

$$\tan \alpha = \frac{I_z}{I_y}\tan\theta$$

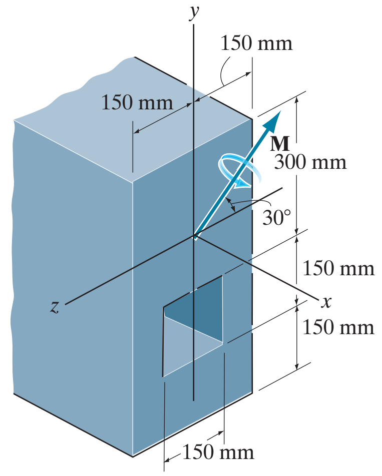
$$\tan \alpha = \frac{5.2132\left(10^{-3}\right)}{1.3078\left(10^{-3}\right)}\tan(-30°)$$

$$\alpha = -66.5°$$ **Ans.**

The orientation of the neutral axis is shown in Fig. *b*.



6–119. If the beam is made from a material having an allowable tensile and compressive stress of $(\sigma_{\text{allow}})_t = 125$ MPa and $(\sigma_{\text{allow}})_c = 150$ MPa, respectively, determine the maximum allowable internal moment **M** that can be applied to the beam.



Internal Moment Components: The y component of **M** is positive since it is directed towards the positive sense of the y axis, whereas the z component of **M**, which is directed towards the negative sense of the z axis, is negative, Fig. a. Thus,

$$M_y = M \sin 30° = 0.5M$$

$$M_z = -M \cos 30° = -0.8660M$$

Section Properties: The location of the centroid of the cross section is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.3(0.6)(0.3) - 0.375(0.15)(0.15)}{0.6(0.3) - 0.15(0.15)} = 0.2893 \text{ m}$$

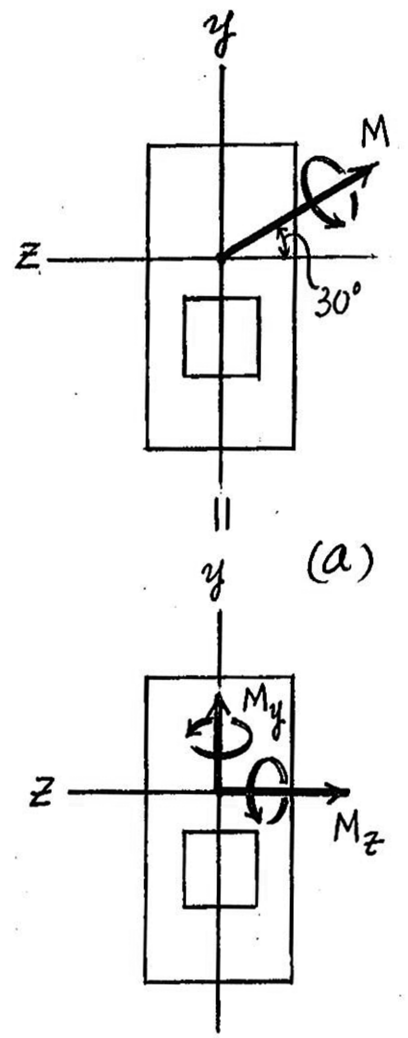
The moments of inertia of the cross section about the principal centroidal y and z axes are

$$I_y = \frac{1}{12}(0.6)\left(0.3^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right) = 1.3078\left(10^{-3}\right) \text{ m}^4$$

$$I_z = \frac{1}{12}(0.3)\left(0.6^3\right) + 0.3(0.6)(0.3 - 0.2893)^2$$

$$- \left[\frac{1}{12}(0.15)\left(0.15^3\right) + 0.15(0.15)(0.375 - 0.2893)^2\right]$$

$$= 5.2132\left(10^{-3}\right)\text{m}^4$$



Bending Stress: By inspection, the maximum bending stress can occur at either corner A or B. For corner A which is in tension,

$$\sigma_A = (\sigma_{\text{allow}})_t = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$125\left(10^6\right) = -\frac{(-0.8660M)(0.2893)}{5.2132\left(10^{-3}\right)} + \frac{0.5M(0.15)}{1.3078\left(10^{-3}\right)}$$

$$M = 1185\,906.82 \text{ N} \cdot \text{m} = 1186 \text{ kN} \cdot \text{m (controls)}$$ **Ans.**

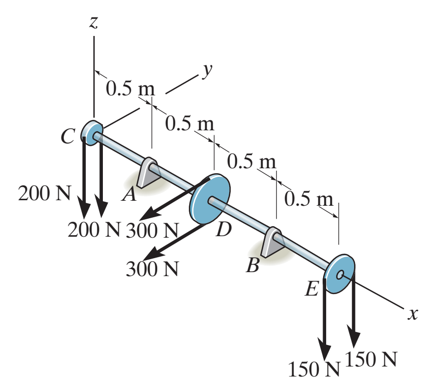
For corner B which is in compression,

$$\sigma_B = (\sigma_{\text{allow}})_c = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y}$$

$$-150\left(10^6\right) = -\frac{(-0.8660M)(-0.3107)}{5.2132\left(10^{-3}\right)} + \frac{0.5M(-0.15)}{1.3078\left(10^{-3}\right)}$$

$$M = 1376\,597.12 \text{ N} \cdot \text{m} = 1377 \text{ kN} \cdot \text{m}$$

***6–120.** The shaft is supported on two journal bearings at A and B which offer no resistance to axial loading. Determine the required diameter d of the shaft if the allowable bending stress for the material is $\sigma_{allow} = 150$ MPa.



The FBD of the shaft is shown in Fig. *a*.

The shaft is subjected to two bending moment components M_z and M_y, Figs. *b* and *c*, respectively.

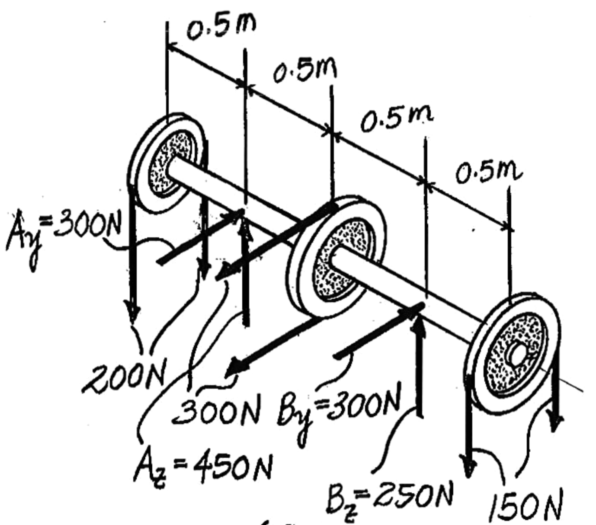
Since all the axes through the centroid of the circular cross-section of the shaft are principal axes, then the resultant moment $M = \sqrt{M_y^{\,2} + M_z^{\,2}}$ can be used for design. The maximum moment occurs at D ($x = 1$m). Then,

$$M_{max} = \sqrt{150^2 + 175^2} = 230.49 \text{ N} \cdot \text{m}$$

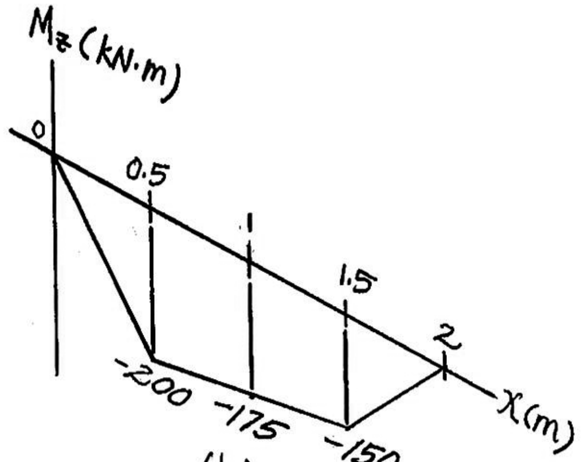
Then,

$$\sigma_{allow} = \frac{M_{max}\,C}{I}; \qquad 150(10^6) = \frac{230.49(d/2)}{\frac{\pi}{4}(d/2)^4}$$

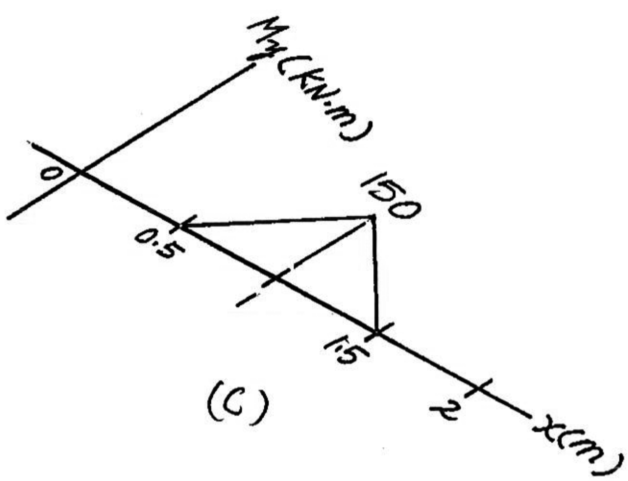
$$d = 0.02501 \text{ m} = 25 \text{ mm} \qquad \textbf{Ans.}$$



(a)

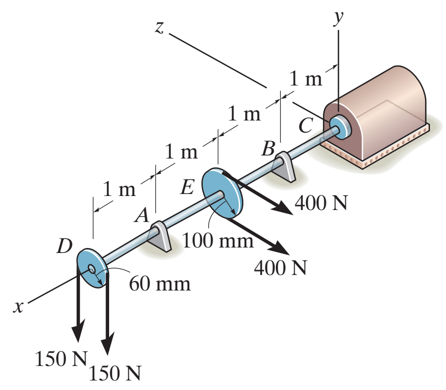


(b)



(c)

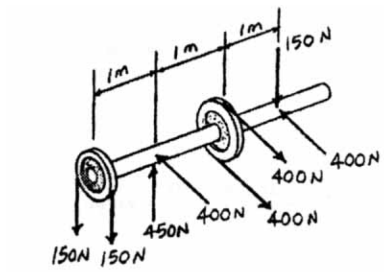
•6–121. The 30-mm-diameter shaft is subjected to the vertical and horizontal loadings of two pulleys as shown. It is supported on two journal bearings at A and B which offer no resistance to axial loading. Furthermore, the coupling to the motor at C can be assumed not to offer any support to the shaft. Determine the maximum bending stress developed in the shaft.

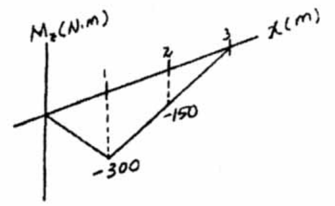


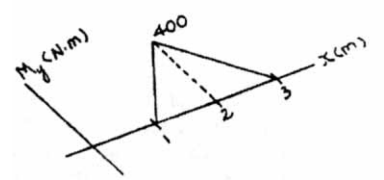
Support Reactions: As shown on FBD.

Internal Moment Components: The shaft is subjected to two bending moment components M_y and M_z. The moment diagram for each component is drawn.

Maximum Bending Stress: Since all the axes through the circle's center for circular shaft are principal axis, then the resultant moment $M = \sqrt{M_y^2 + M_z^2}$ can be used to determine the maximum bending stress. The maximum resultant moment occurs at E $M_{max} = \sqrt{400^2 + 150^2} = 427.2 \text{ N} \cdot \text{m}$.





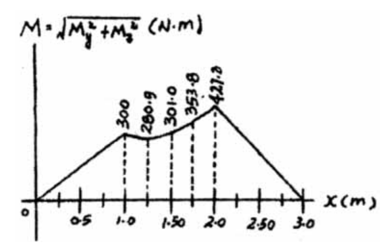


Applying the flexure formula

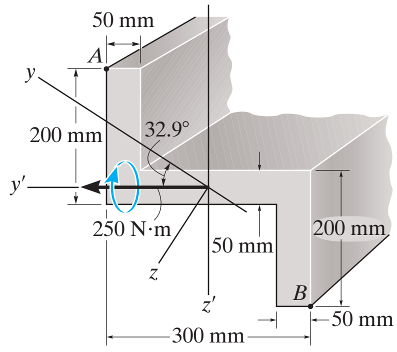
$$\sigma_{max} = \frac{M_{max}\, c}{I}$$

$$= \frac{427.2(0.015)}{\frac{\pi}{4}\left(0.015^4\right)}$$

$$= 161 \text{ MPa}$$ **Ans.**



6–122. Using the techniques outlined in Appendix A, Example A.5 or A.6, the Z section has principal moments of inertia of $I_y = 0.060(10^{-3})\ \text{m}^4$ and $I_z = 0.471(10^{-3})\ \text{m}^4$, computed about the principal axes of inertia y and z, respectively. If the section is subjected to an internal moment of $M = 250\ \text{N}\cdot\text{m}$ directed horizontally as shown, determine the stress produced at point A. Solve the problem using Eq. 6–17.

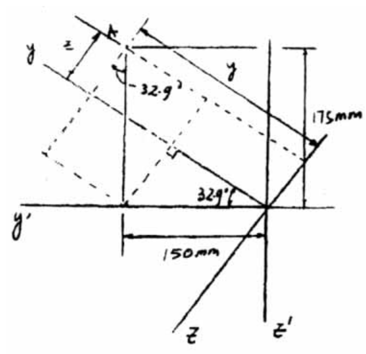


$$M_y = 250\cos 32.9° = 209.9\ \text{N}\cdot\text{m}$$

$$M_z = 250\sin 32.9° = 135.8\ \text{N}\cdot\text{m}$$

$$y = 0.15\cos 32.9° + 0.175\sin 32.9° = 0.2210\ \text{m}$$

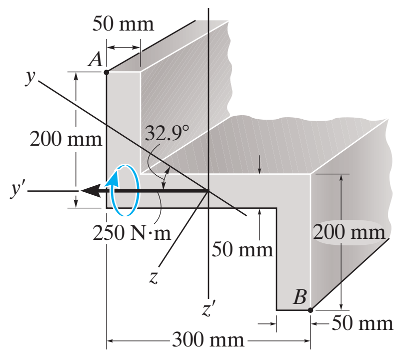
$$z = -(0.175\cos 32.9° - 0.15\sin 32.9°) = -0.06546\ \text{m}$$



$$\sigma_A = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{-135.8(0.2210)}{0.471(10^{-3})} + \frac{209.9(-0.06546)}{60.0(10^{-6})}$$

$$= -293\ \text{kPa} = 293\ \text{kPa (C)}$$ **Ans.**

6–123. Solve Prob. 6–122 using the equation developed in Prob. 6–113.



Internal Moment Components:

$$M_y = 250\ \text{N}\cdot\text{m} \qquad M_z = 0$$

Section Properties:

$$I_y = \frac{1}{12}(0.3)\left(0.05^3\right) + 2\left[\frac{1}{12}(0.05)\left(0.15^3\right) + 0.05(0.15)\left(0.1^2\right)\right]$$

$$= 0.18125\left(10^{-3}\right)\ \text{m}^4$$

$$I_z = \frac{1}{12}(0.05)\left(0.3^3\right) + 2\left[\frac{1}{12}(0.15)\left(0.05^3\right) + 0.15(0.05)\left(0.125^2\right)\right]$$

$$= 0.350(10^{-3})\ \text{m}^4$$

$$I_{yz} = 0.15(0.05)(0.125)(-0.1) + 0.15(0.05)(-0.125)(0.1)$$

$$= -0.1875\left(10^{-3}\right)\ \text{m}^4$$

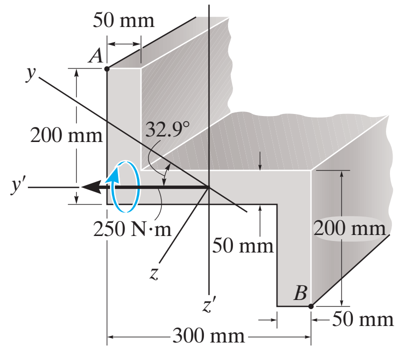
Bending Stress: Using formula developed in Prob. 6-113

$$\sigma = \frac{-(M_z I_y + M_y I_{yz})y + (M_y I_z + M_z I_{yz})z}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = \frac{-[0 + 250(-0.1875)(10^{-3})](0.15) + [250(0.350)(10^{-3}) + 0](-0.175)}{0.18125(10^{-3})(0.350)(10^{-3}) - [0.1875(10^{-3})]^2}$$

$$= -293\ \text{kPa} = 293\ \text{kPa (C)}$$ **Ans.**

***6–124.** Using the techniques outlined in Appendix A, Example A.5 or A.6, the *Z* section has principal moments of inertia of $I_y = 0.060(10^{-3})\ \text{m}^4$ and $I_z = 0.471(10^{-3})\ \text{m}^4$, computed about the principal axes of inertia *y* and *z*, respectively. If the section is subjected to an internal moment of $M = 250\ \text{N}\cdot\text{m}$ directed horizontally as shown, determine the stress produced at point *B*. Solve the problem using Eq. 6–17.



Internal Moment Components:

$$M_{y'} = 250 \cos 32.9° = 209.9\ \text{N}\cdot\text{m}$$

$$M_{z'} = 250 \sin 32.9° = 135.8\ \text{N}\cdot\text{m}$$

Section Property:

$$y' = 0.15 \cos 32.9° + 0.175 \sin 32.9° = 0.2210\ \text{m}$$

$$z' = 0.15 \sin 32.9° - 0.175 \cos 32.9° = -0.06546\ \text{m}$$

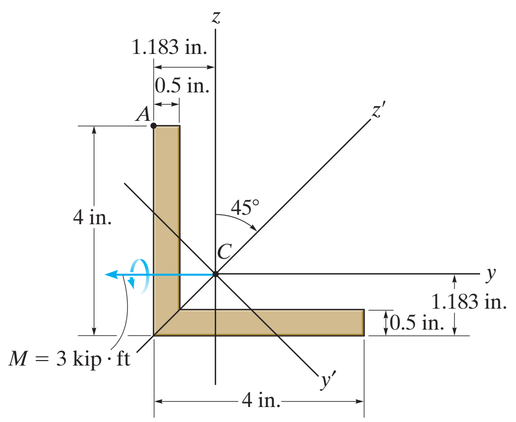
Bending Stress: Applying the flexure formula for biaxial bending

$$\sigma = \frac{M_{z'}y'}{I_{z'}} + \frac{M_{y'}z'}{I_{y'}}$$

$$\sigma_B = \frac{135.8(0.2210)}{0.471(10^{-3})} - \frac{209.9(-0.06546)}{0.060(10^{-3})}$$

$$= 293\ \text{kPa} = 293\ \text{kPa (T)}$$ **Ans.**

•6–125. Determine the bending stress at point *A* of the beam, and the orientation of the neutral axis. Using the method in Appendix A, the principal moments of inertia of the cross section are $I'_z = 8.828\ \text{in}^4$ and $I'_y = 2.295\ \text{in}^4$, where z' and y' are the principal axes. Solve the problem using Eq. 6–17.



Internal Moment Components: Referring to Fig. *a*, the y' and z' components of **M** are negative since they are directed towards the negative sense of their respective axes. Thus,

Section Properties: Referring to the geometry shown in Fig. *b*,

$$z'_A = 2.817 \cos 45° - 1.183 \sin 45° = 1.155 \text{ in.}$$

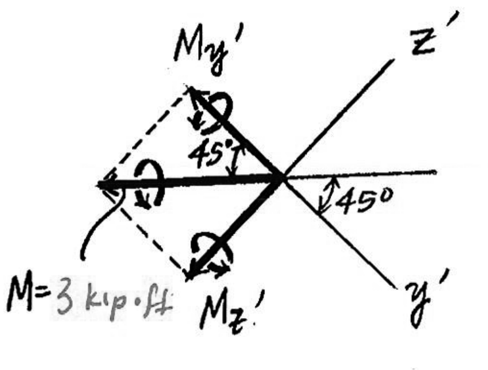
$$y'_A = -(2.817 \sin 45° + 1.183 \cos 45°) = -2.828 \text{ in.}$$

Bending Stress:

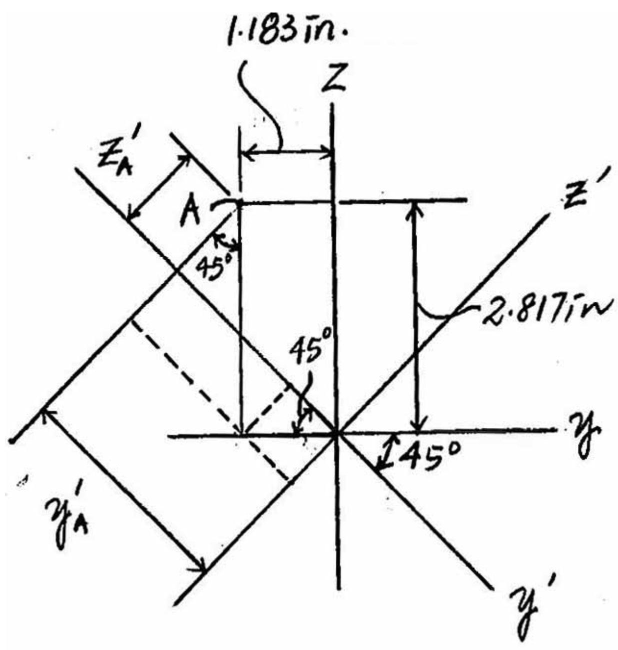
$$\sigma_A = -\frac{M_{z'} y'_A}{I_{z'}} + \frac{M_{y'} z'_A}{I_{y'}}$$

$$= -\frac{(-2.121)(12)(-2.828)}{8.828} + \frac{(-2.121)(12)(1.155)}{2.295}$$

$$= -20.97 \text{ ksi} = 21.0 \text{ ksi (C)}$$ **Ans.**

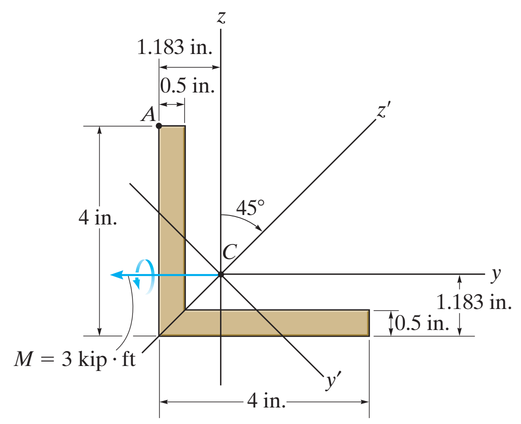


(a)



(b)

6–126. Determine the bending stress at point A of the beam using the result obtained in Prob. 6–113. The moments of inertia of the cross sectional area about the z and y axes are $I_z = I_y = 5.561\ \text{in}^4$ and the product of inertia of the cross sectional area with respect to the z and y axes is $I_{yz} = -3.267\ \text{in}^4$. (See Appendix A)



Internal Moment Components: Since **M** is directed towards the negative sense of the y axis, its y component is negative and it has no z component. Thus,

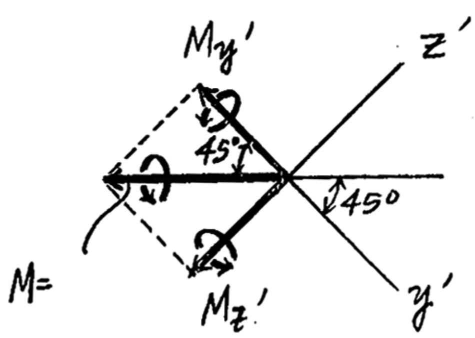
$$M_y = -3\ \text{kip}\cdot\text{ft} \qquad M_z = 0$$

Bending Stress:

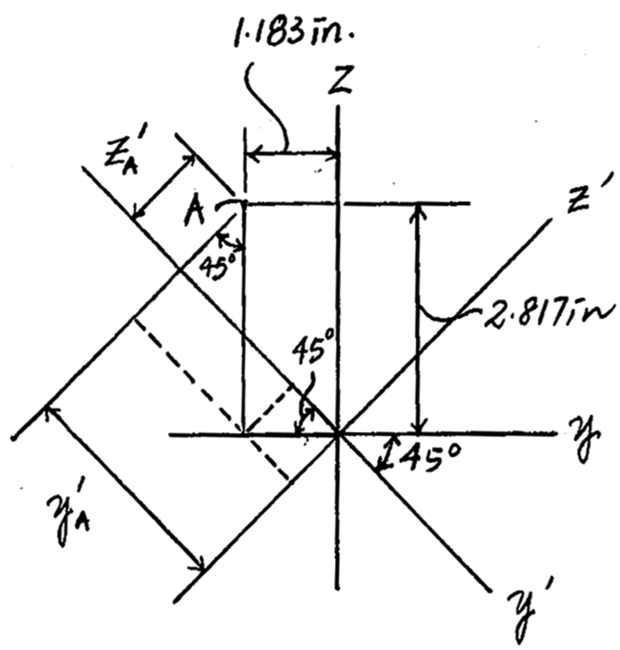
$$\sigma_A = \frac{-\left(M_z I_y + M_y I_{yz}\right) y_A + \left(M_y I_z + M_z I_{yz}\right) z_A}{I_y I_z - I_{yz}^2}$$

$$= \frac{-\left[0(5.561) + (-3)(12)(-3.267)\right](-1.183) + \left[-3(12)(5.561) + 0(-3.267)\right](2.817)}{5.561(5.561) - (-3.267)^2}$$

$$= -20.97\ \text{ksi} = 21.0\ \text{ksi} \qquad \textbf{Ans.}$$

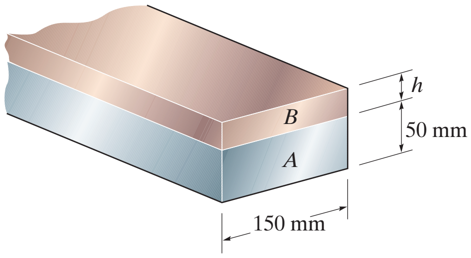


(a)

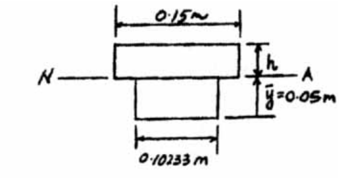


(b)

6–127. The composite beam is made of 6061-T6 aluminum (A) and C83400 red brass (B). Determine the dimension h of the brass strip so that the neutral axis of the beam is located at the seam of the two metals. What maximum moment will this beam support if the allowable bending stress for the aluminum is $(\sigma_{allow})_{al} = 128$ MPa and for the brass $(\sigma_{allow})_{br} = 35$ MPa?



