Problem #1:

Given:

The figure shown

Pin at A: \( D = 20 \text{ mm}; \quad \tau_{\text{allow}} = 30 \text{ MPa} \)

Pins at B and C: \( D = 25 \text{ mm}; \quad \tau_{\text{allow}} = 20 \text{ MPa} \)

Required:

\( P_{\text{max}} \) that can be safely applied

Solution:

Note that BC is a “two-force member”. (How?! So what?!)

The FBD is drawn below. We need to calculate the reactions/forces on the pins in terms of \( P \)

\[ \sum M_A = 0 \quad (\text{Why start with this equation}?!?) \]

\[ \Rightarrow 2P(0.5) + 4P(2) + 4P(3.5) + P(4.5) - F_{BC}\sin 30^\circ(5) = 0 \Rightarrow \]

\[ F_{BC} = 11P \]
\[ \sum F_x = 0 \Rightarrow A_x - 11PC\cos30\degree = 0 \]
\[ \Rightarrow A_x = 9.52628P \]
\[ +\sum F_y = 0 \Rightarrow A_y - 2P - 4P - 4P - P + 11PS\sin30\degree = 0 \Rightarrow \]
\[ \Rightarrow A_y = 5.5P \]

Note that all pins at A, B, and C are in **double** shear. See the original figure.

For the pin at A, we need to take the resultant of \(A_x\) and \(A_y\). (Why?!) 

\[ V_A = R_A = \sqrt{(9.52628P)^2 + (5.5P)^2} = 11P \] (As expected = \(F_{BC}\). How?!)

Set \(\tau_A \equiv 30 = \frac{11P}{2\left(\frac{\pi}{4}\right)(20)^2} \Rightarrow \)

\[ P_{\text{max}}^{(1)} = 1713.6 \text{ N} = 1.714 \text{ kN}. \]

For the pins at B and C, both are under the force \(F_{BC}\). ⇒

\[ \tau_B = \tau_C = \frac{F_{BC}}{A} \Rightarrow \]

\[ \frac{11P}{2\left(\frac{\pi}{4}\right)(25)^2} \equiv 20 \Rightarrow \]

\[ P_{\text{max}}^{(2)} = 1785.0 \text{ N} = 1.785 \text{ kN}. \]

From \(P_{\text{max}}^{(1)}\) and \(P_{\text{max}}^{(2)}\) we choose the **smaller** value for the **maximum allowable** \(P\). (Why?!) 

\[ \Rightarrow \boxed{P_{\text{max}} = 1.714 \text{ kN}} \]

In our case in this problem, we can not tell which one “controls”, the pin at A, or the pins at B and C. (Why?!). Thus we had to check both.

In some cases, it may be possible. (When, and how?!)

+\[ \sum F_x = 0 \Rightarrow A_x - 11PC\cos30\degree = 0 \]

\[ \Rightarrow A_x = 9.52628P \]
Problem #2:

Given:

The figure shown

\[ \tau_{\text{ultimate}} = 400 \text{ MPa}, \text{ factor of safety} = 2.0 \]

Required:

\( D_{\text{required}} \) for the bolts

Solution:

\[ \tau = \frac{V}{A} \]

If we consider the middle plate, the bolts are in double shear; if we consider the upper or lower plate, the bolts are in single shear. See the FBDs below.

Both will give us the same answer. (Why, and how?!)
Taking the lower/upper plate,

\[ \tau = \frac{V}{A} \Rightarrow 200(10)^6 \equiv \frac{40(10)^3}{2 \left( \frac{\pi}{4} \right) D^2} \]

⇒ \[ D_{\text{required}} = 0.01128 \text{ m} = 11.3 \text{ mm} \]

**OR**

Taking the middle plate,

\[ 200(10)^6 \equiv \frac{80(10)^3}{2(2) \left( \frac{\pi}{4} \right) D^2} \]

⇒ \[ D = 11.3 \text{ mm}, \text{ as above} \]
Problem #3:

Given:

The figure shown

60 mm × 60mm oak post; pine block

\( \sigma_{\text{bearing,allow}} = 35 \text{ MPa} \) for the oak
\( \sigma_{\text{bearing,allow}} = 20 \text{ MPa} \) for the pine

Required:

- \( P \)
- If a rigid bearing plate is used between the oak and pine, \( A_{\text{plate}} \) required, so the maximum load can be supported.
- \( P_{\text{max}} \)

Solution:

For the first requirement, clearly the pine block “controls”. (Why, and how?!)

