Problem #1

Given:
The beam shown

Required:
Internal shear force and moment at point C.

Solution

FBD:

Reactions:

\[ F_R = \frac{1}{2} \times W_0 \times L = \frac{W_0 L}{2} \]

\[ + \Sigma F_y = 0; \quad A_y - \frac{W_0 L}{2} = 0 \quad \therefore A_y = \frac{W_0 L}{2} \]

\[ + \Sigma M_A = 0; \quad M_A - \frac{W_0 L}{2} \left( \frac{2}{3} L \right) = 0 \quad \therefore M_A = \frac{W_0 L^2}{3} \]

Using similar triangle,

\[ \frac{W_x}{W_0} = \frac{L/2}{L} \]

\[ \therefore W_x = \frac{W_0}{2} \]
Section at C:

\[ \begin{align*}
\sum F_y &= 0; \quad \frac{W_0L}{2} - \frac{1}{2} \left( \frac{W_0}{2} \right) \left( \frac{L}{2} \right) - V_C = 0 \\
\therefore V_C &= \frac{3}{8} W_0L \\

\sum M_C &= 0; \quad \frac{W_0L^2}{3} - \frac{W_0L}{2} \left( \frac{L}{2} \right) + \frac{1}{2} \left( \frac{W_0}{2} \right) \left( \frac{L}{2} \right) \left( \frac{1}{3} \times \frac{L}{2} \right) + M_C = 0 \\
\therefore M_C &= \frac{5}{48} W_0L^2
\end{align*} \]
Problem #2

Given:
The beam shown in Problem #1

Required:
SFD and BMD by graphical method

Solution
From Problem #1, the reactions are:

\[ A_y = \frac{W_0L}{2}, \quad M_A = \frac{W_0L^2}{3} \]

The SFD and BMD are shown as follows:

\[ \text{Areas} \]

\[ \Delta V_{A\rightarrow B} = A_{\text{load}_{A\rightarrow B}} = -\frac{1}{2}L \times W_0 = -\frac{W_0L}{2} \]

\[ V_B = V_A + \Delta V_{A\rightarrow B} = \frac{W_0L}{2} - \frac{W_0L}{2} = 0 \]

\[ \Delta M_{A\rightarrow B} = \Delta V_{A\rightarrow B} \times \frac{1}{2}L = \frac{2}{3}\left(\frac{W_0L}{2}\right)(L) = \frac{W_0L^2}{3} \]

\[ M_B = M_A + \Delta M_{A\rightarrow B} = -\frac{W_0L^2}{3} + \frac{W_0L^2}{3} = 0 \]
Problem #3

Given:
The beam shown

Required:
SFD and BMD using graphical method

Solution

FBD:

Reactions:
\[ F_y = 0; \quad A_y + B_y = 0 \iff A_y = -B_y \]
\[ \sum M_B = 0; \quad 20 - 50 - A_y(6) = 0 \iff A_y = -5 \text{ KN} \]
\[ \therefore B_y = 5 \text{ KN} \]

The SFD and BMD are shown as follows:
Areas:

\[ V_A = A_y = -5 \text{KN} \]

\[ \Delta V_{A\rightarrow B} = A_{\text{load A} \rightarrow B} = 0 \]

\[ V_B = V_A + \Delta V_{A\rightarrow B} = -5 + 0 = -5 \text{KN} \]

\[ M_A = 50 \text{KNm} \]

\[ \Delta M_{A\rightarrow B} = A_{V_{A\rightarrow B}} = -5 \times 6 = -30 \text{KNm} \]

\[ M_B = M_A + \Delta M_{A\rightarrow B} = 50 - 30 = 20 \text{KNm} \]
Problem 4

Given:
The beam shown:

Required:
The SFD and BMD using graphical method

Solution

To calculate the reactions, we need to separate
the beam at point B,

By considering the right part of point B,

\[ F = 10 \times 2 = 20 \text{KN} \]

\[ 2C_y - (10 \times 2) \times 1 = 0 \quad \therefore C_y = 10 \text{KN} \]

Now, considering the entire beam,

\[ A_y + 10 - 10 \times 3 = 0 \quad \therefore A_y = 20 \text{KN} \]

\[ M_A - (10 \times 3) \times 1.5 + 10 \times 3 = 0 \]

\[ \therefore M_A = 15 \text{KNm} \]
The SFD and BMD are shown below:

To find \( x \), we take similar triangles \( ABD \) and \( AEF \),

\[ \frac{x}{20} = \frac{3}{20 + 10} \]

\[ x = \frac{3}{30} \cdot 20 = 2 \text{ m} \]

Areas

\[ \Delta V_{A \rightarrow C} = A_{\text{load}} A \rightarrow C = -10(3) = -30 \text{ kN} \]

\[ V_C = V_B + \Delta V_{B \rightarrow C} = 20 - 30 = -10 \text{ kN} \]

\[ \Delta M_{A \rightarrow B} = AV_{A \rightarrow B} = \frac{1}{2} \times (20 + 10) \times 1 = 15 \text{ kNm} \]

\[ M_B = M_A + \Delta M_{A \rightarrow B} = -15 + 15 = 0 \]

\[ \Delta M_{B \rightarrow D} = AV_{B \rightarrow D} = \frac{1}{2} \times 10 \times 1 = 5 \text{ kNm} \]

\[ M_D = M_B + \Delta M_{B \rightarrow D} = 0 + 5 = 5 \text{ kNm} \]

\[ \Delta M_{D \rightarrow C} = AV_{D \rightarrow C} = -\frac{1}{2} \times 10 \times 1 = -5 \text{ kNm} \]

\[ M_C = M_D + \Delta M_{D \rightarrow C} = 5 - 5 = 0 \]
Problem #5

Given:
The beam shown

Required:
SFD and BMD using graphical method.