Section Properties:



$$n = \frac{E_{al}}{E_{br}} = \frac{68.9(10^9)}{101(10^9)} = 0.68218$$

$$b_{br} = nb_{al} = 0.68218(0.15) = 0.10233 \text{ m}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$0.05 = \frac{0.025(0.10233)(0.05) + (0.05 + 0.5h)(0.15)h}{0.10233(0.05) + (0.15)h}$$

$$h = 0.04130 \text{ m} = 41.3 \text{ mm}$$ **Ans.**

$$I_{NA} = \frac{1}{12}(0.10233)\left(0.05^3\right) + 0.10233(0.05)(0.05 - 0.025)^2$$

$$+ \frac{1}{12}(0.15)\left(0.04130^3\right) + 0.15(0.04130)(0.070649 - 0.05)^2$$

$$= 7.7851\left(10^{-6}\right) \text{ m}^4$$

Allowable Bending Stress: Applying the flexure formula

Assume failure of red brass

$$(\sigma_{allow})_{br} = \frac{Mc}{I_{NA}}$$

$$35\left(10^6\right) = \frac{M(0.04130)}{7.7851(10^{-6})}$$

$$M = 6598 \text{ N}\cdot\text{m} = 6.60 \text{ kN}\cdot\text{m} \ (\textit{controls!})$$ **Ans.**

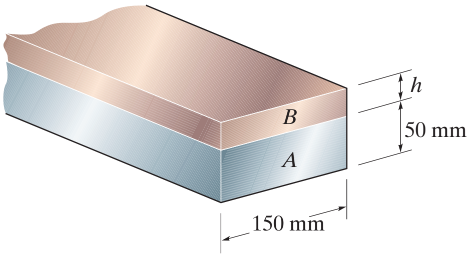
Assume failure of aluminium

$$(\sigma_{allow})_{al} = n\frac{Mc}{I_{NA}}$$

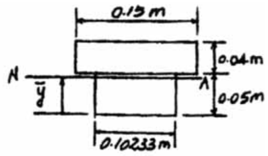
$$128\left(10^6\right) = 0.68218\left[\frac{M(0.05)}{7.7851(10^{-6})}\right]$$

$$M = 29215 \text{ N}\cdot\text{m} = 29.2 \text{ kN}\cdot\text{m}$$

***6–128.** The composite beam is made of 6061-T6 aluminum (A) and C83400 red brass (B). If the height $h = 40$ mm, determine the maximum moment that can be applied to the beam if the allowable bending stress for the aluminum is $(\sigma_{allow})_{al} = 128$ MPa and for the brass $(\sigma_{allow})_{br} = 35$ MPa.



Section Properties: For transformed section.



$$n = \frac{E_{al}}{E_{br}} = \frac{68.9(10^9)}{101.0(10^9)} = 0.68218$$

$$b_{br} = nb_{al} = 0.68218(0.15) = 0.10233 \text{ m}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.025(0.10233)(0.05) + (0.07)(0.15)(0.04)}{0.10233(0.05) + 0.15(0.04)}$$

$$= 0.049289 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.10233)\left(0.05^3\right) + 0.10233(0.05)(0.049289 - 0.025)^2$$

$$+ \frac{1}{12}(0.15)\left(0.04^3\right) + 0.15(0.04)(0.07 - 0.049289)^2$$

$$= 7.45799\left(10^{-6}\right) \text{ m}^4$$

Allowable Bending Stress: Applying the flexure formula

Assume failure of red brass

$$(\sigma_{allow})_{br} = \frac{Mc}{I_{NA}}$$

$$35\left(10^6\right) = \frac{M(0.09 - 0.049289)}{7.45799(10^{-6})}$$

$$M = 6412 \text{ N} \cdot \text{m} = 6.41 \text{ kN} \cdot \text{m} \ (\textit{controls!})$$ **Ans.**

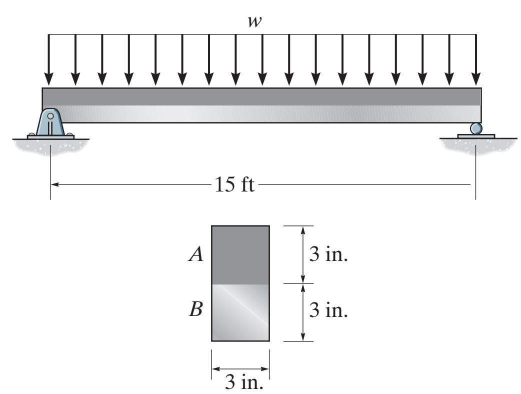
Assume failure of aluminium

$$(\sigma_{allow})_{al} = n\frac{Mc}{I_{NA}}$$

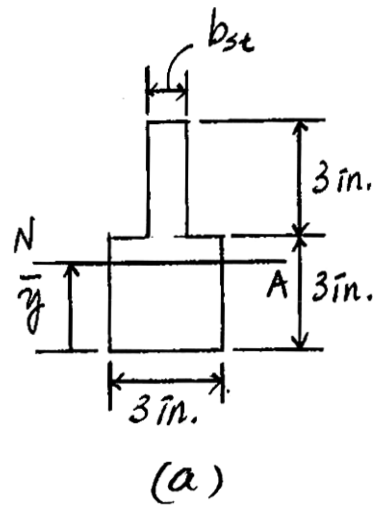
$$128\left(10^6\right) = 0.68218\left[\frac{M(0.049289)}{7.45799(10^{-6})}\right]$$

$$M = 28391 \text{ N} \cdot \text{m} = 28.4 \text{ kN} \cdot \text{m}$$

•**6–129.** Segment *A* of the composite beam is made from 2014-T6 aluminum alloy and segment *B* is A-36 steel. If $w = 0.9$ kip/ft, determine the absolute maximum bending stress developed in the aluminum and steel. Sketch the stress distribution on the cross section.



Maximum Moment: For the simply-supported beam subjected to the uniform distributed load, the maximum moment in the beam is $M_{max} = \frac{wL^2}{8} = \frac{0.9(15^2)}{8}$ $= 25.3125\ kip \cdot ft$.



Section Properties: The cross section will be transformed into that of steel as shown in Fig. *a*. Here, $n = \frac{E_{al}}{E_{st}} = \frac{10.6}{29} = 0.3655$.

Then $b_{st} = nb_{al} = 0.3655(3) = 1.0965$ in. The location of the centroid of the transformed section is

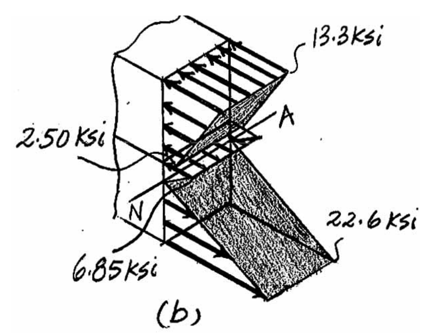
$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{1.5(3)(3) + 4.5(3)(1.0965)}{3(3) + 3(1.0965)} = 2.3030 \text{ in.}$$

The moment of inertia of the transformed section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2 = \frac{1}{12}(3)(3^3) + 3(3)(2.3030 - 1.5)^2$$
$$+ \frac{1}{12}(1.0965)(3^3) + 1.0965(3)(4.5 - 2.3030)^2$$
$$= 30.8991 \text{ in}^4$$

Maximum Bending Stress: For the steel,

$$(\sigma_{max})_{st} = \frac{M_{max} c_{st}}{I} = \frac{25.3125(12)(2.3030)}{30.8991} = 22.6 \text{ ksi}$$ **Ans.**



At the seam,

$$\sigma_{st}\big|_{y=0.6970 \text{ in.}} = \frac{M_{max} y}{I} = \frac{25.3125(12)(0.6970)}{30.8991} = 6.85 \text{ ksi}$$

For the aluminium,

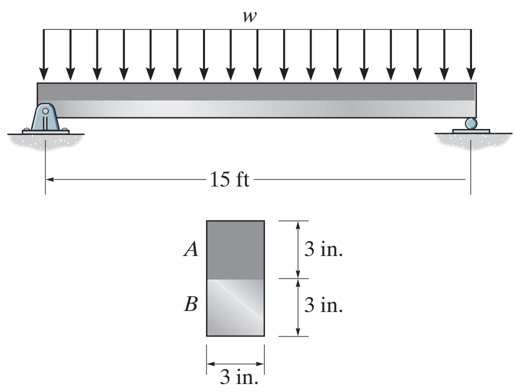
$$(\sigma_{max})_{al} = n\frac{M_{max} c_{al}}{I} = 0.3655\left[\frac{25.3125(12)(6 - 2.3030)}{30.8991}\right] = 13.3 \text{ ksi}$$ **Ans.**

At the seam,

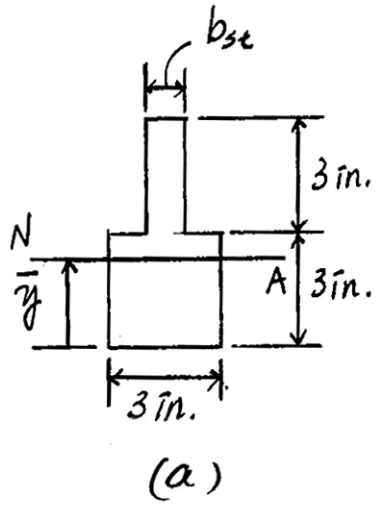
$$\sigma_{al}\big|_{y=0.6970 \text{ in.}} = n\frac{M_{max} y}{I} = 0.3655\left[\frac{25.3125(12)(0.6970)}{30.8991}\right] = 2.50 \text{ ksi}$$

The bending stress across the cross section of the composite beam is shown in Fig. *b*.

6–130. Segment *A* of the composite beam is made from 2014-T6 aluminum alloy and segment *B* is A-36 steel. If the allowable bending stress for the aluminum and steel are $(\sigma_{allow})_{al} = 15$ ksi and $(\sigma_{allow})_{st} = 22$ ksi, determine the maximum allowable intensity *w* of the uniform distributed load.



Maximum Moment: For the simply-supported beam subjected to the uniform distributed load, the maximum moment in the beam is $M_{max} = \frac{wL^2}{8} = \frac{w(15^2)}{8} = 28.125w$.



Section Properties: The cross section will be transformed into that of steel as shown in Fig. *a*. Here, $n = \frac{E_{al}}{E_{st}} = \frac{10.6}{29} = 0.3655$.

Then $b_{st} = nb_{al} = 0.3655(3) = 1.0965$ in. The location of the centroid of the transformed section is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{1.5(3)(3) + 4.5(3)(1.0965)}{3(3) + 3(1.0965)} = 2.3030 \text{ in.}$$

The moment of inertia of the transformed section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2 = \frac{1}{12}(3)(3^3) + 3(3)(2.3030 - 1.5)^2 + \frac{1}{12}(1.0965)(3^3)$$

$$+ 1.0965(3^3) + 1.0965(3)(4.5 - 2.3030)^2$$

$$= 30.8991 \text{ in}^4$$

Bending Stress: Assuming failure of steel,

$$(\sigma_{allow})_{st} = \frac{M_{max}\, c_{st}}{I}; \qquad 22 = \frac{(28.125w)(12)(2.3030)}{30.8991}$$

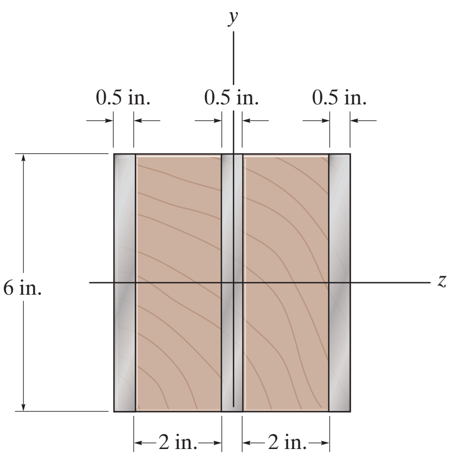
$$w = 0.875 \text{ kip/ft (controls)} \qquad \textbf{Ans.}$$

Assuming failure of aluminium alloy,

$$(\sigma_{allow})_{al} = n\frac{M_{max}\, c_{al}}{I}; \qquad 15 = 0.3655\left[\frac{(28.125w)(12)(6 - 2.3030)}{30.8991}\right]$$

$$w = 1.02 \text{ kip/ft}$$

6–131. The Douglas fir beam is reinforced with A-36 straps at its center and sides. Determine the maximum stress developed in the wood and steel if the beam is subjected to a bending moment of $M_z = 7.50$ kip·ft. Sketch the stress distribution acting over the cross section.



Section Properties: For the transformed section.

$$n = \frac{E_w}{E_{st}} = \frac{1.90(10^3)}{29.0(10^3)} = 0.065517$$

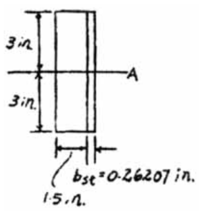
$$b_{st} = nb_w = 0.065517(4) = 0.26207 \text{ in.}$$

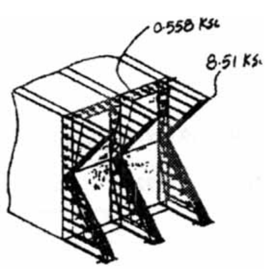
$$I_{NA} = \frac{1}{12}(1.5 + 0.26207)\left(6^3\right) = 31.7172 \text{ in}^4$$

Maximum Bending Stress: Applying the flexure formula

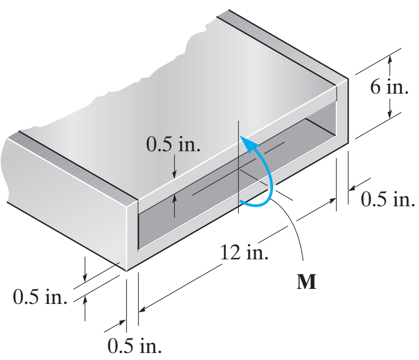
$$(\sigma_{max})_{st} = \frac{Mc}{I} = \frac{7.5(12)(3)}{31.7172} = 8.51 \text{ ksi}$$ **Ans.**

$$(\sigma_{max})_w = n\frac{Mc}{I} = 0.065517\left[\frac{7.5(12)(3)}{31.7172}\right] = 0.558 \text{ ksi}$$ **Ans.**

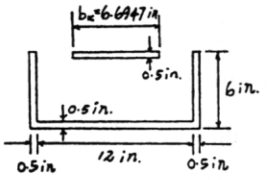




***6–132.** The top plate is made of 2014-T6 aluminum and is used to reinforce a Kevlar 49 plastic beam. Determine the maximum stress in the aluminum and in the Kevlar if the beam is subjected to a moment of $M = 900\ \text{lb} \cdot \text{ft}$.



Section Properties:



$$n = \frac{E_{al}}{E_k} = \frac{10.6(10^3)}{19.0(10^3)} = 0.55789$$

$$b_k = n\, b_{al} = 0.55789(12) = 6.6947 \text{ in.}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.25(13)(0.5) + 2[(3.25)(5.5)(0.5)] + 5.75(6.6947)(0.5)}{13(0.5) + 2(5.5)(0.5) + 6.6947(0.5)}$$

$$= 2.5247 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(13)\left(0.5^3\right) + 13(0.5)(2.5247 - 0.25)^2$$

$$+ \frac{1}{12}(1)\left(5.5^3\right) + 1(5.5)(3.25 - 2.5247)^2$$

$$+ \frac{1}{12}(6.6947)\left(0.5^3\right) + 6.6947(0.5)(5.75 - 2.5247)^2$$

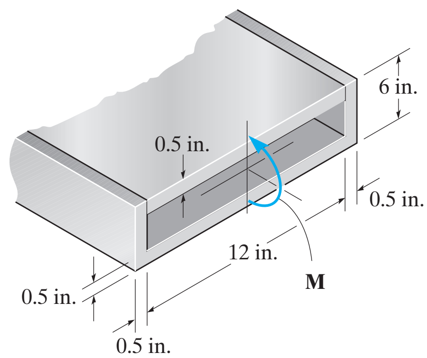
$$= 85.4170 \text{ in}^4$$

Maximum Bending Stress: Applying the flexure formula

$$(\sigma_{\max})_{al} = n\frac{Mc}{I} = 0.55789\left[\frac{900(12)(6 - 2.5247)}{85.4170}\right] = 245 \text{ psi} \qquad \textbf{Ans.}$$

$$(\sigma_{\max})_k = \frac{Mc}{I} = \frac{900(12)(6 - 2.5247)}{85.4168} = 439 \text{ psi} \qquad \textbf{Ans.}$$

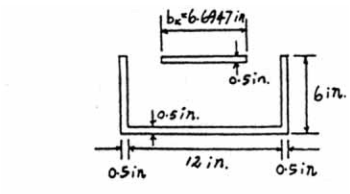
•6–133. The top plate made of 2014-T6 aluminum is used to reinforce a Kevlar 49 plastic beam. If the allowable bending stress for the aluminum is $(\sigma_{allow})_{al} = 40$ ksi and for the Kevlar $(\sigma_{allow})_k = 8$ ksi, determine the maximum moment M that can be applied to the beam.



Section Properties:

$$n = \frac{E_{al}}{E_k} = \frac{10.6(10^3)}{19.0(10^3)} = 0.55789$$

$$b_k = n\, b_{al} = 0.55789(12) = 6.6947 \text{ in.}$$



$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.25(13)(0.5) + 2[(3.25)(5.5)(0.5)] + 5.75(6.6947(0.5)}{13(0.5) + 2(5.5)(0.5) + 6.6947(0.5)}$$

$$= 2.5247 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(13)\left(0.5^3\right) + 13(0.5)(2.5247 - 0.25)^2$$

$$+ \frac{1}{12}(1)\left(5.5^3\right) + 1(5.5)(3.25 - 2.5247)^2$$

$$+ \frac{1}{12}(6.6947)\left(0.5^3\right) + 6.6947(0.5)(5.75 - 2.5247)^2$$

$$= 85.4170 \text{ in}^4$$

Maximum Bending Stress: Applying the flexure formula

Assume failure of aluminium

$$(\sigma_{allow})_{al} = n\frac{Mc}{I}$$

$$40 = 0.55789\left[\frac{M(6 - 2.5247)}{85.4170}\right]$$

$$M = 1762 \text{ kip} \cdot \text{in} = 146.9 \text{ kip} \cdot \text{ft}$$

Assume failure of Kevlar 49

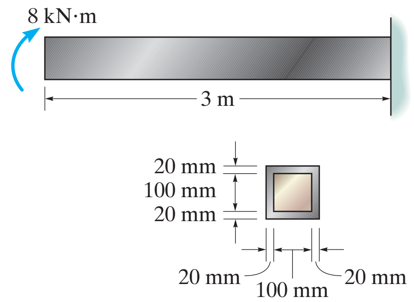
$$(\sigma_{allow})_k = \frac{Mc}{I}$$

$$8 = \frac{M(6 - 2.5247)}{85.4170}$$

$$M = 196.62 \text{ kip} \cdot \text{in}$$

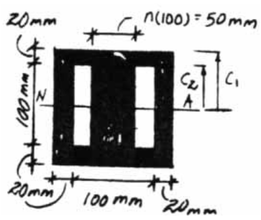
$$= 16.4 \text{ kip} \cdot \text{ft} \qquad (\textit{Controls!})$$ **Ans.**

6–134. The member has a brass core bonded to a steel casing. If a couple moment of 8 kN · m is applied at its end, determine the maximum bending stress in the member. $E_{br} = 100$ GPa, $E_{st} = 200$ GPa.



$$n = \frac{E_{br}}{E_{st}} = \frac{100}{200} = 0.5$$

$$I = \frac{1}{12}(0.14)(0.14)^3 - \frac{1}{12}(0.05)(0.1)^3 = 27.84667(10^{-6})\text{m}^4$$



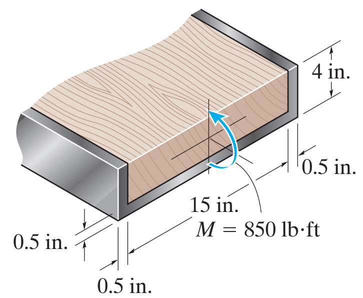
Maximum stress in steel:

$$(\sigma_{st})_{max} = \frac{Mc_1}{I} = \frac{8(10^3)(0.07)}{27.84667(10^{-6})} = 20.1 \text{ MPa} \qquad \text{(max)}$$ **Ans.**

Maximum stress in brass:

$$(\sigma_{br})_{max} = \frac{nMc_2}{I} = \frac{0.5(8)(10^3)(0.05)}{27.84667(10^{-6})} = 7.18 \text{ MPa}$$

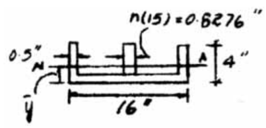
6–135. The steel channel is used to reinforce the wood beam. Determine the maximum stress in the steel and in the wood if the beam is subjected to a moment of $M = 850$ lb · ft. $E_{st} = 29(10^3)$ ksi, $E_w = 1600$ ksi.



$$\bar{y} = \frac{(0.5)(16)(0.25) + 2(3.5)(0.5)(2.25) + (0.8276)(3.5)(2.25)}{0.5(16) + 2(3.5)(0.5) + (0.8276)(3.5)} = 1.1386 \text{ in.}$$

$$I = \frac{1}{12}(16)(0.5^3) + (16)(0.5)(0.8886^2) + 2\left(\frac{1}{12}\right)(0.5)(3.5^3) + 2(0.5)(3.5)(1.1114^2)$$

$$+ \frac{1}{12}(0.8276)(3.5^3) + (0.8276)(3.5)(1.1114^2) = 20.914 \text{ in}^4$$



Maximum stress in steel:

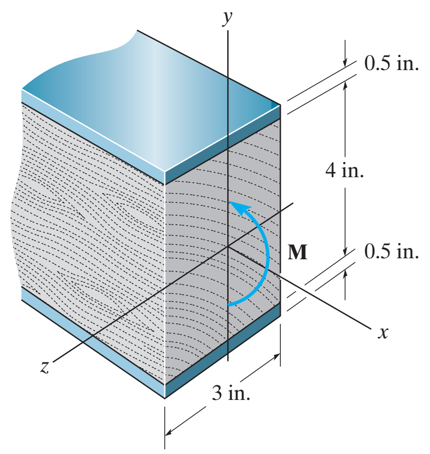
$$(\sigma_{st}) = \frac{Mc}{I} = \frac{850(12)(4 - 1.1386)}{20.914} = 1395 \text{ psi} = 1.40 \text{ ksi}$$ **Ans.**

Maximum stress in wood:

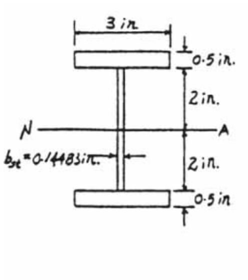
$$(\sigma_w) = n(\sigma_{st})_{max}$$

$$= 0.05517(1395) = 77.0 \text{ psi}$$ **Ans.**

***6–136.** A white spruce beam is reinforced with A-36 steel straps at its top and bottom as shown. Determine the bending moment M it can support if $(\sigma_{allow})_{st} = 22$ ksi and $(\sigma_{allow})_w = 2.0$ ksi.