Thus,

\[ \sigma_b = \frac{P}{A} \equiv (\sigma_{\text{allow, pine}}) \Rightarrow \]

\[ \frac{P_{\text{max}}}{60 \times 60} = 20 \Rightarrow P_{\text{max, allow}} = 72 \text{ kN} \]

We need to find \( P_{\text{max}} \) (which is for the oak) for the second requirement. (Why?!

\[ \frac{P_{\text{max}}}{60 \times 60} = 35 \Rightarrow P_{\text{max}} = 126 \text{ kN} \]

Now, we set \( (\sigma_b)_{\text{pine}} \equiv 20 \text{ MPa} \) (Why?!) \( \Rightarrow \)

\[ \frac{P_{\text{max}}}{A_{\text{bearing, plate}}} \equiv \sigma_{\text{allow, pine}} \]

\[ \frac{126 \times 10^3}{A_{\text{bearing, plate}}} = 20 \Rightarrow A_{\text{bearing, plate}} = 6300 \text{ mm}^2 \]
Problem #4:

Given:

The figure shown
Circular cross section

\( \sigma_{allow_{AB}} = 80 \text{ MPa} \)

\( \sigma_{allow_{BC}} = 130 \text{ MPa} \)

Required:

\( D_{min} \)

Solution:

\[
\sigma = \frac{P}{A} = \frac{N}{A} \quad \text{where } N \text{ is the \underline{internal} force.}
\]

We have two criteria, one for AB, and the other one for BC. By looking at the numbers (loads \& allowable stresses), we \textit{cannot} tell which one controls. (Why, and how?!)  

First, we need to determine the \textit{internal} forces from the FBD’s below.

\[
N_{BC} = -90 \text{ kN} = 90 \text{ kN “C”}
\]
\[ N_{AB} = -90 + 2(30) \left( \frac{4}{5} \right) = -42 \text{kN} = 42 \text{kN} \quad \text{"C"} \]

\[ \sigma_{AB} = \frac{N_{AB}}{A} \Rightarrow \text{set } 80 = \frac{42(10)^3}{\pi \times D_{\text{min}}^2} \Rightarrow \]

\[ D_{\text{min}}^{(1)} = 25.85 \text{ mm} \]

Note that the material behavior in tension is assumed to be the same as in compression.

\[ \sigma_{BC} = \frac{N_{BC}}{A} \Rightarrow \text{set } 130 = \frac{90(10)^3}{\pi \times D_{\text{min}}^2} \Rightarrow \]

\[ D_{\text{min}}^{(2)} = 29.69 \text{ mm} \]

For the minimum required \( D \), we choose the maximum of \( D_{\text{min}}^{(1)} \) and \( D_{\text{min}}^{(2)} \). (Why?!) \( \Rightarrow \]

\[ D_{\text{min}} = 29.7 \text{ mm} \]
Problem #5:

Given:

The figure shown

\( \sigma_{\text{allow}} = 20 \text{ MPa} \) in each cable.

Required:

\( P_{\text{max allow}} \)

Solution:

To find the stresses in the cables, we need to find the internal forces in them. (Why?!) For cable EF, FBD (1) is drawn

\[
\sum F_x = 0 \quad \Rightarrow \\
P - F_{\text{EF}} = 0 \quad \Rightarrow \\
F_{\text{EF}} = P
\]

For the cables AB and CD, FBD (2) is drawn

\[
\sum M_C = 0 \quad \Rightarrow \\
P(2) - F_{\text{AB}}(3) = 0 \\
\Rightarrow F_{\text{AB}} = \frac{2}{3}P \\
\sum F_x = 0 \quad \Rightarrow \\
P - \frac{2}{3}P - F_{\text{CD}} = 0 \quad \Rightarrow \\
F_{\text{CD}} = \frac{1}{3}P\]
Solution of HW # 2

By looking at the numbers (forces & areas), we cannot tell which cable “controls”. (Why, and how?!) Thus we need to check all three cables.

**Cable AB:**

\[ \sigma_{AB} = \frac{F_{AB}}{A} \]

Set \( \sigma_{AB} \equiv 20 \text{ MPa} = \frac{2P}{32} \)

\[ \Rightarrow P_{\text{max}}^{(1)} = 960 \text{ N} \]

**Cable CD:**

\[ \sigma_{CD} \equiv 20 \text{ MPa} = \frac{1P}{20} \]

\[ \Rightarrow P_{\text{max}}^{(2)} = 1200 \text{ N} \]

**Cable EF:**

\[ \sigma_{EF} \equiv 20 \text{ MPa} = \frac{P}{50} \]

\[ \Rightarrow P_{\text{max}}^{(3)} = 1000 \text{ N} \]

For the maximum allowable load \( P \), we choose the minimum of \( P_{\text{max}}^{(1)} \), \( P_{\text{max}}^{(2)} \), and \( P_{\text{max}}^{(3)} \). (Why?!) 

\[ \Rightarrow [P_{\text{max allow}} = 960 \text{ N}] \]

(which is controlled by cable AB)