Solution
FBD:

Reactions:
\[ \sum M_A = 0: \quad B_y (4) - (W_0 \times 2) \times 1 - (2W_0 \times 2) \times 3 = 0 \]
\[ \therefore B_y = 3.5W_0 \]

\[ \sum F_y = 0: \quad A_y + 3.5W_0 - W_0 \times 2 - 2W_0 \times 2 = 0 \]
\[ \therefore A_y = 2.5W_0 \]

The SFD and BMD are shown below:
To find \( \theta \) we need to find the equation where SF = 0

\[
\Rightarrow 2.5W_0 - 2(W_0) - (2W_0)(x - 2) = 0
\]

\[
2.5W_0 - 2W_0 - 2W_0x + 4W_0 = 0
\]

\[
\therefore x = 2.25\text{m}
\]

**Areas**

\[
\Delta V_{A \rightarrow B} = A_{\text{load}} \rightarrow B = -W_0 \times 2 = -2W_0
\]

\[
V_B = V_A + \Delta V_{A \rightarrow B} = 2.5W_0 - 2W_0 = 0.5W_0
\]

\[
\Delta V_{A \rightarrow C} = A_{\text{load}} \rightarrow C = -(W_0 \times 2 - 2W_0 \times 2) = -6W_0
\]

\[
V_C = V_A + \Delta V_{A \rightarrow C} = 2.5W_0 - 6W_0 = -3.5W_0
\]

\[
\Delta M_{A \rightarrow D} = A_{\text{load}} \rightarrow D = \frac{1}{2} (2.5W_0 + 0.5W_0) \times 2 + \frac{1}{2} \times 0.5W_0 \times 0.25
\]

\[
= 3.06W_0
\]

\[
M_0 = M_A + \Delta M_{A \rightarrow D} = 0 + 3.06 = 3.06W_0
\]
Problem 6

Given:
The beam shown

Required:
SFD and BMD using graphical method

Solution
FBD:

Reactions:
$\sum M_A = 0; \quad B_y(3) - 0.3 - (1\times0.6\times3)\times2 = 0$
$\therefore B_y = 0.7 \text{ KN}$
$\sum F_y = 0; \quad A_y + 0.7 - \frac{1}{2} \times 0.6 \times 3 = 0 \quad \therefore A_y = 0.2 \text{ KN}$

The SFD and BMD are shown below,
To find the distance $x$, at a point where $SF = 0$,

$$V_x = 0 = 0.2 - \frac{1}{2} \times 4 \times x$$

$$\Rightarrow W = 0.4x$$

Using similar triangles,

$$\frac{W}{0.6} = \frac{x}{0.3}$$

$$\Rightarrow x = \frac{3W}{0.6} = \frac{3}{0.6} \left( \frac{0.4}{x} \right)$$

$$\therefore x^2 = 2$$

$$\Rightarrow x = 1.41m$$

Areas

$$\Delta V_{A \rightarrow B} = A_{loadA \rightarrow B} = \frac{-0.6 \times 3}{2} = -0.9 \text{ kN}$$

$$V_B = V_A + \Delta V_{A \rightarrow B} = 0.2 - 0.9 = -0.9 \text{ kN}$$

$$\Delta M_{A \rightarrow D} = A_{VA \rightarrow B} = \frac{2}{3} (0.2)(1.41) = 0.188 = 0.19 \text{ kNm}$$

$$M_D = M_A + \Delta M_{A \rightarrow D} = 0 + 0.19 = 0.19 \text{ kNm}$$

$$\Delta M_{D \rightarrow B} = \frac{2}{3} (3.0)(0.2 + 0.7) - 3(0.7) - 0.19$$

$$= -0.49 \text{ kNm}$$

$$M_B = M_D + \Delta M_{D \rightarrow B}$$

$$= 0.19 - 0.49 = -0.3 \text{ kNm}$$
Note:

To determine the distance 'y' at which $M = 0$,

\[ M_x = 0 = 0.2y - \left( \frac{1}{2} \times wx \times y \right) \times \left( \frac{1}{3} \times y \right) = 0 \]

\[ \Rightarrow w = \frac{1.2}{y} \]

Using similar triangles,

\[ \frac{w}{0.6} = \frac{y}{3} \]

\[ \Rightarrow y = \frac{3w}{0.6} = \frac{3}{0.6} \left( \frac{1.2}{y} \right) \]

\[ \Rightarrow y^2 = 6 \]

\[ \therefore y = 2.45 \text{ m} \]