Section Properties: For the transformed section.



$$n = \frac{E_w}{E_{st}} = \frac{1.40(10^3)}{29.0(10^3)} = 0.048276$$

$$b_{st} = nb_w = 0.048276(3) = 0.14483 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(3)\left(5^3\right) - \frac{1}{12}(3 - 0.14483)\left(4^3\right) = 16.0224 \text{ in}^4$$

Allowable Bending Stress: Applying the flexure formula

Assume failure of steel

$$(\sigma_{allow})_{st} = \frac{Mc}{I}$$

$$22 = \frac{M(2.5)}{16.0224}$$

$$M = 141.0 \text{ kip} \cdot \text{in}$$

$$= 11.7 \text{ kip} \cdot \text{ft } (Controls!)$$ **Ans.**

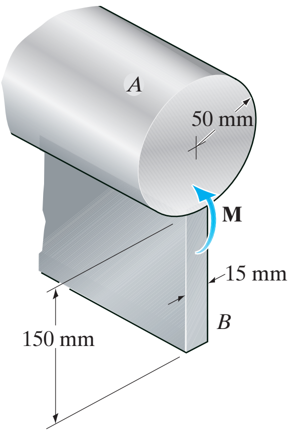
Assume failure of wood

$$(\sigma_{allow})_w = n\frac{My}{I}$$

$$2.0 = 0.048276\left[\frac{M(2)}{16.0224}\right]$$

$$M = 331.9 \text{ kip} \cdot \text{in} = 27.7 \text{ kip} \cdot \text{ft}$$

•6–137. If the beam is subjected to an internal moment of $M = 45\ \text{kN} \cdot \text{m}$, determine the maximum bending stress developed in the A-36 steel section A and the 2014-T6 aluminum alloy section B.



Section Properties: The cross section will be transformed into that of steel as shown in Fig. *a*. Here, $n = \dfrac{E_{al}}{E_{st}} = \dfrac{73.1(10^9)}{200(10^9)} = 0.3655$. Thus, $b_{st} = nb_{al} = 0.3655(0.015) = 0.0054825$ m. The location of the transformed section is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.075(0.15)(0.0054825) + 0.2\left[\pi\left(0.05^2\right)\right]}{0.15(0.0054825) + \pi\left(0.05^2\right)}$$

$$= 0.1882 \text{ m}$$

The moment of inertia of the transformed section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2 = \frac{1}{12}(0.0054825)\left(0.15^3\right) + 0.0054825(0.15)(0.1882 - 0.075)^2$$

$$+ \frac{1}{4}\pi\left(0.05^4\right) + \pi\left(0.05^2\right)(0.2 - 0.1882)^2$$

$$= 18.08\left(10^{-6}\right) \text{m}^4$$

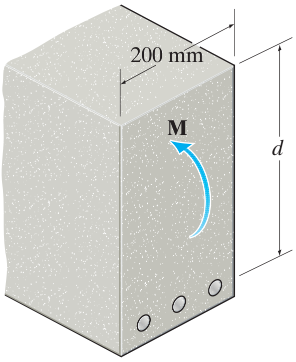
Maximum Bending Stress: For the steel,

$$(\sigma_{\max})_{st} = \frac{Mc_{st}}{I} = \frac{45\left(10^3\right)(0.06185)}{18.08\left(10^{-6}\right)} = 154 \text{ MPa} \qquad \textbf{Ans.}$$

For the aluminum alloy,

$$(\sigma_{\max})_{al} = n\frac{Mc_{al}}{I} = 0.3655\left[\frac{45\left(10^3\right)(0.1882)}{18.08\left(10^{-6}\right)}\right] = 171 \text{ MPa} \qquad \textbf{Ans.}$$

6–138. The concrete beam is reinforced with three 20-mm diameter steel rods. Assume that the concrete cannot support tensile stress. If the allowable compressive stress for concrete is $(\sigma_{\text{allow}})_{\text{con}} = 12.5$ MPa and the allowable tensile stress for steel is $(\sigma_{\text{allow}})_{\text{st}} = 220$ MPa, determine the required dimension d so that both the concrete and steel achieve their allowable stress simultaneously. This condition is said to be 'balanced'. Also, compute the corresponding maximum allowable internal moment **M** that can be applied to the beam. The moduli of elasticity for concrete and steel are $E_{\text{con}} = 25$ GPa and $E_{\text{st}} = 200$ GPa, respectively.



Bending Stress: The cross section will be transformed into that of concrete as shown in Fig. *a*. Here, $n = \frac{E_{st}}{E_{con}} = \frac{200}{25} = 8$. It is required that both concrete and steel achieve their allowable stress simultaneously. Thus,

$$(\sigma_{\text{allow}})_{con} = \frac{Mc_{con}}{I}; \qquad 12.5\left(10^6\right) = \frac{Mc_{con}}{I}$$

$$M = 12.5\left(10^6\right)\left(\frac{I}{c_{con}}\right) \qquad \textbf{(1)}$$

$$(\sigma_{\text{allow}})_{st} = n\frac{Mc_{st}}{I}; \qquad 220\left(10^6\right) = 8\left[\frac{M(d - c_{con})}{I}\right]$$

$$M = 27.5\left(10^6\right)\left(\frac{I}{d - c_{con}}\right) \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2),

$$12.5\left(10^6\right)\left(\frac{I}{c_{con}}\right) = 27.5\left(10^6\right)\left(\frac{I}{d - c_{con}}\right)$$

$$c_{con} = 0.3125d \text{ (3)} \qquad \textbf{(3)}$$

Section Properties: The area of the steel bars is $A_{st} = 3\left[\frac{\pi}{4}\left(0.02^2\right)\right] = 0.3\left(10^{-3}\right)\pi \text{ m}^2$. Thus, the transformed area of concrete from steel is $(A_{con})_t = nA_s = 8\left[0.3\left(10^{-3}\right)\pi\right]$ $= 2.4\left(10^{-3}\right)\pi \text{ m}^2$. Equating the first moment of the area of concrete above and below the neutral axis about the neutral axis,

$$0.2(c_{con})(c_{con}/2) = 2.4\left(10^{-3}\right)\pi\,(d - c_{con})$$

$$0.1c_{con}{}^2 = 2.4\left(10^{-3}\right)\pi d - 2.4\left(10^{-3}\right)\pi c_{con}$$

$$c_{con}{}^2 = 0.024\pi d - 0.024\pi c_{con} \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4),

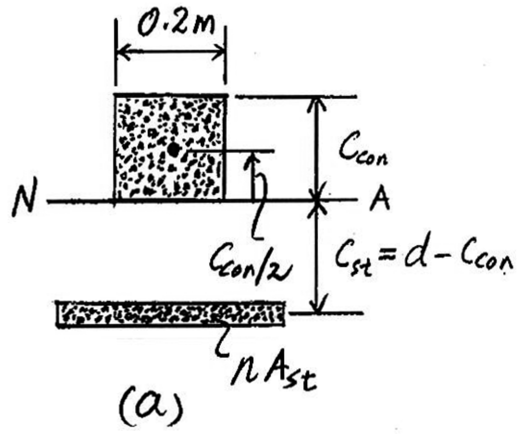
$$d = 0.5308 \text{ m} = 531 \text{ mm} \qquad \textbf{Ans.}$$

$$c_{con} = 0.1659 \text{ m}$$

Thus, the moment of inertia of the transformed section is

$$I = \frac{1}{3}(0.2)\left(0.1659^3\right) + 2.4\left(10^{-3}\right)\pi(0.5308 - 0.1659)^2$$

6–138. Continued



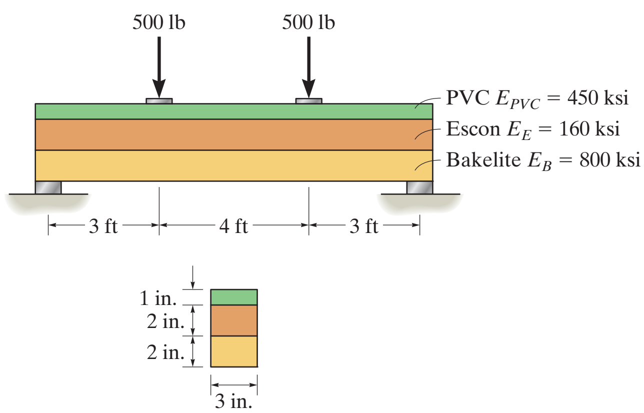
$$= 1.3084\left(10^{-3}\right)\ \text{m}^4$$

Substituting this result into Eq. (1),

$$M = 12.5\left(10^6\right)\left[\frac{1.3084\left(10^{-3}\right)}{0.1659}\right]$$

$$= 98\,594.98\ \text{N}\cdot\text{m} = 98.6\ \text{kN}\cdot\text{m}, \qquad \textbf{Ans.}$$

6–139. The beam is made from three types of plastic that are identified and have the moduli of elasticity shown in the figure. Determine the maximum bending stress in the PVC.



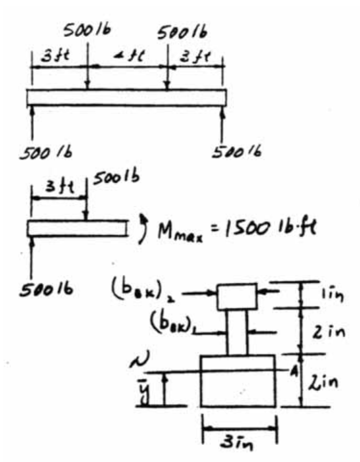
$$(b_{bk})_1 = n_1\, b_{Es} = \frac{160}{800}(3) = 0.6\ \text{in.}$$

$$(b_{bk})_2 = n_2\, b_{pvc} = \frac{450}{800}(3) = 1.6875\ \text{in.}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{(1)(3)(2) + 3(0.6)(2) + 4.5(1.6875)(1)}{3(2) + 0.6(2) + 1.6875(1)} = 1.9346\ \text{in.}$$

$$I = \frac{1}{12}(3)(2^3) + 3(2)(0.9346^2) + \frac{1}{12}(0.6)(2^3) + 0.6(2)(1.0654^2)$$

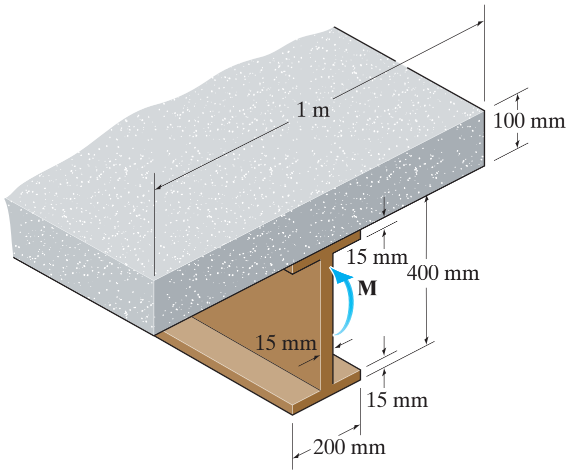
$$+ \frac{1}{12}(1.6875)(1^3) + 1.6875(1)(2.5654^2) = 20.2495\ \text{in}^4$$



$$(\sigma_{\max})_{pvc} = n_2\frac{Mc}{I} = \left(\frac{450}{800}\right)\frac{1500(12)(3.0654)}{20.2495}$$

$$= 1.53\ \text{ksi} \qquad \textbf{Ans.}$$

***6–140.** The low strength concrete floor slab is integrated with a wide-flange A-36 steel beam using shear studs (not shown) to form the composite beam. If the allowable bending stress for the concrete is $(\sigma_{\text{allow}})_{\text{con}} = 10$ MPa, and allowable bending stress for steel is $(\sigma_{\text{allow}})_{\text{st}} = 165$ MPa, determine the maximum allowable internal moment **M** that can be applied to the beam.



Section Properties: The beam cross section will be transformed into that of steel. Here, $n = \dfrac{E_{con}}{E_{st}} = \dfrac{22.1}{200} = 0.1105$. Thus, $b_{st} = nb_{con} = 0.1105(1) = 0.1105$ m. The location of the transformed section is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.0075(0.015)(0.2) + 0.2(0.37)(0.015) + 0.3925(0.015)(0.2) + 0.45(0.1)(0.1105)}{0.015(0.2) + 0.37(0.015) + 0.015(0.2) + 0.1(0.1105)}$$

$$= 0.3222 \text{ m}$$

The moment of inertia of the transformed section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2 = \frac{1}{12}(0.2)\left(0.015^3\right)$$

$$+ 0.2(0.015)(0.3222 - 0.0075)^2$$

$$+ \frac{1}{12}(0.015)\left(0.37^3\right) + 0.015(0.37)(0.3222 - 0.2)^2$$

$$+ \frac{1}{12}(0.2)\left(0.015^3\right) + 0.2(0.015)(0.3925 - 0.3222)^2$$

$$+ \frac{1}{12}(0.1105)\left(0.1^3\right) + 0.1105(0.1)(0.45 - 0.3222)^2$$

$$= 647.93\left(10^{-6}\right) \text{ m}^4$$

Bending Stress: Assuming failure of steel,

$$(\sigma_{\text{allow}})_{st} = \frac{Mc_{st}}{I}; \quad 165\left(10^6\right) = \frac{M(0.3222)}{647.93\left(10^{-6}\right)}$$

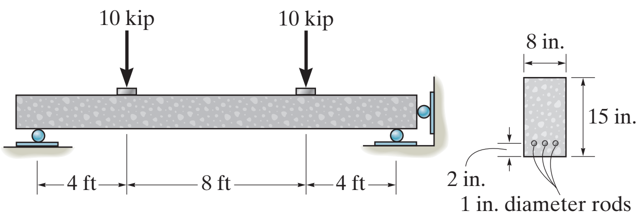
$$M = 331\,770.52 \text{ N}\cdot\text{m} = 332 \text{ kN}\cdot\text{m}$$

Assuming failure of concrete,

$$(\sigma_{\text{allow}})_{con} = n\frac{Mc_{con}}{I}; \qquad 10\left(10^6\right) = 0.1105\left[\frac{M(0.5 - 0.3222)}{647.93\left(10^{-6}\right)}\right]$$

$$M = 329\,849.77 \text{ N}\cdot\text{m} = 330 \text{ kN}\cdot\text{m (controls)} \quad \textbf{Ans.}$$

•6–141. The reinforced concrete beam is used to support the loading shown. Determine the absolute maximum normal stress in each of the A-36 steel reinforcing rods and the absolute maximum compressive stress in the concrete. Assume the concrete has a high strength in compression and yet neglect its strength in supporting tension.



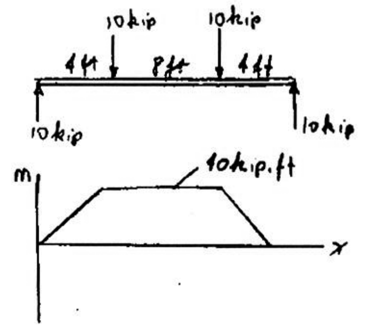
$$M_{max} = (10 \text{ kip})(4 \text{ ft}) = 40 \text{ kip} \cdot \text{ft}$$

$$A_{st} = 3(\pi)(0.5)^2 = 2.3562 \text{ in}^2$$

$$E_{st} = 29.0(10^3) \text{ ksi}$$

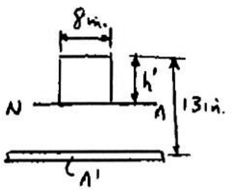
$$E_{con} = 4.20(10^3) \text{ ksi}$$

$$A' = nA_{st} = \frac{29.0(10^3)}{4.20(10^3)}(2.3562) = 16.2690 \text{ in}^2$$



$$\Sigma \bar{y}A = 0; \qquad 8(h')\left(\frac{h'}{2}\right) - 16.2690(13 - h') = 0$$

$$h'^2 + 4.06724h - 52.8741 = 0$$



Solving for the positive root:

$$h' = 5.517 \text{ in.}$$

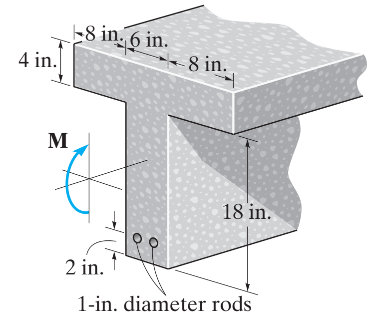
$$I = \left[\frac{1}{12}(8)(5.517)^3 + 8(5.517)(5.517/2)^2\right] + 16.2690(13 - 5.517)^2$$

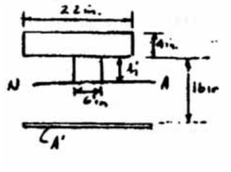
$$= 1358.781 \text{ in}^4$$

$$(\sigma_{con})_{max} = \frac{My}{I} = \frac{40(12)(5.517)}{1358.781} = 1.95 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\sigma_{st})_{max} = n\left(\frac{My}{I}\right) = \left(\frac{29.0(10^3)}{4.20(10^3)}\right)\left(\frac{40(12)(13 - 5.517)}{1358.781}\right) = 18.3 \text{ ksi} \qquad \textbf{Ans.}$$

6–142. The reinforced concrete beam is made using two steel reinforcing rods. If the allowable tensile stress for the steel is $(\sigma_{st})_{allow} = 40$ ksi and the allowable compressive stress for the concrete is $(\sigma_{conc})_{allow} = 3$ ksi, determine the maximum moment M that can be applied to the section. Assume the concrete cannot support a tensile stress. $E_{st} = 29(10^3)$ ksi, $E_{conc} = 3.8(10^3)$ ksi.





$$A_{st} = 2(\pi)(0.5)^2 = 1.5708 \text{ in}^2$$

$$A' = nA_{st} = \frac{29(10^3)}{3.8(10^3)}(1.5708) = 11.9877 \text{ in}^2$$

$$\Sigma \bar{y}A = 0; \qquad 22(4)(h' + 2) + h'(6)(h'/2) - 11.9877(16 - h') = 0$$

$$3h^2 + 99.9877h' - 15.8032 = 0$$

Solving for the positive root:

$$h' = 0.15731 \text{ in.}$$

$$I = \left[\frac{1}{12}(22)(4)^3 + 22(4)(2.15731)^2\right] + \left[\frac{1}{12}(6)(0.15731)^3 + 6(0.15731)(0.15731/2)^2\right]$$

$$+ 11.9877(16 - 0.15731)^2 = 3535.69 \text{ in}^4$$

Assume concrete fails:

$$(\sigma_{con})_{allow} = \frac{My}{I}; \qquad 3 = \frac{M(4.15731)}{3535.69}$$

$$M = 2551 \text{ kip} \cdot \text{in.}$$

Assume steel fails:

$$(\sigma_{st})_{allow} = n\left(\frac{My}{I}\right); \qquad 40 = \left(\frac{29(10^3)}{3.8(10^3)}\right)\left(\frac{M(16 - 0.15731)}{3535.69}\right)$$

$$M = 1169.7 \text{ kip} \cdot \text{in.} = 97.5 \text{ kip} \cdot \text{ft (controls)} \quad \textbf{Ans.}$$

6–143. For the curved beam in Fig. 6–40*a*, show that when the radius of curvature approaches infinity, the curved-beam formula, Eq. 6–24, reduces to the flexure formula, Eq. 6–13.

Normal Stress: Curved-beam formula

$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)} \qquad \text{where } A' = \int_A \frac{dA}{r} \quad \text{and } R = \frac{A}{\int_A \frac{dA}{r}} = \frac{A}{A'}$$

$$\sigma = \frac{M(A - rA')}{Ar(\bar{r}A' - A)} \qquad \textbf{[1]}$$

$$r = \bar{r} + y \qquad \textbf{[2]}$$

$$\bar{r}A' = \bar{r}\int_A \frac{dA}{r} = \int_A \left(\frac{\bar{r}}{\bar{r} + y} - 1 + 1\right) dA$$

$$= \int_A \left(\frac{\bar{r} - \bar{r} - y}{\bar{r} + y} + 1\right) dA$$

$$= A - \int_A \frac{y}{\bar{r} + y}\, dA \qquad \textbf{[3]}$$

Denominator of Eq. [1] becomes,

$$Ar(\bar{r}A' - A) = Ar\left(A - \int_A \frac{y}{\bar{r} + y}\, dA - A\right) = -Ar\int_A \frac{y}{\bar{r} + y}\, dA$$

Using Eq. [2],

$$Ar(\bar{r}A' - A) = -A\int_A \left(\frac{\bar{r}y}{\bar{r} + y} + y - y\right) dA - Ay\int_A \frac{y}{\bar{r} + y}\, dA$$

$$= A\int_A \frac{y^2}{\bar{r} + y}\, dA - A\int_A y\, dA - Ay\int_A \frac{y}{\bar{r} + y}\, dA$$

$$= \frac{A}{\bar{r}}\int_A \left(\frac{y^2}{1 + \frac{y}{\bar{r}}}\right) dA - A\int_A y\, dA - \frac{Ay}{\bar{r}}\int_A \left(\frac{y}{1 + \frac{y}{\bar{r}}}\right) dA$$

But, $\int_A y\, dA = 0$, as $\frac{y}{\bar{r}} \to 0$

Then, $Ar(\bar{r}A' - A) \to \frac{A}{\bar{r}} I$

Eq. [1] becomes $\sigma = \frac{M\bar{r}}{AI}(A - rA')$

Using Eq. [2], $\sigma = \frac{M\bar{r}}{AI}(A - \bar{r}A' - yA')$

Using Eq. [3],

$$\sigma = \frac{M\bar{r}}{AI}\left[A - \left(A - \int_A \frac{y}{\bar{r} + y}\, dA\right) - y\int_A \frac{dA}{\bar{r} + y}\right]$$

$$= \frac{M\bar{r}}{AI}\left[\int_A \frac{y}{\bar{r} + y}\, dA - y\int_A \frac{dA}{\bar{r} + y}\right]$$

6–143. Continued

$$= \frac{M\bar{r}}{AI}\left[\int_A \left(\frac{\frac{y}{\bar{r}}}{1+\frac{y}{\bar{r}}}\right) dA - \frac{y}{\bar{r}}\int_A \left(\frac{dA}{1+\frac{y}{\bar{r}}}\right)\right]$$

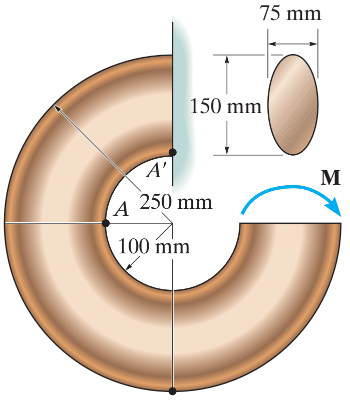
$$= \frac{M\bar{r}}{AI}$$

As $\quad \frac{y}{\bar{r}} \to 0$

$$\int_A \left(\frac{\frac{y}{\bar{r}}}{1+\frac{y}{\bar{r}}}\right) dA = 0 \quad \text{and} \quad \frac{y}{\bar{r}}\int_A \left(\frac{dA}{1+\frac{y}{\bar{r}}}\right) = \frac{y}{\bar{r}}\int_A dA = \frac{yA}{\bar{r}}$$

Therefore, $\quad \sigma = \frac{M\bar{r}}{AI}\left(-\frac{yA}{\bar{r}}\right) = -\frac{My}{I}$ **(Q.E.D.)**

***6–144.** The member has an elliptical cross section. If it is subjected to a moment of $M = 50\ \text{N}\cdot\text{m}$, determine the stress at points A and B. Is the stress at point A', which is located on the member near the wall, the same as that at A? Explain.



$$\int_A \frac{dA}{r} = \frac{2\pi b}{a}\left(\bar{r} - \sqrt{\bar{r}^2 - a^2}\right)$$

$$= \frac{2\pi(0.0375)}{0.075}\left(0.175 - \sqrt{0.175^2 - 0.075^2}\right) = 0.053049301 \text{ m}$$

$$A = \pi ab = \pi(0.075)(0.0375) = 2.8125(10^{-3})\pi$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{2.8125(10^{-3})\pi}{0.053049301} = 0.166556941$$

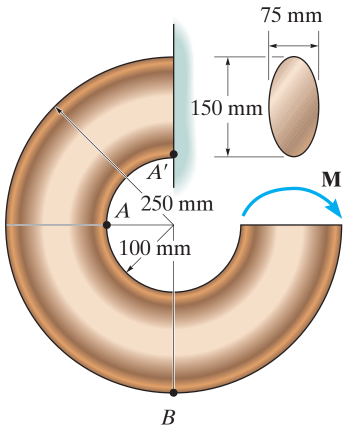
$$\bar{r} - R = 0.175 - 0.166556941 = 0.0084430586$$

$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{50(0.166556941 - 0.1)}{2.8125(10^{-3})\pi\,(0.1)(0.0084430586)} = 446\text{k Pa (T)}$$ **Ans.**

$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{50(0.166556941 - 0.25)}{2.8125(10^{-3})\pi\,(0.25)(0.0084430586)} = 224 \text{ kPa (C)}$$ **Ans.**

No, because of localized stress concentration at the wall. **Ans.**

•6–145. The member has an elliptical cross section. If the allowable bending stress is $\sigma_{allow} = 125$ MPa determine the maximum moment M that can be applied to the member.



