#1 (4.39 modified)

Req:
Steel area so that \( F_{st} = F_{con} \)

Compatibility requirement
\[ \Delta_{st} = \Delta_{con} \]

In terms of forces
\[ \frac{F_{st}L_{st}}{A_{st}E_{st}} = \frac{F_{con}L_{con}}{A_{con}E_{con}} \]

Since forces & lengths are equal
\[ \frac{A_{con}}{A_{st}} = \frac{E_{st}}{E_{con}} \]

Substituting known values
\[ \frac{500 \times 200 - A_{st}}{A_{st}} = \frac{200}{29} \]

Solving for steel area we get:
\[ A_{st} = 12,664 \, \text{mm}^2 \]

\[ \Delta_{st} = \frac{F_{st}L_{st}}{A_{st}E_{st}} = \frac{150 \times 150}{12664 \times 200} = 0.142 \, \text{mm} \]
Req:
Find stresses in steel & aluminum.

Assume gab closes and look at FBD of rigid beam

\[ \Sigma F_y = 0 \]

\[ 2 F_{al} + F_{st} = 160 \]  \( \text{---1} \)

(note column A or C carry the same force!)

Statically Indeterminate Problem
An extra equation can be obtained from deformation geometry

\[ \Delta_{al} = 0.3 + \Delta_{st} \]

Use force def. relation

\[ \frac{F_{al}}{E_{al}} \frac{L_{al}}{A_{al}} = 0.3 + \frac{F_{st}}{E_{st}} \frac{L_{st}}{A_{st}} \]

\[ \frac{F_{al} (125)}{400 (70)} = 0.3 + \frac{F_{st} (125)}{400 (200)} \]

Which yields

\[ F_{st} = \frac{2500}{875} F_{al} = 19.2 \]  \( \text{---2} \)

Solving \( \text{---1} \) \& \( \text{---2} \) we get

\[ F_{st} = 15 \text{ kN} \quad \text{&} \quad F_{al} = 72.5 \text{ kN} \]

The stresses will be

\[ \sigma_{st} = \frac{F_{st}}{A_{st}} = 37.5 \text{ MPa} \quad \text{&} \quad \sigma_{al} = 181 \text{ MPa} \]

Note that stresses are compressive.
#3 Req:
Vertical displacement of point C.

From shown FBD

\[ \tau \geq MA = 0 \]

\[ 5F_{CE} + 2F_{BD} = 10 \times 2.5 = 0 \]

\[ F_{CE} + 0.4F_{BD} = 5 \quad - (1) \]

Statically Indeterminate, need a compatibility equation, use the given deformation geometry (similar triangles)

\[ \frac{\Delta_{BD}}{2} = \frac{\Delta_{CE}}{5} \]

\[ \Delta_{CE} = 2.5 \Delta_{BD} \]

Use force-def. relation

\[ \frac{F_{CE} \cdot L_{CE}}{A_{CE} \cdot E_{CE}} = 2.5 \frac{F_{BD} \cdot L_{BD}}{A_{BD} \cdot E_{BD}} \]

\[ F_{CE} = 1.875F_{BD} \quad - (2) \]

Solving (1) and (2) we get

\[ F_{BD} = 2.2 \text{ kN} \quad F_{CE} = 4.12 \text{ kN} \]

Now calculate the vertical movement of point C which is equal to \( \Delta_{CE} \)

\[ \frac{F_{CE} \cdot L_{CE}}{A \cdot E} = \frac{4.12 \times 2000}{\frac{3}{4} \times (1.5)^2 \times (200)} = 0.233 \text{ mm} \]
#4. Required: stresses in rods

Using equilibrium equation applied to shown FBD

\[ \sum F_y = 0 \]

\[ F_A + F_B + F_C = 20 \quad -1 \]

\[ \sum M_c = 0 \]

\[ 4F_B + 8F_A - 20 \times 2 = 0 \]

\[ F_B + 2F_A = 10 \quad -2 \]

Statically indeterminate, get an extra equation from deformation geometry, which is

\[ \frac{\Delta c - \Delta a}{8} = -\frac{\Delta b - \Delta a}{4} \]

which simplifies to

\[ \Delta b = \frac{1}{2} (\Delta a + \Delta c) \]

Using force-def. relation

\[
\frac{F_B L_B}{\frac{E}{4}(d)^2 E_{BR}} = \frac{1}{2} \left[ \frac{F_A L_A}{\frac{E}{4}(d)^2 E_{AL}} \right]
\]

Substituting

\[ 5.333 F_B = F_A + F_C \quad -3 \]

Solving \(1, 2 \) and \(3\) we get

\[ F_A = 3.421 \text{ kN} \]

\[ F_B = 3.158 \text{ kN} \]

\[ F_C = 13.421 \text{ kN} \]

and

\[ \sigma_A = \frac{F_A}{A_A} = 1.2 \text{ MPa} \]

\[ \sigma_B = \frac{F_B}{A_B} = 4.5 \text{ MPa} \]

\[ \sigma_C = \frac{F_C}{A_C} = 4.7 \text{ MPa} \]
#5 Required

a) Stresses in both materials due to load

b) due to load plus temp change.

Material properties from tables

\[ \alpha_{st} = 12 \times 10^{-6}^\circ C^{-1}, \quad \alpha_{al} = 24 \times 10^{-6}^\circ C^{-1} \]

\[ E_{st} = 2 \times 10^5 \text{Pa}, \quad E_{al} = 68.9 \text{GPa} \]

Solution Assuming gage closes

(1) From shown FBD

\[ \sum F_x = 0 \]

\[ F_{st} + F_{al} = 60 \text{ kN} \]

SI compatibility requirement is

\[ \Delta_{st} + \Delta_{al} = 0.2 \]

In terms of forces

\[ \frac{F_{st}L_{st}}{\alpha_{st}E_{st}} + \frac{(60 - F_{st})L_{al}}{\alpha_{al}E_{al}} = 0.2 \]

\[ \frac{F_{st}(300)}{\alpha(25^2 - 20^2)} + \frac{(60 - F_{st})(450)}{\alpha(25)^2(68.9)} = 0.2 \]

which reduces to

\[ F_{st} = 38.78 \text{ kN} \]

and using (1) we get \( F_{al} = 21.22 \text{ kN} \)

and the resulting stresses are

\[ \sigma_{st} = \frac{F_{st}}{A_{st}} = 54.9 \text{ MPa (compression)} \]

\[ \sigma_{al} = \frac{F_{al}}{A_{al}} = 43.2 \text{ MPa} \]
b) If the temperature is increased then the compatibility equation will be

\[ \Delta_{\text{total}}^{s+} + \Delta_{\text{total}}^{c+} = 0.2 \]

Substituting

\[ \frac{-F_{st}L_{st}}{A_{st}E_{st}} + \alpha_{st} \Delta T L_{st} + \frac{(60-F_{st})L_{al}}{A_{al}E_{al}} + \alpha_{al} \Delta T L_{al} = 0.2 \]

Substituting the values and solving, we get

\[ F_{st} = 48.1 \text{ kN} \]

and from (1)

\[ F_{al} = 11.9 \text{ kN} \]

and the stresses become

\[ \sigma_{st} = 68 \text{ MPa} \]

\[ \sigma_{al} = 24.2 \text{ MPa} \]