PROBLEM #1

Given:
The beam cross-section shown
\[ M = 10 \text{KN.m} \]

Required:
\[ \theta_{\text{max}} \text{ in the beam} \]

SOLUTION

Since the beam has symmetrical cross-section, it implies that the N.A. is at the Centre.

\[ I_{\text{N.A.}} = 2 \left[ \frac{1}{12} (0.3)(0.4)^3 \right] + 2 \left[ \frac{1}{12} (0.14)(0.02)^3 + 0.14(0.02)(0.15)^2 \right] \]

\[ = 0.50963 \times 10^{-3} \text{ m}^4 \]
\[ \sigma_{\text{max}} = \frac{Mc}{I} \]
\[ = \frac{(10 \times 10^3)(0.2)}{0.500963 \times 10^{-3}} = 3.92 \text{MPa} \]

Location: \( \sigma_{\text{max}} \) is at the top or bottom of the cross-section.
PROBLEM #2

Given:
The beam and its cross-section shown

Required:
- Stress at the top and bottom
- Stress distribution along the vertical axis.

SOLUTION

FBD:

Support reactions:
\[ \sum M = 0; \ -15(1.5) - 7.5(4) + By = 0 \]
\[ ; \ By = 8.75 \text{ kN} (\uparrow) \]

\[ \sum F_y = 0; \ -15 - 7.5 + 8.75 + Ay = 0 \]
\[ ; \ Ay = 13.75 \text{ kN} (\uparrow) \]

To find the internal moment at a section passing through point C,
\[ \sum M_c = 0; \ -M - \left( \frac{1}{2} \times 2.5 \times 1.5 \right)(0.5) + 8.75(1.5) = 0 \]
\[ ; \ M_c = 12 \times 1875 \text{ kNm} (\uparrow) \]
The moment is shown in the beam cross-section as follows:

[Diagram of beam with moment and forces labeled]

This moment will cause compression above N.A. and tension below the N.A.

Location of N.A.

[Diagram of beam with dimensions labeled]

(All dimensions in m)

\[ \bar{y} = \frac{(0.15)(0.03)(0.165) + (0.03)(0.15)(0.075)}{(0.15)(0.03) + (0.15)(0.03)} = 0.12 \text{ m} \]

\[ \Rightarrow I_{N.A} = \left[ \frac{(0.03)(0.15)^3}{12} + (0.03)(0.15)(0.045)^2 \right] + \left[ \frac{(0.15)(0.03)^3}{12} + (0.03)(0.15)(0.045)^2 \right] \]

\[ = 1.755 \times 10^{-5} + 9.45 \times 10^{-6} \]

\[ = 2.7 \times 10^{-5} \text{ m}^4 \]

Stresses:

\[ \sigma_{\text{top}} = -\frac{12 \times 1875 \times 10^3}{2.7 \times 10^{-5}} [0.06] = -27.08 \text{ MPa} \text{ "Compression"} \]

\[ \sigma_{\text{bottom}} = -\frac{12 \times 1875 \times 10^3}{2.7 \times 10^{-5}} [-0.12] = +54.17 \text{ MPa} \text{ "Tension"} \]
Stress distribution

Top

Bottom

0.06m

0.12m

2.708 MPa

54.17 MPa
PROBLEM #3

Given:
- The beam and its cross-section as shown
- $O_{\text{ultimate}} = 180 \text{ MPa}$
- F.S. = 3

Required:
- a) $P_{\text{max}}$ for H-shape cross-section
- b) $P_{\text{max}}$ for I-shape cross-section
- c) To compare (a) and (b)

SOLUTION

$O_{\text{allowable}} = \frac{O_{\text{ultimate}}}{\text{F.S.}} = \frac{180}{3} = 60 \text{ MPa}$