$a = 0.075 \text{ m}; \qquad b = 0.0375 \text{ m}$

$A = \pi(0.075)(0.0375) = 0.0028125\,\pi$

$$\int_A \frac{dA}{r} = \frac{2\pi b}{a}\left(\bar{r} - \sqrt{\bar{r}^2 - a^2}\right) = \frac{2\pi(0.0375)}{0.075}\left(0.175 - \sqrt{0.175^2 - 0.075^2}\right)$$

$$= 0.053049301 \text{ m}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.0028125\pi}{0.053049301} = 0.166556941 \text{ m}$$

$$\bar{r} - R = 0.175 - 0.166556941 = 8.4430586(10^{-3}) \text{ m}$$

$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

Assume tension failure.

$$125(10^6) = \frac{M(0.166556941 - 0.1)}{0.0028125\pi(0.1)(8.4430586)(10^{-3})}$$

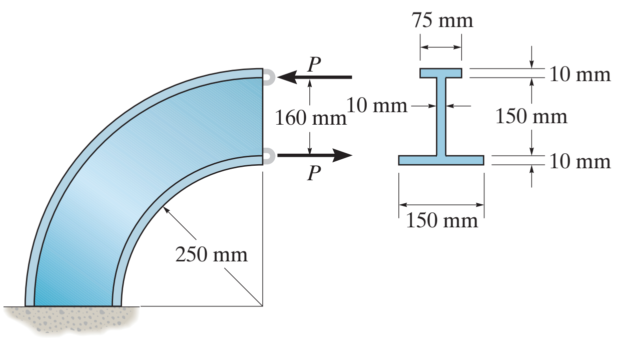
$M = 14.0 \text{ kN} \cdot \text{m}$ (controls) **Ans.**

Assume compression failure:

$$-125(10^6) = \frac{M(0.166556941 - 0.25)}{0.0028125\pi(0.25)(8.4430586)(10^{-3})}$$

$M = 27.9 \text{ kN} \cdot \text{m}$

6–146. Determine the greatest magnitude of the applied forces P if the allowable bending stress is $(\sigma_{allow})_c = 50$ MPa in compression and $(\sigma_{allow})_t = 120$ MPa in tension.



Internal Moment: $M = 0.160P$ is positive since it tends to increase the beam's radius of curvature.

Section Properties:

$$\bar{r} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.255(0.15)(0.01) + 0.335(0.15)(0.01) + 0.415(0.075)(0.01)}{0.15(0.01) + 0.15(0.01) + 0.075(0.01)}$$

$$= 0.3190 \text{ m}$$

$$A = 0.15(0.01) + 0.15(0.01) + 0.075(0.01) = 0.00375 \text{ m}^2$$

$$\Sigma \int_A \frac{dA}{r} = 0.15 \ln \frac{0.26}{0.25} + 0.01 \ln \frac{0.41}{0.26} + 0.075 \ln \frac{0.42}{0.41}$$

$$= 0.012245 \text{ m}$$

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}} = \frac{0.00375}{0.012245} = 0.306243 \text{ m}$$

$$\bar{r} - R = 0.319 - 0.306243 = 0.012757 \text{ m}$$

Allowable Normal Stress: Applying the curved-beam formula

Assume tension failure

$$(\sigma_{allow})_t = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

$$120\left(10^6\right) = \frac{0.16P(0.306243 - 0.25)}{0.00375(0.25)(0.012757)}$$

$$P = 159482 \text{ N} = 159.5 \text{ kN}$$

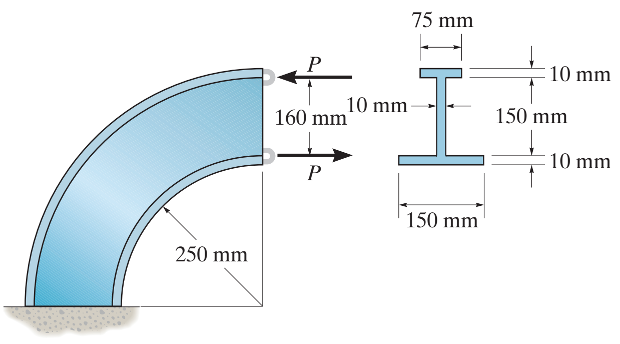
Assume compression failure

$$(\sigma_{allow})_t = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

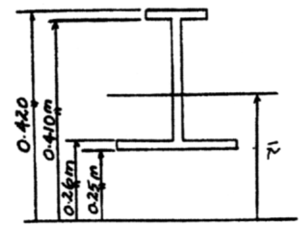
$$-50\left(10^6\right) = \frac{0.16P(0.306243 - 0.42)}{0.00375(0.42)(0.012757)}$$

$$P = 55195 \text{ N} = 55.2 \text{ kN} \ (\textit{Controls !}) \qquad \textbf{Ans.}$$

6–147. If $P = 6$ kN, determine the maximum tensile and compressive bending stresses in the beam.



Internal Moment: $M = 0.160(6) = 0.960\text{ kN}\cdot\text{m}$ is positive since it tends to increase the beam's radius of curvature.



Section Properties:

$$\bar{r} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.255(0.15)(0.01) + 0.335(0.15)(0.01) + 0.415(0.075)(0.01)}{0.15(0.01) + 0.15(0.01) + 0.075(0.01)}$$

$$= 0.3190 \text{ m}$$

$$A = 0.15(0.01) + 0.15(0.01) + 0.075(0.01) = 0.00375 \text{ m}^2$$

$$\Sigma \int_A \frac{dA}{r} = 0.15 \ln \frac{0.26}{0.25} + 0.01 \ln \frac{0.41}{0.26} + 0.075 \ln \frac{0.42}{0.41}$$

$$= 0.012245 \text{ m}$$

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}} = \frac{0.00375}{0.012245} = 0.306243 \text{ m}$$

$$\bar{r} - R = 0.319 - 0.306243 = 0.012757 \text{ m}$$

Normal Stress: Applying the curved-beam formula

$$(\sigma_{\max})_t = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

$$= \frac{0.960(10^3)(0.306243 - 0.25)}{0.00375(0.25)(0.012757)}$$

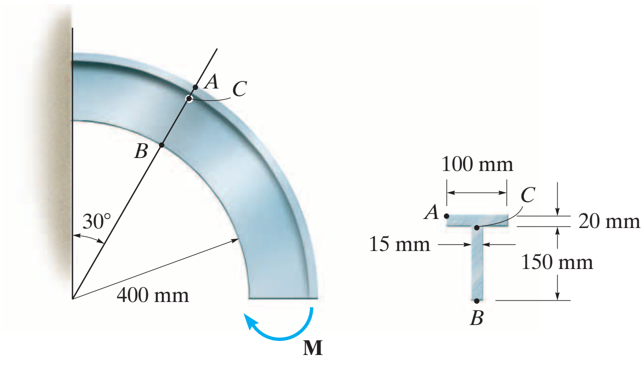
$$= 4.51 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\sigma_{\max})_c = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

$$= \frac{0.960(10^3)(0.306243 - 0.42)}{0.00375(0.42)(0.012757)}$$

$$= -5.44 \text{ MPa} \qquad \textbf{Ans.}$$

***6–148.** The curved beam is subjected to a bending moment of $M = 900\ \text{N} \cdot \text{m}$ as shown. Determine the stress at points A and B, and show the stress on a volume element located at each of these points.



Internal Moment: $M = -900\ \text{N} \cdot \text{m}$ is negative since it tends to decrease the beam's radius curvature.

Section Properties:

$$\Sigma A = 0.15(0.015) + 0.1(0.02) = 0.00425\ \text{m}^2$$

$$\Sigma \bar{r} A = 0.475(0.15)(0.015) + 0.56(0.1)(0.02) = 2.18875(10^{-3})\ \text{m}^3$$

$$\bar{r} = \frac{\Sigma \bar{r} A}{\Sigma A} = \frac{2.18875\ (10^{-3})}{0.00425} = 0.5150\ \text{m}$$

$$\Sigma \int_A \frac{dA}{r} = 0.015 \ln \frac{0.55}{0.4} + 0.1 \ln \frac{0.57}{0.55} = 8.348614(10^{-3})\ \text{m}$$

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}} = \frac{0.00425}{8.348614(10^{-3})} = 0.509067\ \text{m}$$

$$\bar{r} - R = 0.515 - 0.509067 = 5.933479(10^{-3})\ \text{m}$$

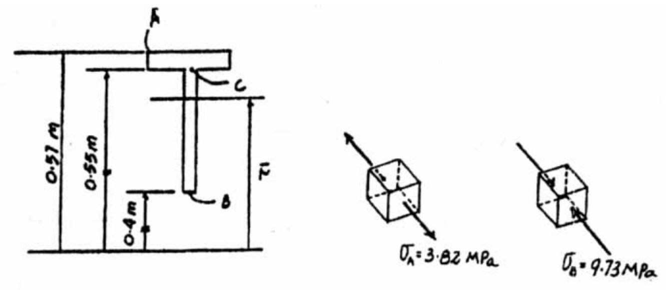
Normal Stress: Applying the curved-beam formula

$$\sigma_A = \frac{M(R - r_A)}{A r_A (\bar{r} - R)} = \frac{-900(0.509067 - 0.57)}{0.00425(0.57)(5.933479)(10^{-3})}$$

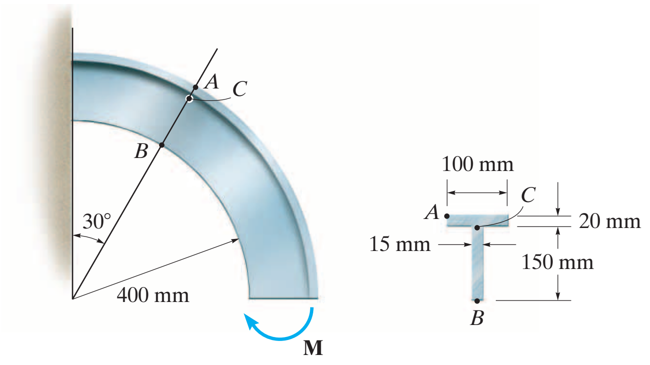
$$= 3.82\ \text{MPa (T)} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{M(R - r_B)}{A r_B (\bar{r} - R)} = \frac{-900(0.509067 - 0.4)}{0.00425(0.4)(5.933479)(10^{-3})}$$

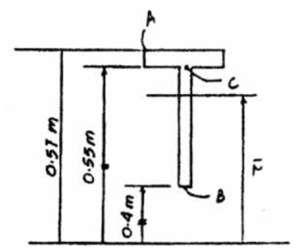
$$= -9.73\ \text{MPa} = 9.73\ \text{MPa (C)} \qquad \textbf{Ans.}$$



•6–149. The curved beam is subjected to a bending moment of $M = 900\ \text{N}\cdot\text{m}$. Determine the stress at point C.



Internal Moment: $M = -900\ \text{N}\cdot\text{m}$ is negative since it tends to decrease the beam's radius of curvature.



Section Properties:

$$\Sigma A = 0.15(0.015) + 0.1(0.02) = 0.00425\ \text{m}^2$$

$$\Sigma \bar{r} A = 0.475(0.15)(0.015) + 0.56(0.1)(0.02) = 2.18875(10^{-3})\ \text{m}$$

$$\bar{r} = \frac{\Sigma \bar{r} A}{\Sigma A} = \frac{2.18875\,(10^{-3})}{0.00425} = 0.5150\ \text{m}$$

$$\Sigma \int_A \frac{dA}{r} = 0.015 \ln \frac{0.55}{0.4} + 0.1 \ln \frac{0.57}{0.55} = 8.348614(10^{-3})\ \text{m}$$

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}} = \frac{0.00425}{8.348614(10^{-3})} = 0.509067\ \text{m}$$

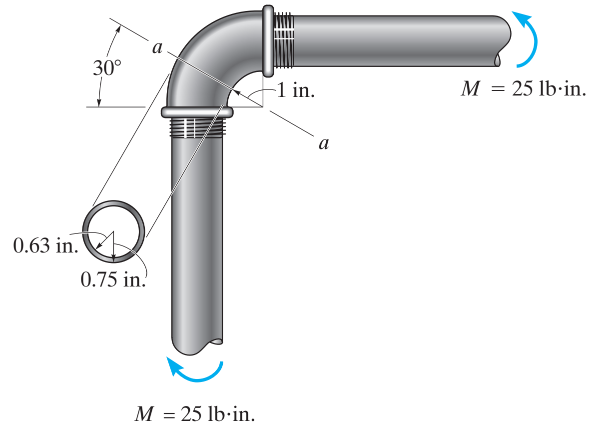
$$\bar{r} - R = 0.515 - 0.509067 = 5.933479(10^{-3})\ \text{m}$$

Normal Stress: Applying the curved-beam formula

$$\sigma_C = \frac{M(R - r_C)}{A r_C(\bar{r} - R)} = \frac{-900(0.509067 - 0.55)}{0.00425(0.55)(5.933479)(10^{-3})}$$

$$= 2.66\ \text{MPa (T)}$$ **Ans.**

6–150. The elbow of the pipe has an outer radius of 0.75 in. and an inner radius of 0.63 in. If the assembly is subjected to the moments of $M = 25$ lb·in., determine the maximum stress developed at section a–a.



$$\int_A \frac{dA}{r} = \Sigma 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$$

$$= 2\pi\left(1.75 - \sqrt{1.75^2 - 0.75^2}\right) - 2\pi\left(1.75 - \sqrt{1.75^2 - 0.63^2}\right)$$

$$= 0.32375809 \text{ in.}$$

$$A = \pi(0.75^2) - \pi(0.63^2) = 0.1656\,\pi$$

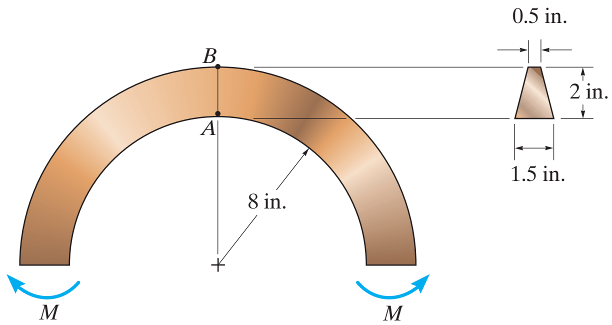
$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.1656\,\pi}{0.32375809} = 1.606902679 \text{ in.}$$

$$\bar{r} - R = 1.75 - 1.606902679 = 0.14309732 \text{ in.}$$

$$(\sigma_{max})_t = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{25(1.606902679 - 1)}{0.1656\,\pi(1)(0.14309732)} = 204 \text{ psi (T)}$$ **Ans.**

$$(\sigma_{max})_c = = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{25(1.606902679 - 2.5)}{0.1656\pi(2.5)(0.14309732)} = 120 \text{ psi (C)}$$ **Ans.**

6–151. The curved member is symmetric and is subjected to a moment of $M = 600$ lb·ft. Determine the bending stress in the member at points A and B. Show the stress acting on volume elements located at these points.



$$A = 0.5(2) + \frac{1}{2}(1)(2) = 2 \text{ in}^2$$

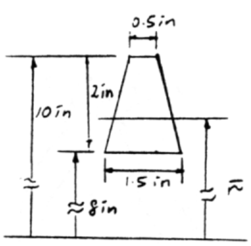
$$\bar{r} = \frac{\Sigma \bar{r} A}{\Sigma A} = \frac{9(0.5)(2) + 8.6667\left(\frac{1}{2}\right)(1)(2)}{2} = 8.83333 \text{ in.}$$

$$\int_A \frac{dA}{r} = 0.5 \ln\frac{10}{8} + \left[\frac{1(10)}{(10 - 8)}\left[\ln\frac{10}{8}\right] - 1\right] = 0.22729 \text{ in.}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{2}{0.22729} = 8.7993 \text{ in.}$$

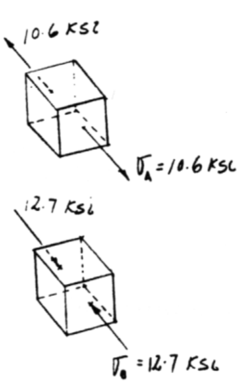
$$\bar{r} - R = 8.83333 - 8.7993 = 0.03398 \text{ in.}$$

$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

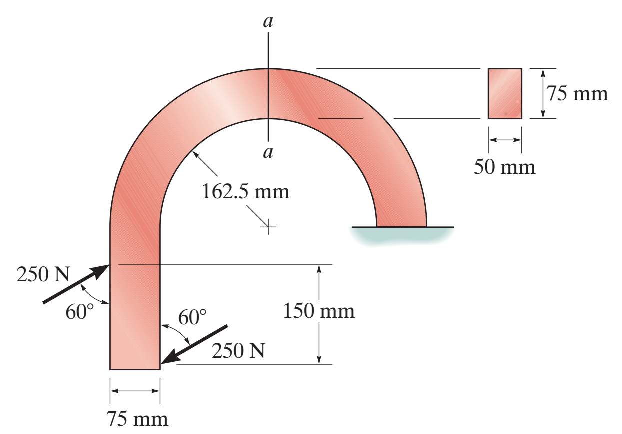


$$\sigma_A = \frac{600(12)(8.7993 - 8)}{2(8)(0.03398)} = 10.6 \text{ ksi (T)}$$ **Ans.**

$$\sigma_B = \frac{600(12)(8.7993 - 10)}{2(10)(0.03398)} = -12.7 \text{ ksi} = 12.7 \text{ ksi (C)}$$ **Ans.**



***6–152.** The curved bar used on a machine has a rectangular cross section. If the bar is subjected to a couple as shown, determine the maximum tensile and compressive stress acting at section $a-a$. Sketch the stress distribution on the section in three dimensions.



$$\circlearrowleft + \Sigma M_O = 0; \qquad M - 250 \cos 60° (0.075) - 250 \sin 60° (0.15) = 0$$

$$M = 41.851 \text{ N} \cdot \text{m}$$

$$\int_A \frac{dA}{r} = b \ln \frac{r_2}{r_1} = 0.05 \ln \frac{0.2375}{0.1625} = 0.018974481 \text{ m}$$

$$A = (0.075)(0.05) = 3.75(10^{-3}) \text{ m}^2$$

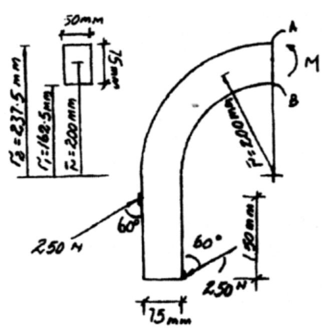
$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{3.75(10^{-3})}{0.018974481} = 0.197633863 \text{ m}$$

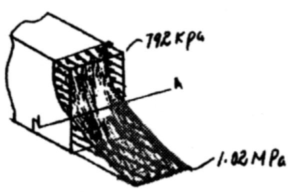
$$\bar{r} - R = 0.2 - 0.197633863 = 0.002366137$$

$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{41.851(0.197633863 - 0.2375)}{3.75(10^{-3})(0.2375)(0.002366137)} = -791.72 \text{ kPa}$$

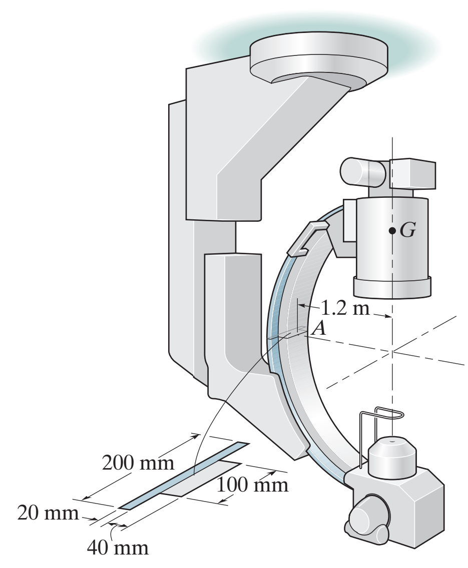
$$= 792 \text{ kPa (C)}$$ **Ans.**

$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{41.851\ (0.197633863 - 0.1625)}{3.75(10^{-3})(0.1625)(0.002366137)} = 1.02 \text{ MPa (T)}$$ **Ans.**





•6–153. The ceiling-suspended C-arm is used to support the X-ray camera used in medical diagnoses. If the camera has a mass of 150 kg, with center of mass at G, determine the maximum bending stress at section A.



Section Properties:

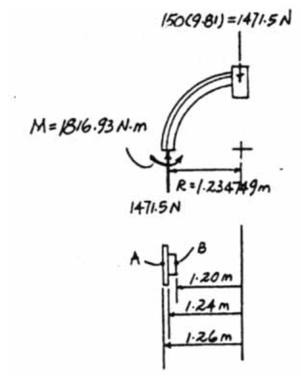
$$\bar{r} = \frac{\Sigma \bar{r} A}{\Sigma A} = \frac{1.22(0.1)(0.04) + 1.25(0.2)(0.02)}{0.1(0.04) + 0.2(0.02)} = 1.235 \text{ m}$$

$$\Sigma \int_A \frac{dA}{r} = 0.1 \ln \frac{1.24}{1.20} + 0.2 \ln \frac{1.26}{1.24} = 6.479051\left(10^{-3}\right)\text{m}$$

$$A = 0.1(0.04) + 0.2(0.02) = 0.008 \text{ m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.008}{6.479051\,(10^{-3})} = 1.234749 \text{ m}$$

$$\bar{r} - R = 1.235 - 1.234749 = 0.251183\left(10^{-3}\right) \text{ m}$$



Internal Moment: The internal moment must be computed about the neutral axis as shown on FBD. $M = -1816.93 \text{ N} \cdot \text{m}$ is negative since it tends to decrease the beam's radius of curvature.

Maximum Normal Stress: Applying the curved-beam formula

$$\sigma_A = \frac{M(R - r_A)}{A r_A (\bar{r} - R)}$$

$$= \frac{-1816.93(1.234749 - 1.26)}{0.008(1.26)(0.251183)(10^{-3})}$$

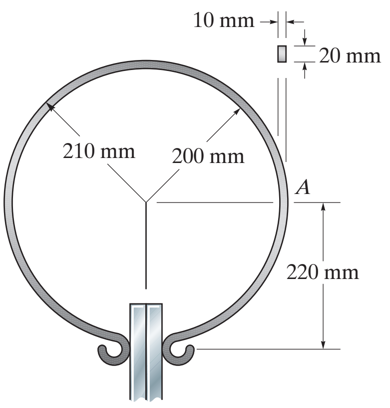
$$= 18.1 \text{ MPa (T)}$$

$$\sigma_B = \frac{M(R - r_B)}{A r_B (\bar{r} - R)}$$

$$= \frac{-1816.93(1.234749 - 1.20)}{0.008(1.20)(0.251183)(10^{-3})}$$

$$= -26.2 \text{ MPa} = 26.2 \text{ MPa (C)} \qquad \textbf{(Max)} \qquad \textbf{Ans.}$$

6–154. The circular spring clamp produces a compressive force of 3 N on the plates. Determine the maximum bending stress produced in the spring at A. The spring has a rectangular cross section as shown.



Internal Moment: As shown on FBD, $M = 0.660\ \text{N}\cdot\text{m}$ is positive since it tends to increase the beam's radius of curvature.

Section Properties:

$$\bar{r} = \frac{0.200 + 0.210}{2} = 0.205\ \text{m}$$

$$\int_A \frac{dA}{r} = b \ln \frac{r_2}{r_1} = 0.02 \ln \frac{0.21}{0.20} = 0.97580328\left(10^{-3}\right)\ \text{m}$$

$$A = (0.01)(0.02) = 0.200\left(10^{-3}\right)\ \text{m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.200(10^{-3})}{0.97580328(10^{-3})} = 0.204959343\ \text{m}$$

$$\bar{r} - R = 0.205 - 0.204959343 = 0.040657\left(10^{-3}\right)\ \text{m}$$

Maximum Normal Stress: Applying the curved-beam formula

$$\sigma_C = \frac{M(R - r_2)}{A r_2(\bar{r} - R)}$$

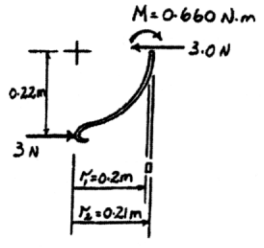
$$= \frac{0.660(0.204959343 - 0.21)}{0.200(10^{-3})(0.21)(0.040657)(10^{-3})}$$

$$= -1.95\text{MPa} = 1.95\ \text{MPa (C)}$$

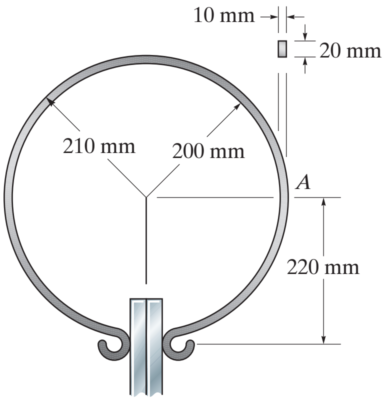
$$\sigma_t = \frac{M(R - r_1)}{A r_1(\bar{r} - R)}$$

$$= \frac{0.660(0.204959343 - 0.2)}{0.200(10^{-3})(0.2)(0.040657)(10^{-3})}$$

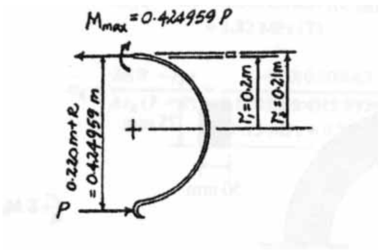
$$= 2.01\ \text{MPa}\ (\mathbf{T}) \qquad \text{(Max)}$$ **Ans.**



6–155. Determine the maximum compressive force the spring clamp can exert on the plates if the allowable bending stress for the clamp is $\sigma_{\text{allow}} = 4$ MPa.



Section Properties:



$$\bar{r} = \frac{0.200 + 0.210}{2} = 0.205 \text{ m}$$

$$\int_A \frac{dA}{r} = b \ln \frac{r_2}{r_1} = 0.02 \ln \frac{0.21}{0.20} = 0.97580328\left(10^{-3}\right) \text{ m}$$

$$A = (0.01)(0.02) = 0.200\left(10^{-3}\right) \text{ m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.200(10^{-3})}{0.97580328(10^{-3})} = 0.204959 \text{ m}$$

$$\bar{r} - R = 0.205 - 0.204959343 = 0.040657\left(10^{-3}\right) \text{ m}$$

Internal Moment: The internal moment must be computed about the neutral axis as shown on FBD. $M_{\text{max}} = 0.424959P$ is positive since it tends to increase the beam's radius of curvature.

Allowable Normal Stress: Applying the curved-beam formula

Assume compression failure

$$\sigma_c = \sigma_{\text{allow}} = \frac{M(R - r_2)}{Ar_2(\bar{r} - R)}$$

$$-4\left(10^6\right) = \frac{0.424959P(0.204959 - 0.21)}{0.200(10^{-3})(0.21)(0.040657)(10^{-3})}$$

$$P = 3.189 \text{ N}$$

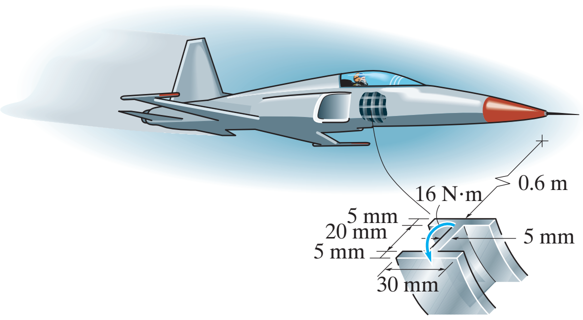
Assume tension failure

$$\sigma_t = \sigma_{\text{allow}} = \frac{M(R - r_1)}{Ar_1(\bar{r} - R)}$$

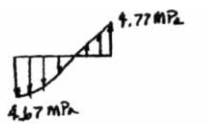
$$4\left(10^6\right) = \frac{0.424959P(0.204959 - 0.2)}{0.200(10^{-3})(0.2)(0.040657)(10^{-3})}$$

$$P = 3.09 \text{ N } (\textit{Controls !}) \qquad \textbf{Ans.}$$

***6–156.** While in flight, the curved rib on the jet plane is subjected to an anticipated moment of $M = 16\ \text{N}\cdot\text{m}$ at the section. Determine the maximum bending stress in the rib at this section, and sketch a two-dimensional view of the stress distribution.



$$\int_A dA/r = (0.03)\ln\frac{0.605}{0.6} + (0.005)\ln\frac{0.625}{0.605} + (0.03)\ln\frac{0.630}{0.625} = 0.650625(10^{-3})\text{ in.}$$



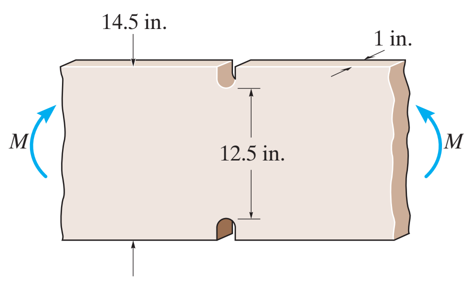
$$A = 2(0.005)(0.03) + (0.02)(0.005) = 0.4(10^{-3})\text{ in}^2$$

$$R = \frac{A}{\int_A dA/r} = \frac{0.4(10^{-3})}{0.650625(10^{-3})} = 0.6147933$$

$$(\sigma_c)_{\max} = \frac{M(R - r_c)}{Ar_A(\bar{r} - R)} = \frac{16(0.6147933 - 0.630)}{0.4(10^{-3})(0.630)(0.615 - 0.6147933)} = -4.67\text{ MPa}$$

$$(\sigma_s)_{\max} = \frac{M(R - r_s)}{Ar_A(\bar{r} - R)} = \frac{16(0.6147933 - 0.6)}{0.4(10^{-3})(0.6)(0.615 - 0.6147933)} = 4.77\text{ MPa}$$ **Ans.**

•6–157. If the radius of each notch on the plate is $r = 0.5$ in., determine the largest moment that can be applied. The allowable bending stress for the material is $\sigma_{\text{allow}} = 18$ ksi.



$$b = \frac{14.5 - 12.5}{2} = 1.0\text{ in.}$$

$$\frac{b}{r} = \frac{1}{0.5} = 2.0 \qquad \frac{r}{h} = \frac{0.5}{12.5} = 0.04$$

From Fig. 6-44:

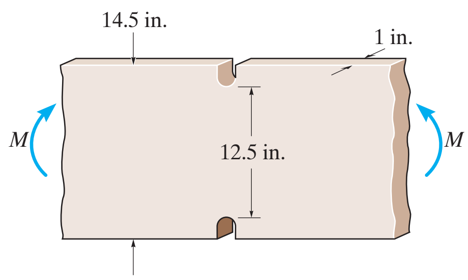
$$K = 2.60$$

$$\sigma_{\max} = K\frac{Mc}{I}$$

$$18(10^3) = 2.60\left[\frac{(M)(6.25)}{\frac{1}{12}(1)(12.5)^3}\right]$$

$$M = 180\,288\text{ lb}\cdot\text{in.} = 15.0\text{ kip}\cdot\text{ft}$$ **Ans.**

6–158. The symmetric notched plate is subjected to bending. If the radius of each notch is $r = 0.5$ in. and the applied moment is $M = 10$ kip·ft, determine the maximum bending stress in the plate.



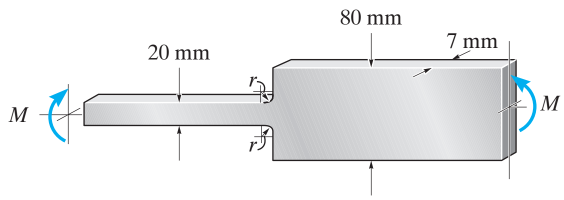
$$\frac{b}{r} = \frac{1}{0.5} = 2.0 \qquad \frac{r}{h} = \frac{0.5}{12.5} = 0.04$$

From Fig. 6-44:

$$K = 2.60$$

$$\sigma_{max} = K\frac{Mc}{I} = 2.60\left[\frac{(10)(12)(6.25)}{\frac{1}{12}(1)(12.5)^3}\right] = 12.0 \text{ ksi}$$ **Ans.**

6–159. The bar is subjected to a moment of $M = 40$ N·m. Determine the smallest radius r of the fillets so that an allowable bending stress of $\sigma_{allow} = 124$ MPa is not exceeded.



Allowable Bending Stress:

$$\sigma_{allow} = K\frac{Mc}{I}$$

$$124\left(10^6\right) = K\left[\frac{40(0.01)}{\frac{1}{12}(0.007)(0.02^3)}\right]$$

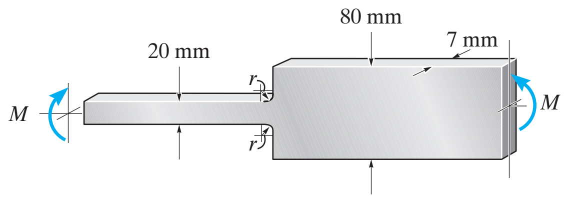
$$K = 1.45$$

Stress Concentration Factor: From the graph in the text with $\frac{w}{h} = \frac{80}{20} = 4$ and $K = 1.45$, then $\frac{r}{h} = 0.25$.

$$\frac{r}{20} = 0.25$$

$$r = 5.00 \text{ mm}$$ **Ans.**

***6–160.** The bar is subjected to a moment of $M = 17.5$ N·m. If $r = 5$ mm, determine the maximum bending stress in the material.



Stress Concentration Factor: From the graph in the text with $\frac{w}{h} = \frac{80}{20} = 4$ and $\frac{r}{h} = \frac{5}{20} = 0.25$, then $K = 1.45$.

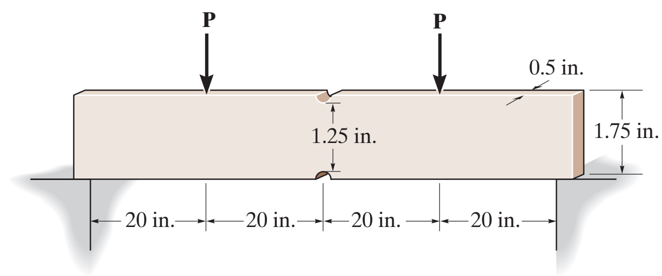
Maximum Bending Stress:

$$\sigma_{max} = K\frac{Mc}{I}$$

$$= 1.45\left[\frac{17.5(0.01)}{\frac{1}{12}(0.007)(0.02^3)}\right]$$

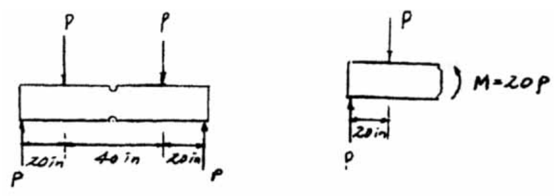
$$= 54.4 \text{ MPa}$$ **Ans.**

•6–161. The simply supported notched bar is subjected to two forces **P**. Determine the largest magnitude of **P** that can be applied without causing the material to yield. The material is A-36 steel. Each notch has a radius of $r = 0.125$ in.



$$b = \frac{1.75 - 1.25}{2} = 0.25$$

$$\frac{b}{r} = \frac{0.25}{0.125} = 2; \qquad \frac{r}{h} = \frac{0.125}{1.25} = 0.1$$

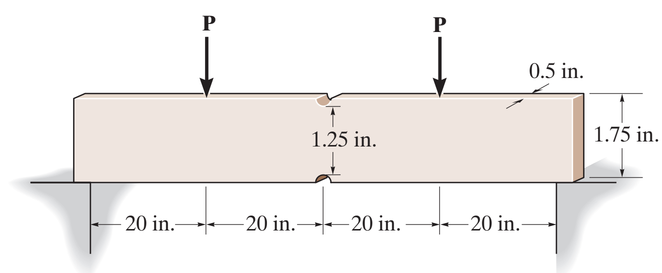


From Fig. 6-44. $K = 1.92$

$$\sigma_Y = K\frac{Mc}{I}; \qquad 36 = 1.92\left[\frac{20P(0.625)}{\frac{1}{12}(0.5)(1.25)^3}\right]$$

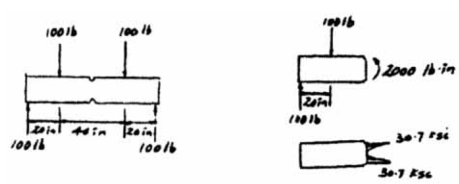
$P = 122$ lb **Ans.**

6–162. The simply supported notched bar is subjected to the two loads, each having a magnitude of $P = 100$ lb. Determine the maximum bending stress developed in the bar, and sketch the bending-stress distribution acting over the cross section at the center of the bar. Each notch has a radius of $r = 0.125$ in.



$$b = \frac{1.75 - 1.25}{2} = 0.25$$

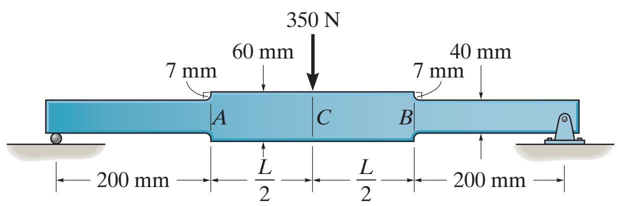
$$\frac{b}{r} = \frac{0.25}{0.125} = 2; \qquad \frac{r}{h} = \frac{0.125}{1.25} = 0.1$$



From Fig. 6-44, $K = 1.92$

$$\sigma_{max} = K\frac{Mc}{I} = 1.92\left[\frac{2000(0.625)}{\frac{1}{12}(0.5)(1.25)^3}\right] = 29.5 \text{ ksi}$$ **Ans.**

6–163. Determine the length L of the center portion of the bar so that the maximum bending stress at A, B, and C is the same. The bar has a thickness of 10 mm.



$$\frac{w}{h} = \frac{60}{40} = 1.5 \qquad \frac{r}{h} = \frac{7}{40} = 0.175$$

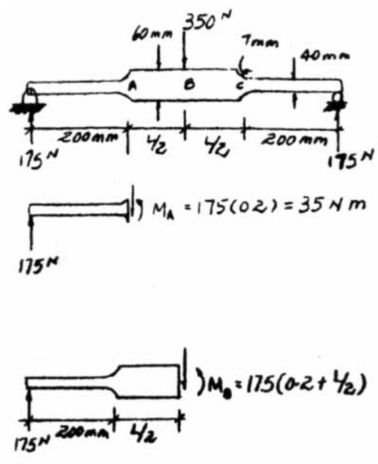
From Fig. 6-43, $K = 1.5$

$$(\sigma_A)_{max} = K\frac{M_A c}{I} = 1.5\left[\frac{(35)(0.02)}{\frac{1}{12}(0.01)(0.04^3)}\right] = 19.6875\text{ MPa}$$

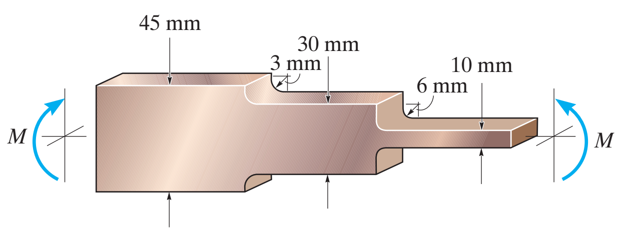
$$(\sigma_B)_{max} = (\sigma_A)_{max} = \frac{M_B c}{I}$$

$$19.6875(10^6) = \frac{175(0.2 + \frac{L}{2})(0.03)}{\frac{1}{12}(0.01)(0.06^3)}$$

$$L = 0.95\text{ m} = 950\text{ mm}$$ **Ans.**



***6–164.** The stepped bar has a thickness of 15 mm. Determine the maximum moment that can be applied to its ends if it is made of a material having an allowable bending stress of $\sigma_{allow} = 200$ MPa.



Stress Concentration Factor:

For the smaller section with $\frac{w}{h} = \frac{30}{10} = 3$ and $\frac{r}{h} = \frac{6}{10} = 0.6$, we have $K = 1.2$ obtained from the graph in the text.

For the larger section with $\frac{w}{h} = \frac{45}{30} = 1.5$ and $\frac{r}{h} = \frac{3}{30} = 0.1$, we have $K = 1.75$ obtained from the graph in the text.

Allowable Bending Stress:

For the smaller section

$$\sigma_{max} = \sigma_{allow} = K\frac{Mc}{I};$$

$$200\left(10^6\right) = 1.2\left[\frac{M(0.005)}{\frac{1}{12}(0.015)(0.01^3)}\right]$$

$$M = 41.7\text{ N}\cdot\text{m} \quad (\textit{Controls !})$$ **Ans.**

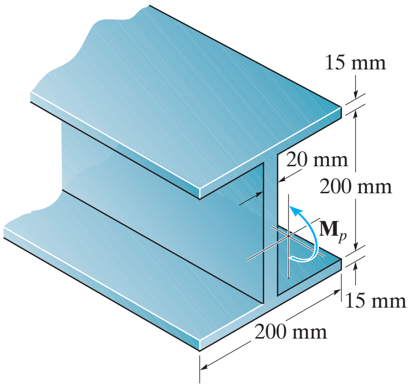
For the larger section

$$\sigma_{max} = \sigma_{allow} = K\frac{Mc}{I};$$

$$200\left(10^6\right) = 1.75\left[\frac{M(0.015)}{\frac{1}{12}(0.015)(0.03^3)}\right]$$

$$M = 257\text{ N}\cdot\text{m}$$

•6–165. The beam is made of an elastic plastic material for which $\sigma_Y = 250$ MPa. Determine the residual stress in the beam at its top and bottom after the plastic moment $\mathbf{M}_p$ is applied and then released.



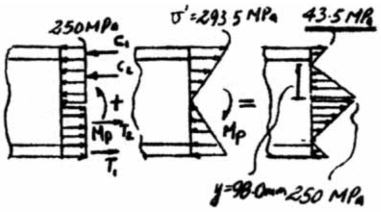
$$I_x = \frac{1}{12}(0.2)(0.23)^3 - \frac{1}{12}(0.18)(0.2)^3 = 82.78333(10^{-6})\text{m}^4$$

$$C_1 = T_1 = \sigma_Y(0.2)(0.015) = 0.003\sigma_Y$$

$$C_2 = T_2 = \sigma_Y(0.1)(0.02) = 0.002\sigma_Y$$

$$M_p = 0.003\sigma_Y(0.215) + 0.002\sigma_Y(0.1) = 0.000845\,\sigma_Y$$

$$= 0.000845(250)(10^6) = 211.25\text{ kN}\cdot\text{m}$$

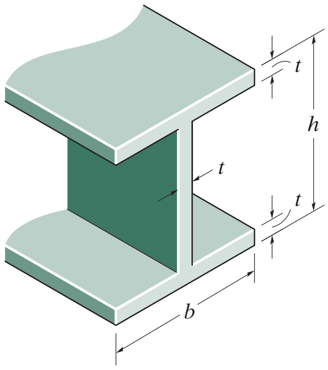


$$\sigma = \frac{M_p c}{I} = \frac{211.25(10^3)(0.115)}{82.78333(10^{-6})} = 293.5\text{ MPa}$$

$$\frac{y}{250} = \frac{0.115}{293.5}; \qquad y = 0.09796\text{ m} = 98.0\text{ mm}$$

$$\sigma_{top} = \sigma_{bottom} = 293.5 - 250 = 43.5\text{ MPa} \qquad \textbf{Ans.}$$

6–166. The wide-flange member is made from an elastic-plastic material. Determine the shape factor.

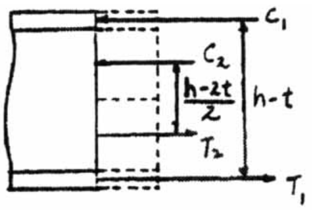


Plastic analysis:

$$T_1 = C_1 = \sigma_Y bt; \qquad T_2 = C_2 = \sigma_Y\left(\frac{h-2t}{2}\right)t$$

$$M_P = \sigma_Y bt(h-t) + \sigma_Y\left(\frac{h-2t}{2}\right)(t)\left(\frac{h-2t}{2}\right)$$

$$= \sigma_Y\left[bt(h-t) + \frac{t}{4}(h-2t)^2\right]$$



Elastic analysis:

$$I = \frac{1}{12}bh^3 - \frac{1}{12}(b-t)(h-2t)^3$$

$$= \frac{1}{12}[bh^3 - (b-t)(h-2t)^3]$$

$$M_Y = \frac{\sigma_y I}{c} = \frac{\sigma_Y\left(\frac{1}{12}\right)[bh^3 - (b-t)(h-2t)^3]}{\frac{h}{2}}$$

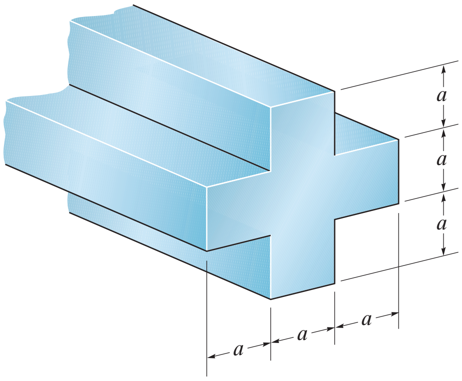
$$= \frac{bh^3 - (b-t)(h-2t)^3}{6h}\sigma_Y$$

Shape factor:

$$k = \frac{M_P}{M_Y} = \frac{[bt(h-t) + \frac{t}{4}(h-2t)^2]\sigma_Y}{\frac{bh^3-(b-t)(h-2t)^3}{6h}\sigma_Y}$$

$$= \frac{3h}{2}\left[\frac{4bt(h-t) + t(h-2t)^2}{bh^3 - (b-t)(h-2t)^3}\right] \qquad \textbf{Ans.}$$

6–167. Determine the shape factor for the cross section.



Maximum Elastic Moment: The moment of inertia about neutral axis must be determined first.

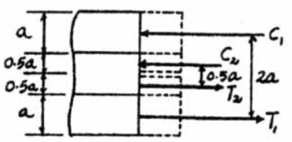
$$I_{NA} = \frac{1}{12}(a)(3a)^3 + \frac{1}{12}(2a)\left(a^3\right) = 2.41667a^4$$

Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y\,(2.41667a^4)}{1.5a} = 1.6111a^3\sigma_Y$$

Plastic Moment:



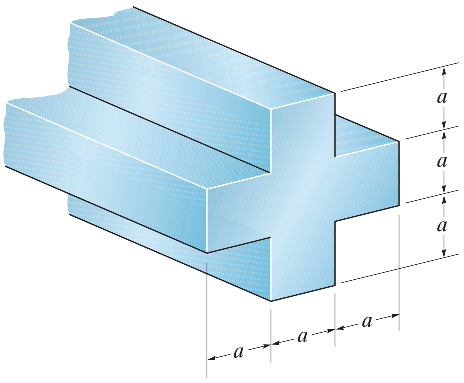
$$M_P = \sigma_Y\,(a)(a)(2a) + \sigma_Y\,(0.5a)(3a)(0.5a)$$

$$= 2.75a^3\sigma_Y$$

Shape Factor:

$$k = \frac{M_P}{M_Y} = \frac{2.75a^3\sigma_Y}{1.6111a^3\sigma_Y} = 1.71$$ **Ans.**

***6–168.** The beam is made of elastic perfectly plastic material. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. Take $a = 2$ in. and $\sigma_Y = 36$ ksi.



Maximum Elastic Moment: The moment of inertia about neutral axis must be determined first.

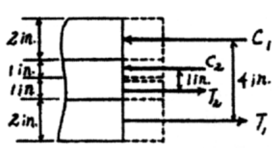
$$I_{NA} = \frac{1}{12}(2)\left(6^3\right) + \frac{1}{12}(4)\left(2^3\right) = 38.667\ \text{in}^4$$

Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \ = \frac{M_Y\,c}{I}$$

$$M_Y = \frac{\sigma_Y\,I}{c} = \frac{36(38.667)}{3}$$

$$= 464\ \text{kip}\cdot\text{in} = 38.7\ \text{kip}\cdot\text{ft}$$ **Ans.**

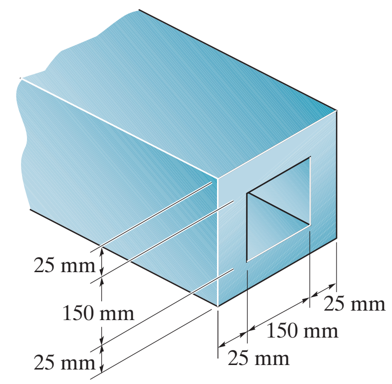


Plastic Moment:

$$M_P = 36(2)(2)(4) + 36(1)(6)(1)$$

$$= 792\ \text{kip}\cdot\text{in} = 66.0\ \text{kip}\cdot\text{ft}$$ **Ans.**

•6–169. The box beam is made of an elastic perfectly plastic material for which $\sigma_Y = 250$ MPa. Determine the residual stress in the top and bottom of the beam after the plastic moment $\mathbf{M}_p$ is applied and then released.