$\Sigma M = 0;
-P(0.5) - P(0.8) + M = 0
\Rightarrow M = 1.3P$

$\Sigma F_y = 0; Ay + P - P = 0
\Rightarrow Ay = 2P$

Hence, $M_{\text{max}} = 1.3P$
a) For H-cross section

\[ I_{N,A} = 2 \left( \frac{(0.015)(0.01)^3}{12} \right) + \left( \frac{0.075 \times 0.015^3}{12} \right) \]

\[ = 2.5 \times 10^{-6} + 2.109375 \times 10^{-8} \]

\[ = 2.52109375 \times 10^{-6} \text{ m}^4 \]

\[ \Rightarrow P_{\text{allow}} = P_{\text{max}} = 60 \times 10^6 = \frac{1.2 \times 0.05}{2.52109375 \times 10^{-6}} \]

\[ \Rightarrow P_{\text{max}} = 2.327 \text{ KN} \]

b) For I-cross section,
\[ I_{n.a} = 2 \left[ \frac{(0.1)(0.015)^3}{12} + \frac{(0.1)(0.015)(0.045)^2}{12} \right] + \frac{(0.015)(0.075)^3}{12} \]

\[ = 6.13125 \times 10^{-6} + 5.2734375 \times 10^{-7} \]

\[ = 6.65859375 \times 10^{-6} \text{ m}^4 \]

\[ \sigma_{allow} = \sigma_{max.} = 60 \times 10^6 = \frac{(1.3 P)(0.0525)}{6.65859375 \times 10^{-6}} \]

\[ \Rightarrow P_{max.} = 5.854 \text{ kN} \]

C) Since \( P_{max(I)} > P_{max(H)} \), hence the I-shape cross section is better.
**PROBLEM #4**

**Given:**

The beam cross section shown,

\( \sigma_{\text{allow(tension)}} = 40 \text{ MPa} \)

\( \sigma_{\text{allow(compr.)}} = 70 \text{ MPa} \)

**Required**

- Positive \( M_{\text{max}} \).
- Negative \( M_{\text{max}} \).

**SOLUTION**

- Location of \( N\cdot A \),

\[
\bar{y} = \frac{(0.15)(0.02)(0.01) + (0.009)(0.15)(0.095)}{(0.15)(0.02) + (0.009)(0.15)}
\]

\[= 0.03638 \text{ m} \]

\[
I_{N\cdot A} = \frac{(0.15)(0.02)^3 + (0.15)(0.02)(0.02638)^2}{12} \]

\[+ \frac{(0.009)(0.15)^3 + (0.009)(0.15)(0.05862)^2}{12} \]

\[= 9.35797 \times 10^{-6} \text{ m}^4 \]
- Positive moment

\[
\sigma_{\text{max}} = \sigma_{\text{allow}} = \frac{40 \times 10^6}{9.35797 \times 10^{-6}} = \frac{M \times 0.03638}{9.35797 \times 10^{-6}}
\]

\[
\Rightarrow M_1 = 10.289 \text{ KN.m}
\]

\[
\sigma_{\text{allow}} = 70 \times 10^6 = \frac{M \times 0.13362}{9.35797 \times 10^{-6}}
\]

\[
\Rightarrow M_2 = 4.902 \text{ KNm}
\]

Hence, \( \text{Positive } M_{\text{max}} = 4.902 \text{ KNm} \)

- Negative moment

\[
\sigma_{\text{max}} = \sigma_{\text{allow}} = \frac{40 \times 10^6}{9.35797 \times 10^{-6}} = \frac{M \times 0.13362}{9.35797 \times 10^{-6}}
\]

\[
\Rightarrow M_1 = 2.801 \text{ KNm}
\]

\[
\sigma_{\text{allow}} = 70 \times 10^6 = \frac{M \times 0.03638}{9.35797 \times 10^{-6}}
\]

\[
\Rightarrow M_2 = 1.806 \text{ KNm}
\]

Hence, \( \text{Negative } M_{\text{max}} = 2.801 \text{ KNm} \)