Plastic Moment:

$$M_P = 250\left(10^6\right)(0.2)(0.025)(0.175) + 250\left(10^6\right)(0.075)(0.05)(0.075)$$

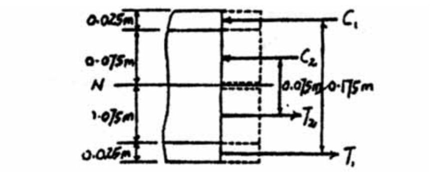
$$= 289062.5 \text{ N} \cdot \text{m}$$

Modulus of Rupture: The modulus of rupture σ_r can be determined using the flexure formula with the application of reverse, plastic moment $M_P = 289062.5 \text{ N} \cdot \text{m}$.

$$I = \frac{1}{12}(0.2)\left(0.2^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right)$$

$$= 91.14583\left(10^{-6}\right) \text{ m}^4$$

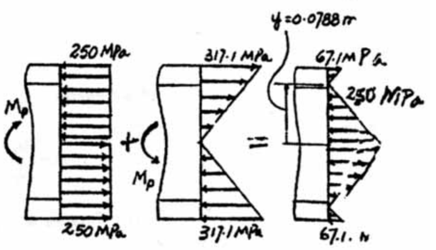
$$\sigma_r = \frac{M_P\, c}{I} = \frac{289062.5\,(0.1)}{91.14583\left(10^{-6}\right)} = 317.41 \text{ MPa}$$



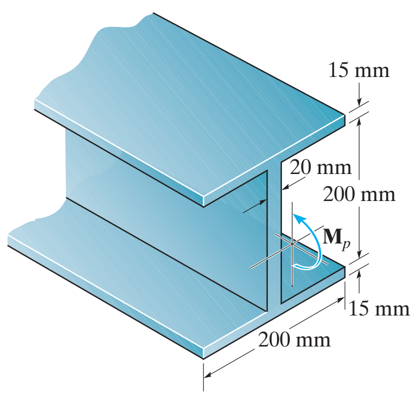
Residual Bending Stress: As shown on the diagram.

$$\sigma'_{\text{top}} = \sigma'_{\text{bot}} = \sigma_r - \sigma_Y$$

$$= 317.14 - 250 = 67.1 \text{ MPa} \qquad \textbf{Ans.}$$



6–170. Determine the shape factor for the wide-flange beam.



$$I_x = \frac{1}{12}(0.2)(0.23)^3 - \frac{1}{12}(0.18)(0.2)^3 = 82.78333\left(10^{-6}\right) \text{ m}^4$$

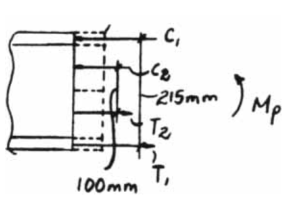
$$C_1 = T_1 = \sigma_Y(0.2)(0.015) = 0.003\sigma_Y$$

$$C_2 = T_2 = \sigma_Y(0.1)(0.02) = 0.002\sigma_Y$$

$$M_p = 0.003\sigma_Y(0.215) + 0.002\sigma_Y(0.1) = 0.000845\,\sigma_Y$$

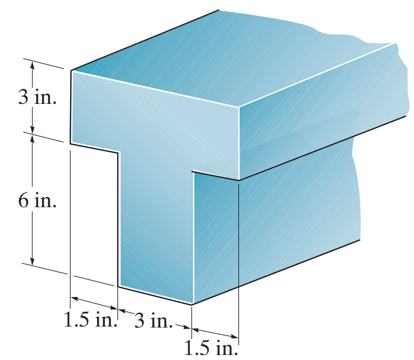
$$\sigma_Y = \frac{M_Y\, c}{I}$$

$$M_Y = \frac{\sigma_Y\left(82.78333)10^{-6}\right)}{0.115} = 0.000719855\,\sigma_Y$$



$$k = \frac{M_p}{M_Y} = \frac{0.000845\sigma_Y}{0.000719855\sigma_Y} = 1.17 \qquad \textbf{Ans.}$$

6–171. Determine the shape factor of the beam's cross section.



Referring to Fig. *a*, the location of centroid of the cross-section is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{7.5(3)(6) + 3(6)(3)}{3(6) + 6(3)} = 5.25 \text{ in.}$$

The moment of inertia of the cross-section about the neutral axis is

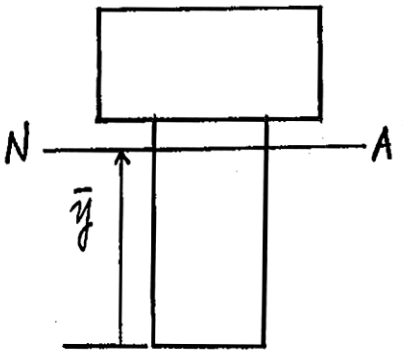
$$I = \frac{1}{12}(3)\left(6^3\right) + 3(6)(5.25 - 3)^2 + \frac{1}{12}(6)\left(3^3\right) + 6(3)(7.5 - 5.25)^2$$

$$= 249.75 \text{ in}^4$$

Here $\sigma_{max} = \sigma_Y$ and $c = \bar{y} = 5.25$ in. Thus

$$\sigma_{max} = \frac{Mc}{I}; \qquad \sigma_Y = \frac{M_Y(5.25)}{249.75}$$

$$M_Y = 47.571\sigma_Y$$



(a)

Referring to the stress block shown in Fig. *b*,

$$\int_A \sigma dA = 0; \qquad T - C_1 - C_2 = 0$$

$$d(3)\sigma_Y - (6 - d)(3)\sigma_Y - 3(6)\sigma_Y = 0$$

$$d = 6 \text{ in.}$$

Since $d = 6$ in., $c_1 = 0$, Fig. *c*. Here

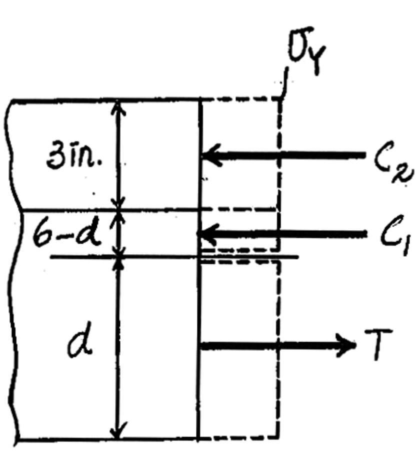
$$T = C = 3(6)\,\sigma_Y = 18\,\sigma_Y$$

Thus,

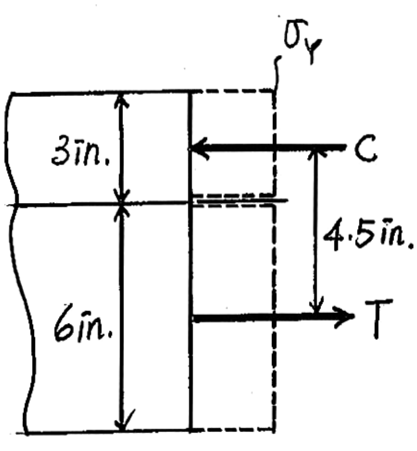
$$M_P = T(4.5) = 18\,\sigma_Y\,(4.5) = 81\,\sigma_Y$$

Thus,

$$k = \frac{M_P}{M_Y} = \frac{81\,\sigma_Y}{47.571\,\sigma_Y} = 1.70$$ **Ans.**

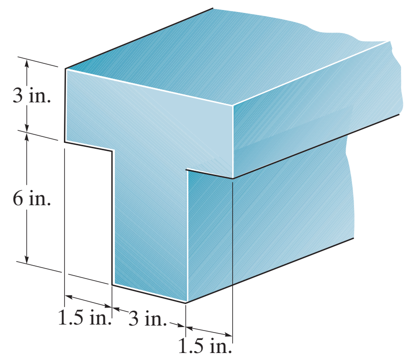


(b)



(c)

***6–172.** The beam is made of elastic-perfectly plastic material. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. Take $\sigma_Y = 36$ ksi.



Referring to Fig. *a*, the location of centroid of the cross-section is

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{7.5(3)(6) + 3(6)(3)}{3(6) + 6(3)} = 5.25 \text{ in.}$$

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(3)(6^3) + 3(6)(5.25 - 3)^2 + \frac{1}{12}(6)(3^3) + 6(3)(7.5 - 5.25)^2$$

$$= 249.75 \text{ in}^4$$

Here, $\sigma_{max} = \sigma_Y = 36$ ksi and $c = \bar{y} = 5.25$ in. Then

$$\sigma_{max} = \frac{Mc}{I}; \qquad 36 = \frac{M_Y(5.25)}{249.75}$$

$$M_Y = 1712.57 \text{ kip} \cdot \text{in} = 143 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

Referring to the stress block shown in Fig. *b*,

$$\int_A \sigma dA = 0; \qquad T - C_1 - C_2 = 0$$

$$d(3)(36) - (6 - d)(3)(36) - 3(6)(36) = 0$$

$$d = 6 \text{ in.}$$

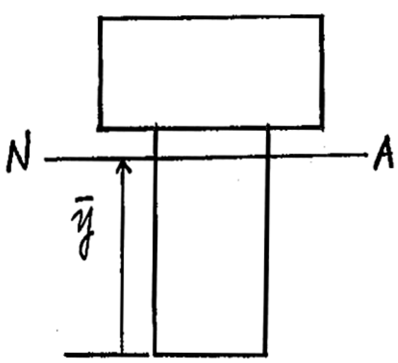
Since $d = 6$ in., $C_1 = 0$,

Here,

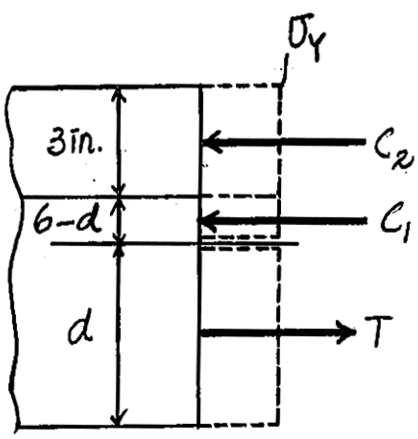
$$T = C = 3(6)(36) = 648 \text{ kip}$$

Thus,

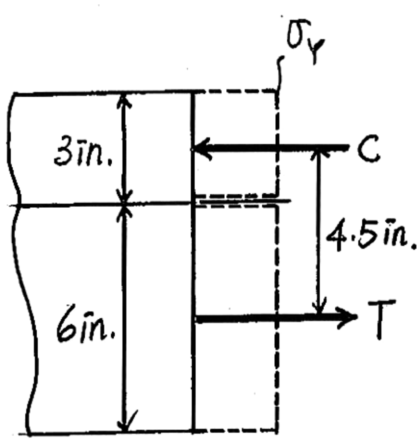
$$M_P = T(4.5) = 648(4.5) = 2916 \text{ kip} \cdot \text{in} = 243 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$



(a)

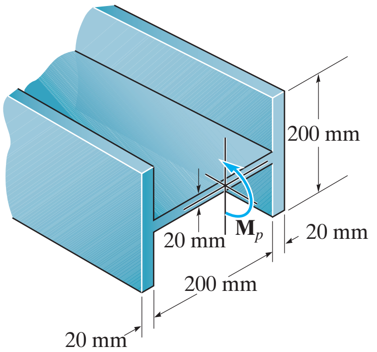


(b)



(c)

•6–173. Determine the shape factor for the cross section of the H-beam.



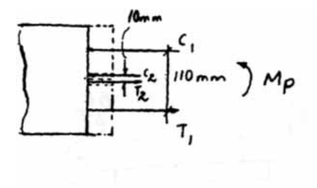
$$I_x = \frac{1}{12}(0.2)(0.02^3) + 2\left(\frac{1}{12}\right)(0.02)(0.2^3) = 26.8(10^{-6})\text{m}^4$$

$$C_1 = T_1 = \sigma_Y(2)(0.09)(0.02) = 0.0036\sigma_y$$

$$C_2 = T_2 = \sigma_Y(0.01)(0.24) = 0.0024\sigma_y$$

$$M_p = 0.0036\sigma_Y(0.11) + 0.0024\sigma_Y(0.01) = 0.00042\sigma_Y$$

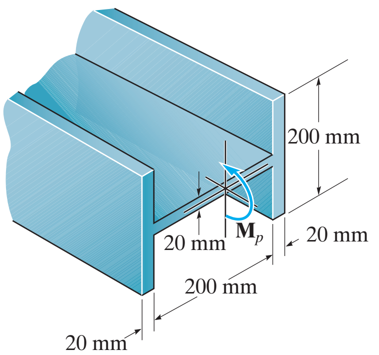
$$\sigma_Y = \frac{M_Y c}{I}$$



$$M_Y = \frac{\sigma_Y(26.8)(10^{-6})}{0.1} = 0.000268\sigma_Y$$

$$k = \frac{M_p}{M_Y} = \frac{0.00042\sigma_Y}{0.000268\sigma_Y} = 1.57 \qquad \textbf{Ans.}$$

6–174. The H-beam is made of an elastic-plastic material for which $\sigma_Y = 250$ MPa. Determine the residual stress in the top and bottom of the beam after the plastic moment $\mathbf{M}_p$ is applied and then released.

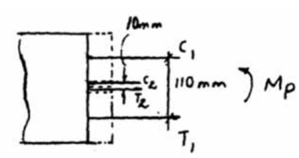


$$I_x = \frac{1}{12}(0.2)(0.02^3) + 2\left(\frac{1}{12}\right)(0.02)(0.2^3) = 26.8(10^{-6})\text{m}^4$$

$$C_1 = T_1 = \sigma_Y(2)(0.09)(0.02) = 0.0036\sigma_y$$

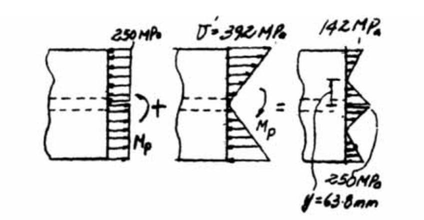
$$C_2 = T_2 = \sigma_Y(0.01)(0.24) = 0.0024\sigma_y$$

$$M_p = 0.0036\sigma_Y(0.11) + 0.0024\sigma_Y(0.01) = 0.00042\sigma_Y$$



$$M_p = 0.00042(250)\left(10^6\right) = 105 \text{ kN}\cdot\text{m}$$

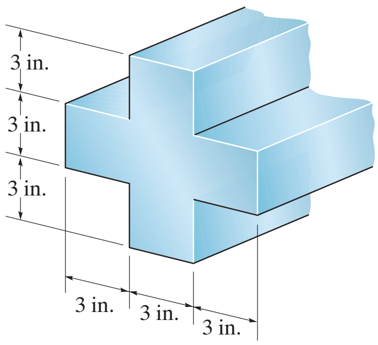
$$\sigma' = \frac{M_p c}{I} = \frac{105(10^3)(0.1)}{26.8(10^{-6})} = 392 \text{ MPa}$$



$$\frac{y}{250} = \frac{0.1}{392}; \qquad y = 0.0638 = 63.8 \text{ mm}$$

$$\sigma_T = \sigma_B = 392 - 250 = 142 \text{ MPa} \qquad \textbf{Ans.}$$

6–175. Determine the shape factor of the cross section.



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(3)(9^3) + \frac{1}{12}(6)(3^3) = 195.75 \text{ in}^4$$

Here, $\sigma_{max} = \sigma_Y$ and $c = 4.5$ in. Then

$$\sigma_{max} = \frac{Mc}{I}; \qquad \sigma_Y = \frac{M_Y(4.5)}{195.75}$$

$$M_Y = 43.5\,\sigma_Y$$

Referring to the stress block shown in Fig. *a*,

$$T_1 = C_1 = 3(3)\sigma_Y = 9\,\sigma_Y$$

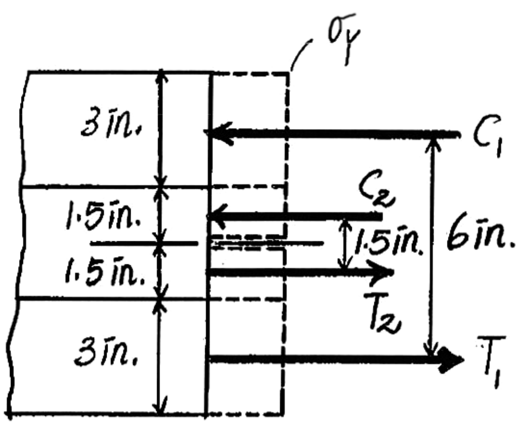
$$T_2 = C_2 = 1.5(9)\sigma_Y = 13.5\,\sigma_Y$$

Thus,

$$M_P = T_1(6) + T_2(1.5)$$

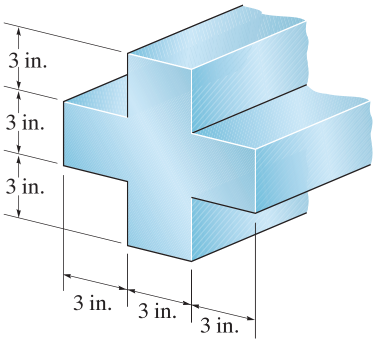
$$= 9\sigma_Y(6) + 13.5\sigma_Y(1.5) = 74.25\,\sigma_Y$$

$$k = \frac{M_P}{M_Y} = \frac{74.25\,\sigma_Y}{43.5\,\sigma_Y} = 1.71$$ **Ans.**



(a)

***6–176.** The beam is made of elastic-perfectly plastic material. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. Take $\sigma_Y = 36$ ksi.



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(3)(9^3) + \frac{1}{12}(6)(3^3) = 195.75 \text{ in}^4$$

Here, $\sigma_{max} = \sigma_Y = 36$ ksi and $c = 4.5$ in. Then

$$\sigma_{max} = \frac{Mc}{I}; \qquad 36 = \frac{M_Y(4.5)}{195.75}$$

$$M_Y = 1566 \text{ kip} \cdot \text{in} = 130.5 \text{ kip} \cdot \text{ft}$$ **Ans.**

Referring to the stress block shown in Fig. *a*,

$$T_1 = C_1 = 3(3)(36) = 324 \text{ kip}$$

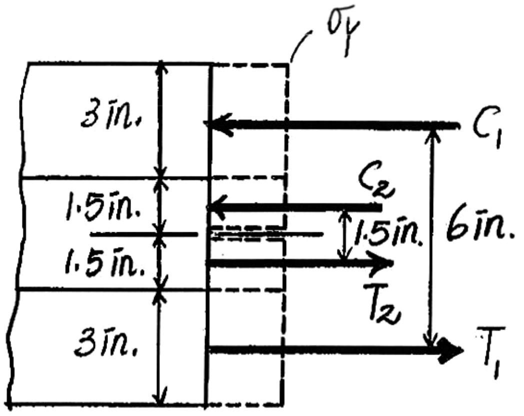
$$T_2 = C_2 = 1.5(9)(36) = 486 \text{ kip}$$

Thus,

$$M_P = T_1(6) + T_2(1.5)$$

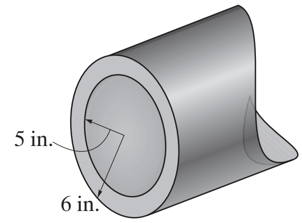
$$= 324(6) + 486(1.5)$$

$$= 2673 \text{ kip} \cdot \text{in.} = 222.75 \text{ kip} \cdot \text{ft} = 223 \text{ kip} \cdot \text{ft}$$ **Ans.**



(a)

•6–177. Determine the shape factor of the cross section for the tube.



The moment of inertia of the tube's cross-section about the neutral axis is

$$I = \frac{\pi}{4}\left(r_o^4 - r_i^4\right) = \frac{\pi}{4}\left(6^4 - 5^4\right) = 167.75\ \pi\ \text{in}^4$$

Here, $\sigma_{max} = \sigma_Y$ and $C = r_o = 6$ in,

$$\sigma_{max} = \frac{Mc}{I}; \qquad \sigma_Y = \frac{M_Y\,(6)}{167.75\ \pi}$$

$$M_Y = 87.83\ \sigma_Y$$

The plastic Moment of the table's cross-section can be determined by super posing the moment of the stress block of the solid circular cross-section with radius $r_o = 6$ in and $r_i = 5$ in. as shown in Figure a, Here,

$$T_1 = C_1 = \frac{1}{2}\pi(6^2)\sigma_Y = 18\pi\sigma_Y$$

$$T_2 = C_2 = \frac{1}{2}\pi(5^2)\sigma_Y = 12.5\pi\ \sigma_Y$$

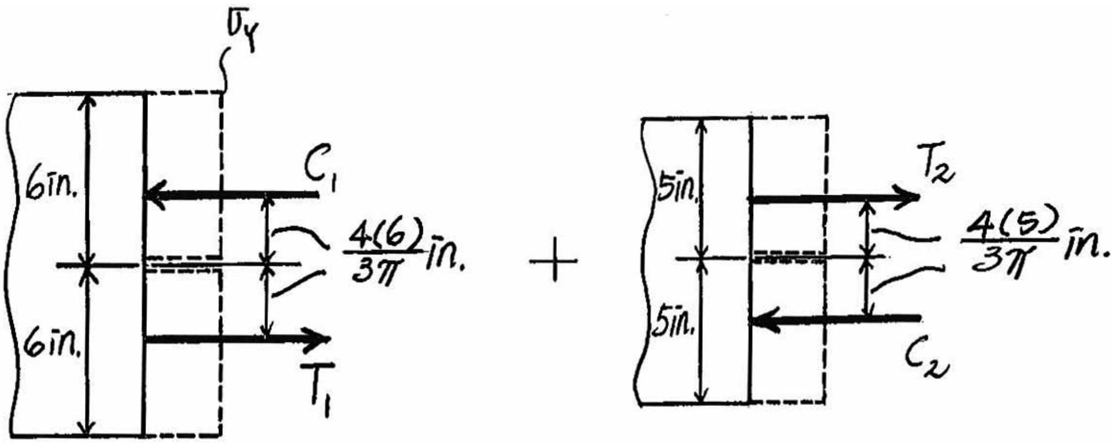
Thus,

$$M_P = T_1\left\{2\left[\frac{4(6)}{3\pi}\right]\right\} - T_2\left\{2\left[\frac{4(5)}{3\pi}\right]\right\}$$

$$= (18\pi\sigma_Y)\left(\frac{16}{\pi}\right) - 12.5\pi\sigma_Y\left(\frac{40}{3\pi}\right)$$

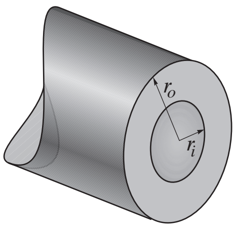
$$= 121.33\ \sigma_Y$$

$$k = \frac{M_P}{M_Y} = \frac{121.33\ \sigma_Y}{87.83\ \sigma_Y} = 1.38$$ **Ans.**



(a)

6–178. The beam is made from elastic-perfectly plastic material. Determine the shape factor for the thick-walled tube.



Maximum Elastic Moment. The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{\pi}{4}\left(r_o{}^4 - r_i^4\right)$$

With $c = r_o$ and $\sigma_{\max} = \sigma_Y$,

$$\sigma_{\max} = \frac{Mc}{I}; \qquad \sigma_Y = \frac{M_Y(r_o)}{\frac{\pi}{4}\left(r_o{}^4 - r_i{}^4\right)}$$

$$M_Y = \frac{\pi}{4r_o}\left(r_o{}^4 - r_i{}^4\right)\sigma_Y$$

Plastic Moment. The plastic moment of the cross section can be determined by superimposing the moment of the stress block of the solid beam with radius r_0 and r_i as shown in Fig. *a*, Referring to the stress block shown in Fig. *a*,

$$T_1 = c_1 = \frac{\pi}{2} r_o{}^2\sigma_Y$$

$$T_2 = c_2 = \frac{\pi}{2} r_i{}^2\sigma_Y$$

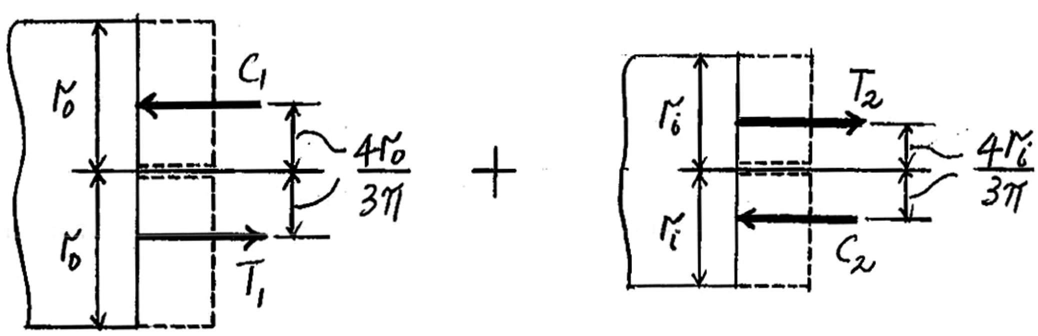
$$M_P = T_1\left[2\left(\frac{4r_o}{3\pi}\right)\right] - T_2\left[2\left(\frac{4r_i}{3\pi}\right)\right]$$

$$= \frac{\pi}{2} r_o{}^2\sigma_Y\left(\frac{8r_o}{3\pi}\right) - \frac{\pi}{2}r_i{}^2\sigma_Y\left(\frac{8r_i}{3\pi}\right)$$

$$= \frac{4}{3}\left(r_o{}^3 - r_i{}^3\right)\sigma_Y$$

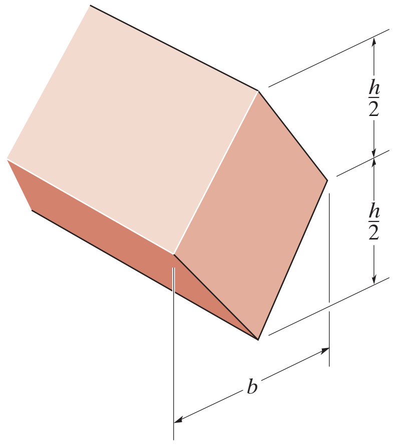
Shape Factor.

$$k = \frac{M_P}{M_Y} = \frac{\frac{4}{3}\left(r_o{}^3 - r_i{}^3\right)\sigma_Y}{\frac{\pi}{4r_o}\left(r_o{}^4 - r_i{}^4\right)\sigma_Y} = \frac{16r_o\left(r_o{}^3 - r_i{}^3\right)}{3\pi\left(r_o{}^4 - r_i{}^4\right)} \qquad \textbf{Ans.}$$



(a)

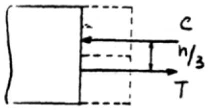
6–179. Determine the shape factor for the member.



Plastic analysis:

$$T = C = \frac{1}{2}(b)\left(\frac{h}{2}\right)\sigma_Y = \frac{b\,h}{4}\sigma_Y$$

$$M_P = \frac{b\,h}{4}\sigma_Y\left(\frac{h}{3}\right) = \frac{b\,h^2}{12}\sigma_Y$$



Elastic analysis:

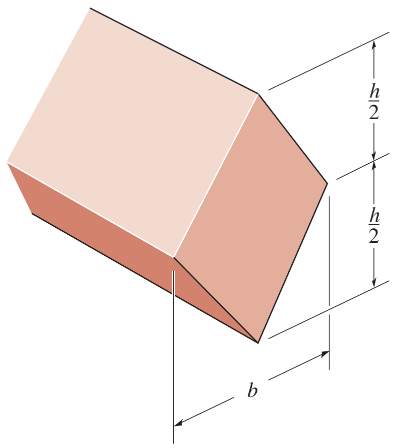
$$I = 2\left[\frac{1}{12}(b)\left(\frac{h}{2}\right)^3\right] = \frac{b\,h^3}{48}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y\left(\frac{bh^3}{48}\right)}{\frac{h}{2}} = \frac{b\,h^2}{24}\sigma_Y$$

Shape factor:

$$k = \frac{M_p}{M_Y} = \frac{\frac{bh^2}{12}\sigma_Y}{\frac{bh^2}{24}\sigma_Y} = 2$$ **Ans.**

***6–180.** The member is made from an elastic-plastic material. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. Take $b = 4$ in., $h = 6$ in., $\sigma_Y = 36$ ksi.



Elastic analysis:

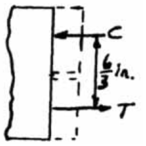
$$I = 2\left[\frac{1}{12}(4)(3)^3\right] = 18 \text{ in}^4$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{36(18)}{3} = 216 \text{ kip} \cdot \text{in.} = 18 \text{ kip} \cdot \text{ft}$$ **Ans.**

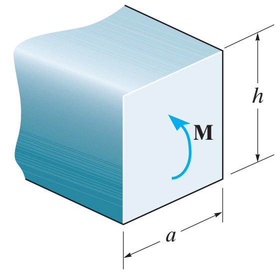
Plastic analysis:

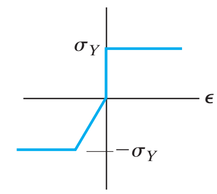
$$T = C = \frac{1}{2}(4)(3)(36) = 216 \text{ kip}$$

$$M_p = 2160\left(\frac{6}{3}\right) = 432 \text{ kip} \cdot \text{in.} = 36 \text{ kip} \cdot \text{ft}$$ **Ans.**



•6–181. The beam is made of a material that can be assumed perfectly plastic in tension and elastic perfectly plastic in compression. Determine the maximum bending moment M that can be supported by the beam so that the compressive material at the outer edge starts to yield.



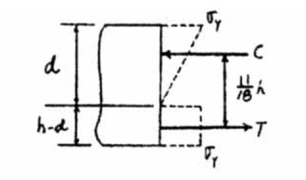


$$\int_A \sigma \, dA = 0; \qquad C - T = 0$$

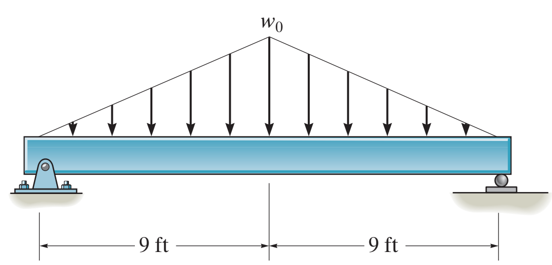
$$\frac{1}{2}\sigma_Y(d)(a) - \sigma_Y(h - d)a = 0$$

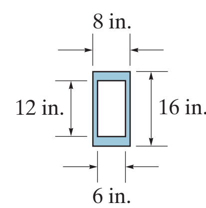
$$d = \frac{2}{3}h$$

$$M = \frac{1}{2}\sigma_Y\left(\frac{2}{3}h\right)(a)\left(\frac{11}{18}h\right) = \frac{11a\,h^2}{54}\sigma_Y \qquad \textbf{Ans.}$$



6–182. The box beam is made from an elastic-plastic material for which $\sigma_Y = 25$ ksi. Determine the intensity of the distributed load w_0 that will cause the moment to be (a) the largest elastic moment and (b) the largest plastic moment.





Elastic analysis:

$$I = \frac{1}{12}(8)(16^3) - \frac{1}{12}(6)(12^3) = 1866.67 \text{ in}^4$$

$$M_{\max} = \frac{\sigma_Y I}{c}; \qquad 27w_0(12) = \frac{25(1866.67)}{8}$$

$w_0 = 18.0$ kip/ft **Ans.**

Plastic analysis:

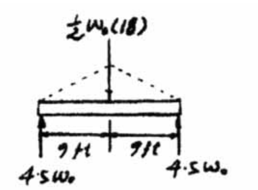
$$C_1 = T_1 = 25(8)(2) = 400 \text{ kip}$$

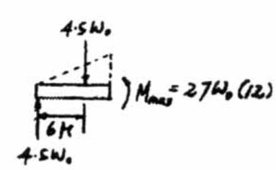
$$C_2 = T_2 = 25(6)(2) = 300 \text{ kip}$$

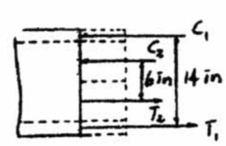
$$M_P = 400(14) + 300(6) = 7400 \text{ kip} \cdot \text{in.}$$

$$27w_0(12) = 7400$$

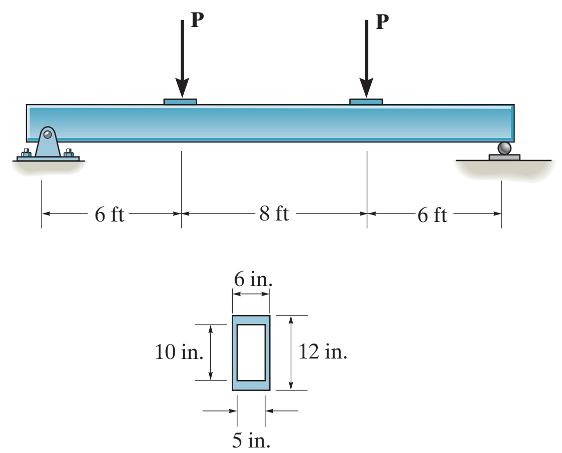
$w_0 = 22.8$ kip/ft **Ans.**







6–183. The box beam is made from an elastic-plastic material for which $\sigma_Y = 36$ ksi. Determine the magnitude of each concentrated force **P** that will cause the moment to be (a) the largest elastic moment and (b) the largest plastic moment.



From the moment diagram shown in Fig. *a*, $M_{max} = 6\,P$.

The moment of inertia of the beam's cross-section about the neutral axis is

$$I = \frac{1}{12}(6)(12^3) - \frac{1}{12}(5)(10^3) = 447.33 \text{ in}^4$$

Here, $\sigma_{max} = \sigma_Y = 36$ ksi and $c = 6$ in.

$$\sigma_{max} = \frac{Mc}{I}; \qquad 36 = \frac{M_Y\,(6)}{447.33}$$

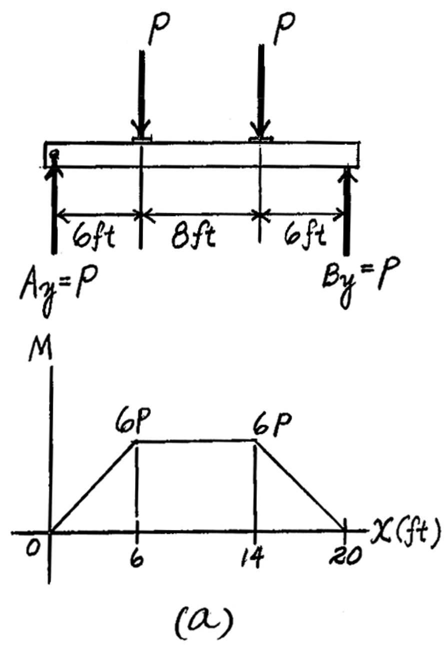
$$M_Y = 2684 \text{ kip} \cdot \text{in} = 223.67 \text{ kip} \cdot \text{ft}$$

It is required that

$$M_{max} = M_Y$$

$$6P = 223.67$$

$$P = 37.28 \text{ kip} = 37.3 \text{ kip} \qquad \textbf{Ans.}$$



Referring to the stress block shown in Fig. *b*,

$$T_1 = C_1 = 6(1)(36) = 216 \text{ kip}$$

$$T_2 = C_2 = 5(1)(36) = 180 \text{ kip}$$

Thus,

$$M_P = T_1(11) + T_2(5)$$

$$= 216(11) + 180(5)$$

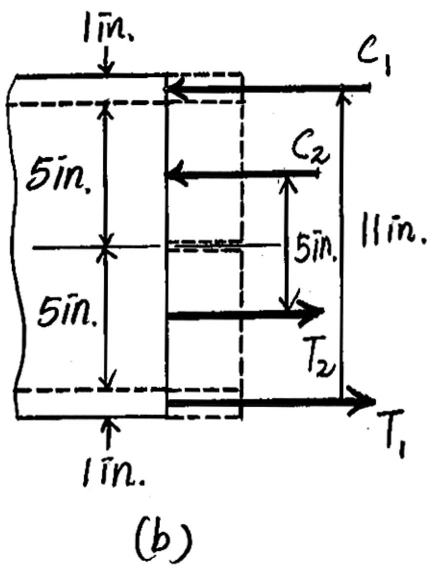
$$= 3276 \text{ kip} \cdot \text{in} = 273 \text{ kip} \cdot \text{ft}$$

It is required that

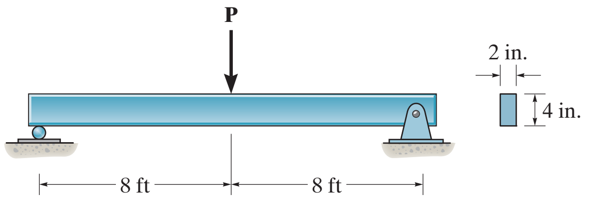
$$M_{max} = M_P$$

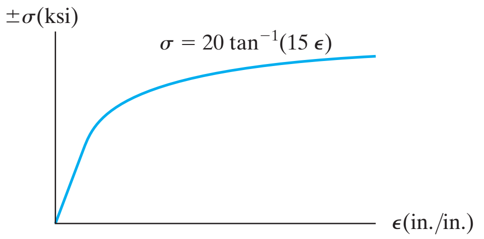
$$6P = 273$$

$$P = 45.5 \text{ kip} \qquad \textbf{Ans.}$$

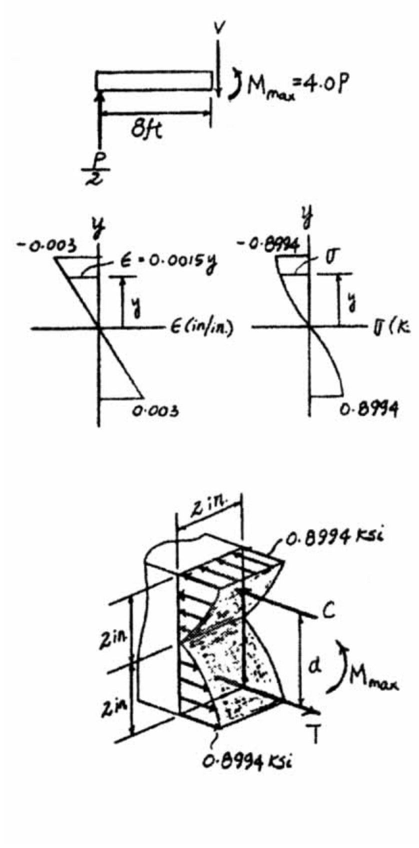


***6–184.** The beam is made of a polyester that has the stress–strain curve shown. If the curve can be represented by the equation $\sigma = [20\tan^{-1}(15\epsilon)]$ ksi, where $\tan^{-1}(15\epsilon)$ is in radians, determine the magnitude of the force **P** that can be applied to the beam without causing the maximum strain in its fibers at the critical section to exceed $\epsilon_{max} = 0.003$ in./in.





Maximum Internal Moment: The maximum internal moment $M = 4.00P$ occurs at the mid span as shown on FBD.



Stress–Strain Relationship: Using the stress–strain relationship. the bending stress can be expressed in terms of y using $\varepsilon = 0.0015y$.

$$\sigma = 20\tan^{-1}(15\varepsilon)$$

$$= 20\tan^{-1}[15(0.0015y)]$$

$$= 20\tan^{-1}(0.0225y)$$

When $\varepsilon_{max} = 0.003$ in./in., $y = 2$ in. and $\sigma_{max} = 0.8994$ ksi

Resultant Internal Moment: The resultant internal moment M can be evaluated from the integal $\int_A y\sigma dA$.

$$M = 2\int_A y\sigma dA$$

$$= 2\int_0^{2in} y\left[20\tan^{-1}(0.0225y)\right](2dy)$$

$$= 80\int_0^{2in} y\tan^{-1}(0.0225y)\,dy$$

$$= 80\left[\frac{1+(0.0225)^2y^2}{2(0.0225)^2}\tan^{-1}(0.0225y) - \frac{y}{2(0.0225)}\right]\Bigg|_0^{2in.}$$

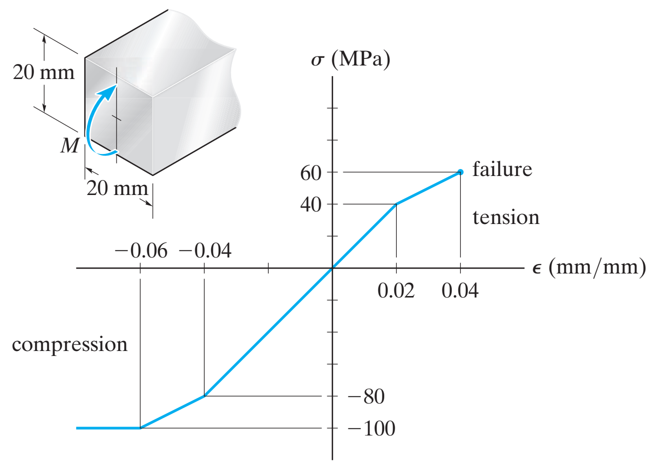
$$= 4.798 \text{ kip}\cdot\text{in}$$

Equating

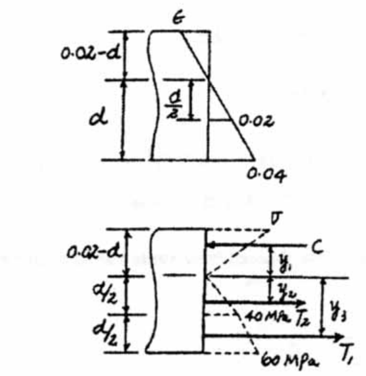
$$M = 4.00P(12) = 4.798$$

$$P = 0.100 \text{ kip} = 100 \text{ lb} \qquad \textbf{Ans.}$$

•6–185. The plexiglass bar has a stress–strain curve that can be approximated by the straight-line segments shown. Determine the largest moment M that can be applied to the bar before it fails.



Ultimate Moment:



$$\int_A \sigma\, dA = 0; \qquad C - T_2 - T_1 = 0$$

$$\sigma\left[\frac{1}{2}(0.02 - d)(0.02)\right] - 40\left(10^6\right)\left[\frac{1}{2}\left(\frac{d}{2}\right)(0.02)\right] - \frac{1}{2}(60 + 40)\left(10^6\right)\left[(0.02)\frac{d}{2}\right] = 0$$

$$\sigma - 50\sigma\, d - 3500(10^6)d = 0$$

Assume.$\sigma = 74.833$ MPa; $d = 0.010334$ m

From the strain diagram,

$$\frac{\varepsilon}{0.02 - 0.010334} = \frac{0.04}{0.010334} \qquad \varepsilon = 0.037417 \text{ mm/mm}$$

From the stress–strain diagram,

$$\frac{\sigma}{0.037417} = \frac{80}{0.04} \qquad \sigma = 74.833 \text{ MPa (OK! Close to assumed value)}$$

Therefore,

$$C = 74.833\left(10^6\right)\left[\frac{1}{2}(0.02 - 0.010334)(0.02)\right] = 7233.59 \text{ N}$$

$$T_1 = \frac{1}{2}(60 + 40)\left(10^6\right)\left[(0.02)\left(\frac{0.010334}{2}\right)\right] = 5166.85 \text{ N}$$

$$T_2 = 40\left(10^6\right)\left[\frac{1}{2}(0.02)\left(\frac{0.010334}{2}\right)\right] = 2066.74 \text{ N}$$

$$y_1 = \frac{2}{3}(0.02 - 0.010334) = 0.0064442 \text{ m}$$

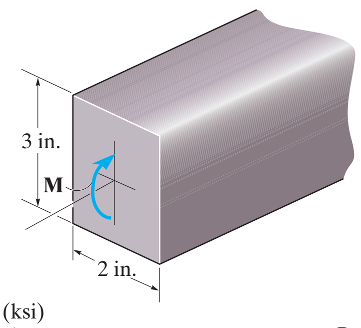
$$y_2 = \frac{2}{3}\left(\frac{0.010334}{2}\right) = 0.0034445 \text{ m}$$

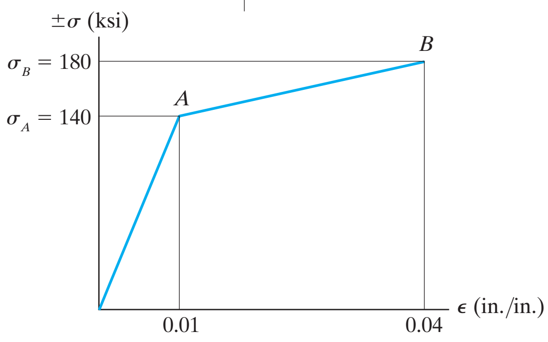
$$y_3 = \frac{0.010334}{2} + \left[1 - \frac{1}{3}\left(\frac{2(40) + 60}{40 + 60}\right)\right]\left(\frac{0.010334}{2}\right) = 0.0079225\text{m}$$

$$M = 7233.59(0.0064442) + 2066.74(0.0034445) + 5166.85(0.0079255)$$

$$= 94.7 \text{ N}\cdot\text{m}$$ **Ans.**

6–186. The stress–strain diagram for a titanium alloy can be approximated by the two straight lines. If a strut made of this material is subjected to bending, determine the moment resisted by the strut if the maximum stress reaches a value of (a) σ_A and (b) σ_B.





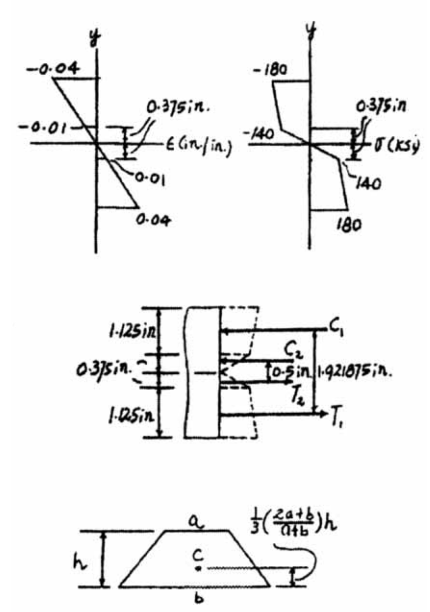
a) ***Maximum Elastic Moment :*** Since the stress is linearly related to strain up to point A, the flexure formula can be applied.

$$\sigma_A = \frac{Mc}{I}$$

$$M = \frac{\sigma_A I}{c}$$

$$= \frac{140\left[\frac{1}{12}(2)(3^3)\right]}{1.5}$$

$$= 420 \text{ kip}\cdot\text{in} = 35.0 \text{ kip}\cdot\text{ft}$$ **Ans.**



b) ***The Ultimate Moment :***

$$C_1 = T_1 = \frac{1}{2}(140 + 180)(1.125)(2) = 360 \text{ kip}$$

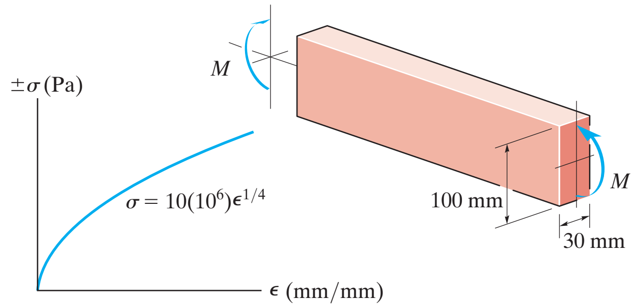
$$C_2 = T_2 = \frac{1}{2}(140)(0.375)(2) = 52.5 \text{ kip}$$

$$M = 360(1.921875) + 52.5(0.5)$$

$$= 718.125 \text{ kip}\cdot\text{in} = 59.8 \text{ kip}\cdot\text{ft}$$ **Ans.**

Note: The centroid of a trapezodial area was used in calculation of moment.

6–187. A beam is made from polypropylene plastic and has a stress–strain diagram that can be approximated by the curve shown. If the beam is subjected to a maximum tensile and compressive strain of $\epsilon = 0.02$ mm/mm, determine the maximum moment M.



$\varepsilon_{max} = 0.02$

$\sigma_{max} = 10\left(10^6\right)(0.02)^{1/4} = 3.761$ MPa

$$\frac{0.02}{0.05} = \frac{\varepsilon}{y}$$

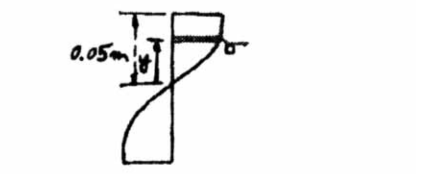
$$\varepsilon = 0.4\,y$$

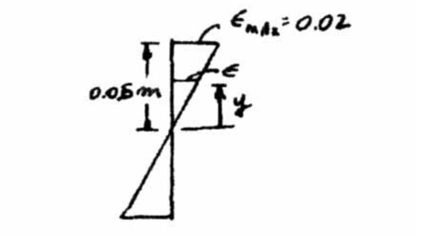
$\sigma = 10\left(10^6\right)(0.4)^{1/4} y^{1/4}$

$$M = \int_A y\,\sigma\,dA = 2\int_0^{0.05} y(7.9527)\left(10^6\right)y^{1/4}(0.03)dy$$

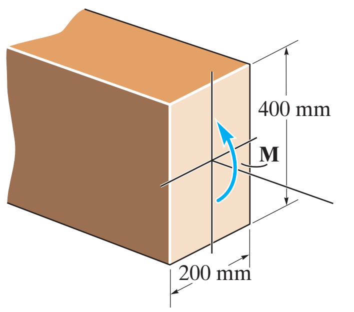
$$M = 0.47716\left(10^6\right)\int_0^{0.05} y^{5/4}dy = 0.47716\left(10^6\right)\left(\frac{4}{5}\right)(0.05)^{9/4}$$

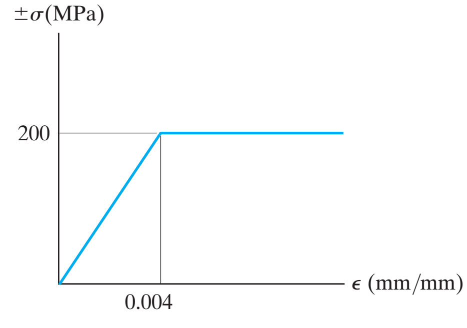
$M = 251\ \text{N}\cdot\text{m}$ **Ans.**





***6–188.** The beam has a rectangular cross section and is made of an elastic-plastic material having a stress–strain diagram as shown. Determine the magnitude of the moment **M** that must be applied to the beam in order to create a maximum strain in its outer fibers of $\epsilon_{max} = 0.008$.

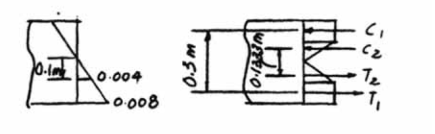




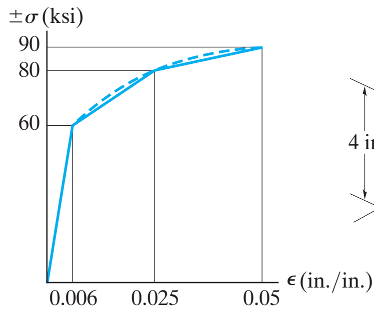
$C_1 = T_1 = 200\left(10^6\right)(0.1)(0.2) = 4000$ kN

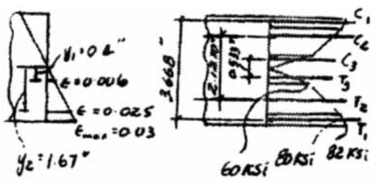
$C_2 = T_2 = \frac{1}{2}(200)\left(10^6\right)(0.1)(0.2) = 2000$ kN

$M = 4000(0.3) + 2000(0.1333) = 1467\ \text{kN}\cdot\text{m} = 1.47\ \text{MN}\cdot\text{m}$ **Ans.**



•6–189. The bar is made of an aluminum alloy having a stress–strain diagram that can be approximated by the straight line segments shown. Assuming that this diagram is the same for both tension and compression, determine the moment the bar will support if the maximum strain at the top and bottom fibers of the beam is $\epsilon_{max} = 0.03$.





$$\frac{\sigma - 80}{0.03 - 0.025} = \frac{90 - 80}{0.05 - 0.025}; \qquad \sigma = 82 \text{ ksi}$$

$$C_1 = T_1 = \frac{1}{2}(0.3333)(80 + 82)(3) = 81 \text{ kip}$$

$$C_2 = T_2 = \frac{1}{2}(1.2666)(60 + 80)(3) = 266 \text{ kip}$$

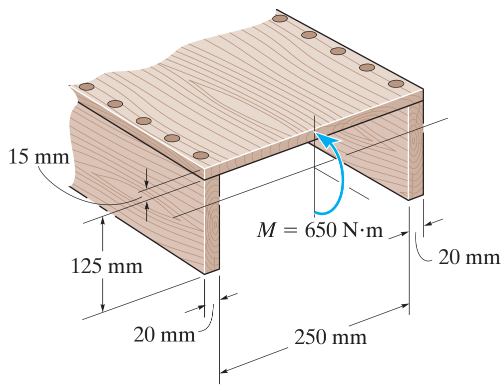
$$C_3 = T_3 = \frac{1}{2}(0.4)(60)(3) = 36 \text{ kip}$$

$$M = 81(3.6680) + 266(2.1270) + 36(0.5333)$$

$$= 882.09 \text{ kip} \cdot \text{in.} = 73.5 \text{ kip} \cdot \text{ft}$$ **Ans.**

Note: The centroid of a trapezodial area was used in calculation of moment areas.

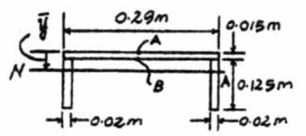
6–190. The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 650 \text{ N} \cdot \text{m}$, determine the resultant force the bending stress produces on the top board.



Section Properties:

$$\bar{y} = \frac{0.0075(0.29)(0.015) + 2[0.0775(0.125)(0.02)]}{0.29(0.015) + 2(0.125)(0.02)}$$

$$= 0.044933 \text{ m}$$



$$I_{NA} = \frac{1}{12}(0.29)\left(0.015^3\right) + 0.29(0.015)(0.044933 - 0.0075)^2$$

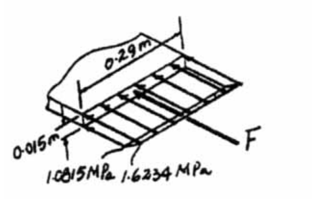
$$+ \frac{1}{12}(0.04)\left(0.125^3\right) + 0.04(0.125)(0.0775 - 0.044933)^2$$

$$= 17.99037\left(10^{-6}\right) \text{m}^4$$

Bending Stress: Applying the flexure formula $\sigma = \frac{My}{I}$

$$\sigma_B = \frac{650(0.044933 - 0.015)}{17.99037(10^{-6})} = 1.0815 \text{ MPa}$$

$$\sigma_A = \frac{650(0.044933)}{17.99037(10^{-6})} = 1.6234 \text{ MPa}$$

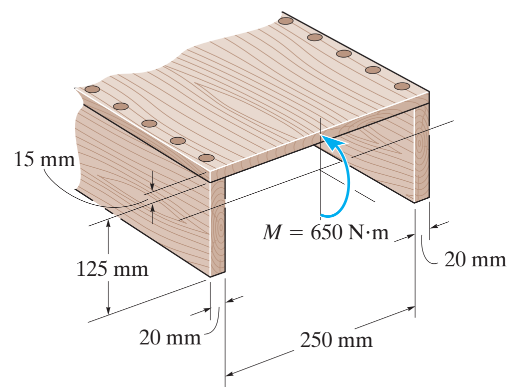


Resultant Force:

$$F_R = \frac{1}{2}(1.0815 + 1.6234)\left(10^6\right)(0.015)(0.29)$$

$$= 5883 \text{ N} = 5.88 \text{ kN}$$ **Ans.**

6–191. The beam is made from three boards nailed together as shown. Determine the maximum tensile and compressive stresses in the beam.



Section Properties:

$$\bar{y} = \frac{0.0075(0.29)(0.015) + 2[0.0775(0.125)(0.02)]}{0.29(0.015) + 2(0.125)(0.02)}$$

$$= 0.044933 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.29)\left(0.015^3\right) + 0.29(0.015)(0.044933 - 0.0075)^2$$

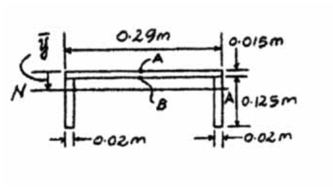
$$+ \frac{1}{12}(0.04)\left(0.125^3\right) + 0.04(0.125)(0.0775 - 0.044933)^2$$

$$= 17.99037\left(10^{-6}\right) \text{ m}^4$$

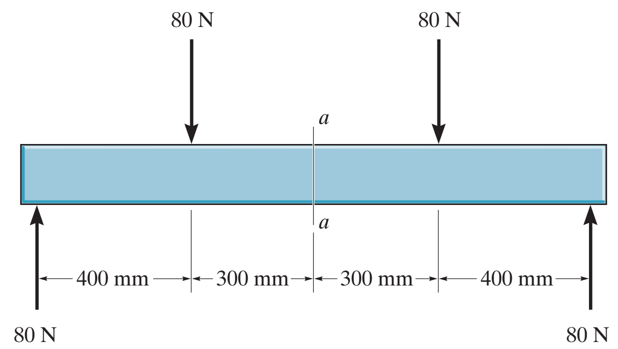
Maximum Bending Stress: Applying the flexure formula $\sigma = \frac{My}{I}$

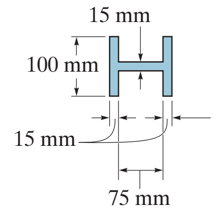
$$(\sigma_{\max})_t = \frac{650(0.14 - 0.044933)}{17.99037(10^{-6})} = 3.43 \text{ MPa } (\mathbf{T})$$ **Ans.**

$$(\sigma_{\max})_c = \frac{650(0.044933)}{17.99037(10^{-6})} = 1.62 \text{ MPa (C)}$$ **Ans.**



***6–192.** Determine the bending stress distribution in the beam at section *a–a*. Sketch the distribution in three dimensions acting over the cross section.



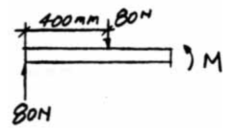


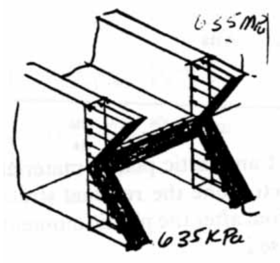
$$\zeta+\Sigma M = 0; \qquad M - 80(0.4) = 0$$

$$M = 32\ \text{N}\cdot\text{m}$$

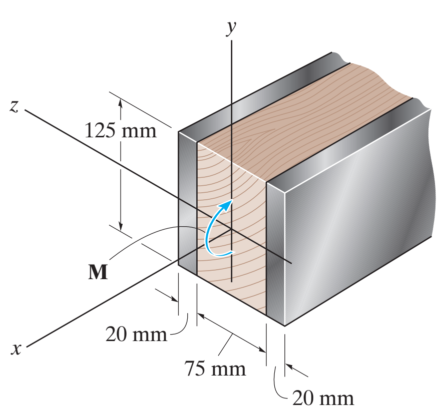
$$I_z = \frac{1}{12}(0.075)(0.015^3) + 2\left(\frac{1}{12}\right)(0.015)(0.1^3) = 2.52109(10^{-6})\text{m}^4$$

$$\sigma_{\text{max}} = \frac{Mc}{I} = \frac{32(0.05)}{2.52109(10^{-6})} = 635\ \text{kPa}$$ **Ans.**



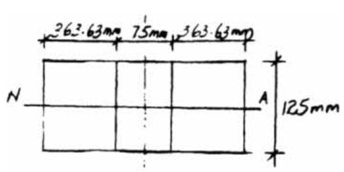


•6–193. The composite beam consists of a wood core and two plates of steel. If the allowable bending stress for the wood is $(\sigma_{\text{allow}})_w = 20$ MPa, and for the steel $(\sigma_{\text{allow}})_{st} = 130$ MPa, determine the maximum moment that can be applied to the beam. $E_w = 11$ GPa, $E_{st} = 200$ GPa.



$$n = \frac{E_{st}}{E_w} = \frac{200(10^9)}{11(10^9)} = 18.182$$

$$I = \frac{1}{12}(0.80227)(0.125^3) = 0.130578(10^{-3})\text{m}^4$$



Failure of wood :

$$(\sigma_w)_{\text{max}} = \frac{Mc}{I}$$

$$20(10^6) = \frac{M(0.0625)}{0.130578(10^{-3})}; \qquad M = 41.8\ \text{kN}\cdot\text{m}$$

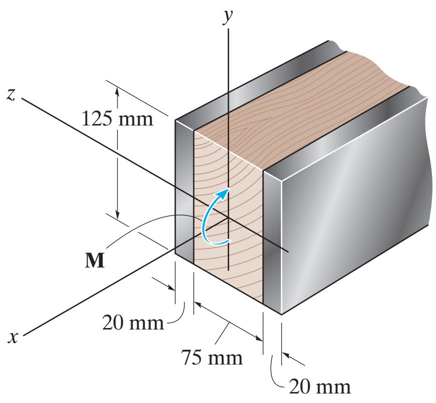
Failure of steel :

$$(\sigma_{st})_{\text{max}} = \frac{nMc}{I}$$

$$130(10^6) = \frac{18.182(M)(0.0625)}{0.130578(10^{-3})}$$

$$M = 14.9\ \text{kN}\cdot\text{m} \quad \text{(controls)}$$ **Ans.**

6–194. Solve Prob. 6–193 if the moment is applied about the y axis instead of the z axis as shown.



$$n = \frac{11(10^9)}{200(10^4)} = 0.055$$

$$I = \frac{1}{12}(0.125)(0.115^3) - \frac{1}{12}(0.118125)(0.075^3) = 11.689616(10^{-6})$$

Failure of wood :

$$(\sigma_w)_{\max} = \frac{nMc_2}{I}$$

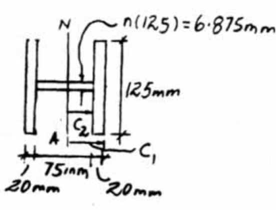
$$20(10^6) = \frac{0.055(M)(0.0375)}{11.689616(10^{-6})}; \qquad M = 113 \text{ kN} \cdot \text{m}$$

Failure of steel :

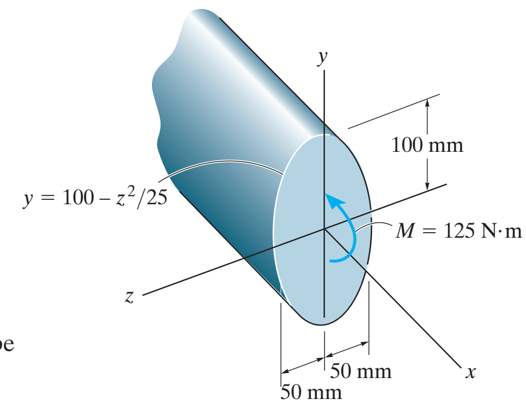
$$(\sigma_{st})_{\max} = \frac{Mc_1}{I}$$

$$130(10^6) = \frac{M(0.0575)}{11.689616(10^{-6})}$$

$$M = 26.4 \text{ kN} \cdot \text{m} \quad \text{(controls)}$$ **Ans.**



6–195. A shaft is made of a polymer having a parabolic cross section. If it resists an internal moment of $M = 125\ \text{N}\cdot\text{m}$, determine the maximum bending stress developed in the material (a) using the flexure formula and (b) using integration. Sketch a three-dimensional view of the stress distribution acting over the cross-sectional area. *Hint:* The moment of inertia is determined using Eq. A–3 of Appendix A.



Maximum Bending Stress: The moment of inertia about y axis must be determined first in order to use Flexure Formula

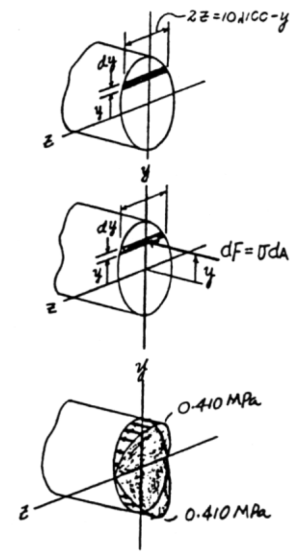
$$I = \int_A y^2\, dA$$

$$= 2\int_0^{100\text{mm}} y^2\,(2z)\, dy$$

$$= 20\int_0^{100\text{mm}} y^2\sqrt{100 - y}\, dy$$

$$= 20\left[-\frac{3}{2}\,y^2(100 - y)^{\frac{3}{2}} - \frac{8}{15}\,y(100 - y)^{\frac{5}{2}} - \frac{16}{105}(100 - y)^{\frac{7}{2}}\right]\Bigg|_0^{100\text{mm}}$$

$$= 30.4762\left(10^{-6}\right)\text{mm}^4 = 30.4762\left(10^{-6}\right)\text{m}^4$$



Thus,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{125(0.1)}{30.4762(10^{-6})} = 0.410\ \text{MPa}$$ **Ans.**

Maximum Bending Stress: Using integration

$$dM = 2[y(\sigma\, dA)] = 2\left\{y\left[\left(\frac{\sigma_{\max}}{100}\right)y\right](2z\, dy)\right\}$$

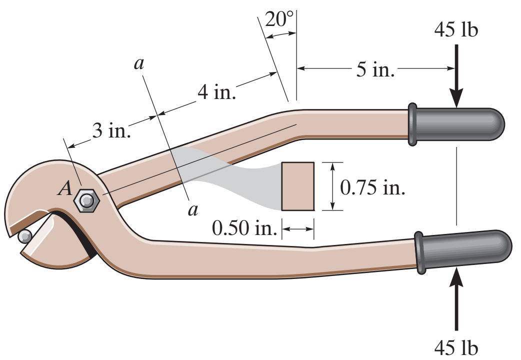
$$M = \frac{\sigma_{\max}}{5}\int_0^{100\text{mm}} y^2\sqrt{100 - y}\, dy$$

$$125\left(10^3\right) = \frac{\sigma_{\max}}{5}\left[-\frac{3}{2}\,y^2(100 - y)^{\frac{3}{2}} - \frac{8}{15}\,y(100 - y)^{\frac{5}{2}} - \frac{16}{105}\,(100 - y)^{\frac{7}{2}}\right]\Bigg|_0^{100\text{mm}}$$

$$125\left(10^3\right) = \frac{\sigma_{\max}}{5}\,(1.5238)\left(10^6\right)$$

$$\sigma_{\max} = 0.410\ \text{N/mm}^2 = 0.410\ \text{MPa}$$ **Ans.**

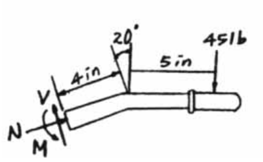
***6–196.** Determine the maximum bending stress in the handle of the cable cutter at section *a–a*. A force of 45 lb is applied to the handles. The cross-sectional area is shown in the figure.



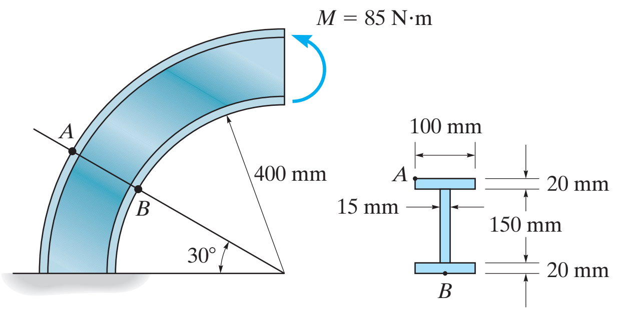
$\circlearrowleft + \Sigma M = 0; \qquad M - 45(5 + 4\cos 20°) = 0$

$M = 394.14 \text{ lb} \cdot \text{in.}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{394.14(0.375)}{\frac{1}{12}(0.5)(0.75^3)} = 8.41 \text{ ksi}$$ **Ans.**



•6–197. The curved beam is subjected to a bending moment of $M = 85$ N · m as shown. Determine the stress at points *A* and *B* and show the stress on a volume element located at these points.

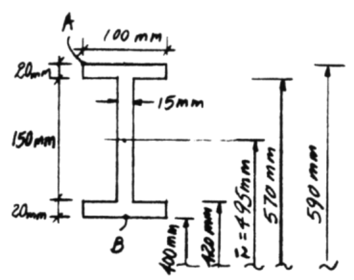


$$\int_A \frac{dA}{r} = b \ln \frac{r_2}{r_1} = 0.1 \ln \frac{0.42}{0.40} + 0.015 \ln \frac{0.57}{0.42} + 0.1 \ln \frac{0.59}{0.57}$$

$$= 0.012908358 \text{ m}$$

$$A = 2(0.1)(0.02) + (0.15)(0.015) = 6.25(10^{-3}) \text{ m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{6.25(10^{-3})}{0.012908358} = 0.484182418 \text{ m}$$

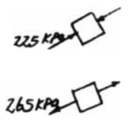


$$\bar{r} - R = 0.495 - 0.484182418 = 0.010817581 \text{ m}$$

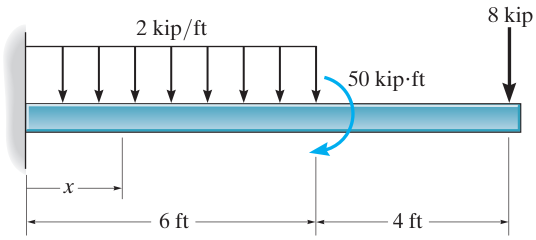
$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{85(0.484182418 - 0.59)}{6.25(10^{-3})(0.59)(0.010817581)} = -225.48 \text{ kPa}$$

$\sigma_A = 225$ kPa (C) **Ans.**

$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{85(0.484182418 - 0.40)}{6.25(10^{-3})(0.40)(0.010817581)} = 265 \text{ kPa (T)}$$ **Ans.**



6–198. Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x, where $0 \leq x < 6$ ft.

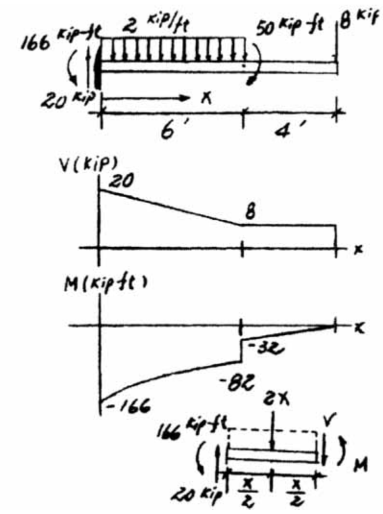


$+\uparrow \Sigma F_y = 0; \qquad 20 - 2x - V = 0$

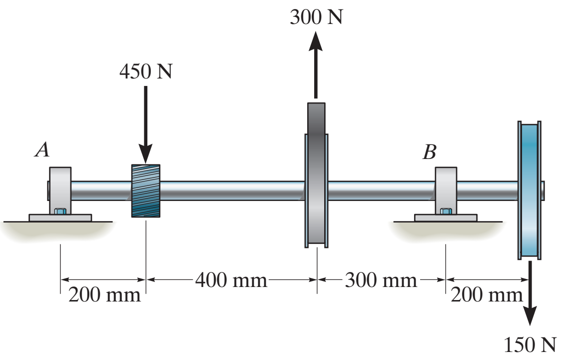
$$V = 20 - 2x \qquad \textbf{Ans.}$$

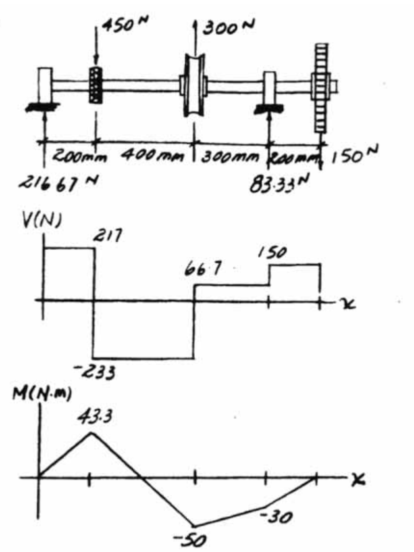
$\circlearrowleft + \Sigma M_{NA} = 0; \qquad 20x - 166 - 2x\left(\frac{x}{2}\right) - M = 0$

$$M = -x^2 + 20x - 166 \qquad \textbf{Ans.}$$

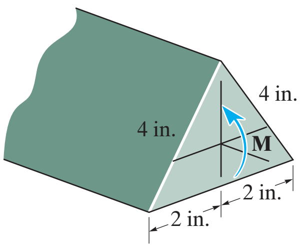


6–199. Draw the shear and moment diagrams for the shaft if it is subjected to the vertical loadings of the belt, gear, and flywheel. The bearings at A and B exert only vertical reactions on the shaft.





***6–200.** A member has the triangular cross section shown. Determine the largest internal moment M that can be applied to the cross section without exceeding allowable tensile and compressive stresses of $(\sigma_{\text{allow}})_t = 22$ ksi and $(\sigma_{\text{allow}})_c = 15$ ksi, respectively.



$$\bar{y} \text{ (From base)} = \frac{1}{3}\sqrt{4^2 - 2^2} = 1.1547 \text{ in.}$$

$$I = \frac{1}{36}(4)(\sqrt{4^2 - 2^2})^3 = 4.6188 \text{ in}^4$$

Assume failure due to tensile stress :

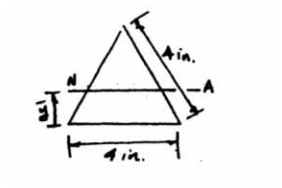
$$\sigma_{\max} = \frac{My}{I}; \qquad 22 = \frac{M(1.1547)}{4.6188}$$

$$M = 88.0 \text{ kip} \cdot \text{in.} = 7.33 \text{ kip} \cdot \text{ft}$$

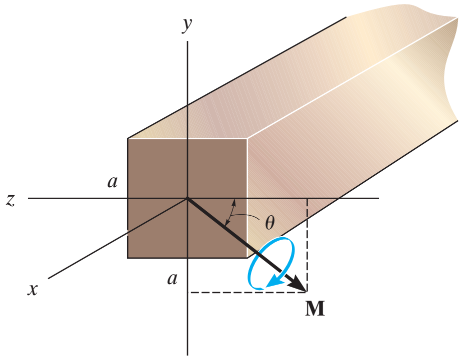
Assume failure due to compressive stress:

$$\sigma_{\max} = \frac{Mc}{I}; \qquad 15 = \frac{M(3.4641 - 1.1547)}{4.6188}$$

$M = 30.0 \text{ kip} \cdot \text{in.} = 2.50 \text{ kip} \cdot \text{ft}$ (controls) **Ans.**



•6–201. The strut has a square cross section a by a and is subjected to the bending moment **M** applied at an angle θ as shown. Determine the maximum bending stress in terms of a, M, and θ. What angle θ will give the largest bending stress in the strut? Specify the orientation of the neutral axis for this case.



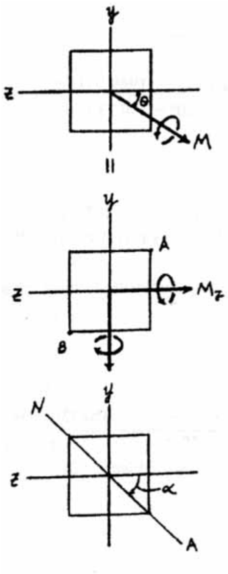
Internal Moment Components:

$$M_z = -M\cos\theta \qquad M_y = -M\sin\theta$$

Section Property:

$$I_y = I_z = \frac{1}{12}a^4$$

Maximum Bending Stress: By Inspection, Maximum bending stress occurs at A and B. Applying the flexure formula for biaxial bending at point A



$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$= -\frac{-M\cos\theta\left(\frac{a}{2}\right)}{\frac{1}{12}a^4} + \frac{-M\sin\theta\left(-\frac{a}{2}\right)}{\frac{1}{12}a^4}$$

$$= \frac{6M}{a^3}(\cos\theta + \sin\theta) \qquad \textbf{Ans.}$$

$$\frac{d\sigma}{d\theta} = \frac{6M}{a^3}(-\sin\theta + \cos\theta) = 0$$

$$\cos\theta - \sin\theta = 0$$

$$\theta = 45° \qquad \textbf{Ans.}$$

Orientation of Neutral Axis:

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\tan\alpha = (1)\tan(45°)$$

$$\alpha = 45° \qquad \textbf{Ans.}$$