King Fahd University of Petroleum & Minerals

CIVIL ENGINEERING DEPARTMENT

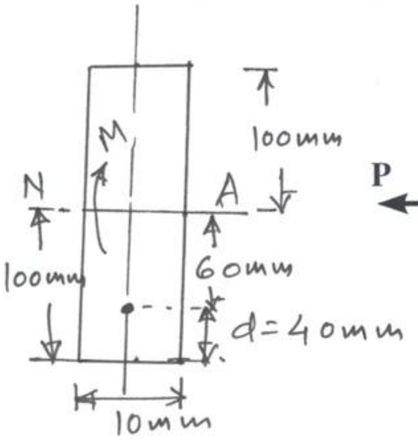
CE 203 STRUCTURAL MECHANICS I

First Semester 2012 / 2013 (**121**)

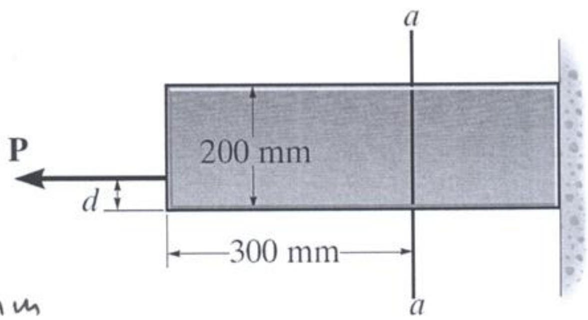
HOMEWORK NO. 13 (Key Solution)

- **Textbook Sections Covered:** 8.2 (Compound Stress)
- **DUE DATE: Monday, 17 December 2012**

1 – The given thin plate is subjected to the shown load. Determine the largest value of P that can be safely applied. Also , determine the location of the Neutral Axis. *Given: allowable tensile stress is 20 MPa , allowable compressive stress is 24 MPa, plate thickness = 10mm , d = 40 mm.*



(Not by scale)



$M_{about\ NA} = 60P$ (Will cause comp. stress above NA and tensile stress below NA).

$I_{NA} = \frac{10 \times 200^3}{12} = 6.67 \times 10^6\ mm^4$

$A = 10 \times 200 = 2000\ mm^2$

$\sigma_{max,c} = \frac{P}{A} - \frac{60P \times 100}{I} = \sigma_{allow,c}$

$\Rightarrow \frac{P}{2000} - \frac{60P \times 100}{6.67 \times 10^6} = -24$

$\Rightarrow P = 60,068\ N = 60.06\ KN$

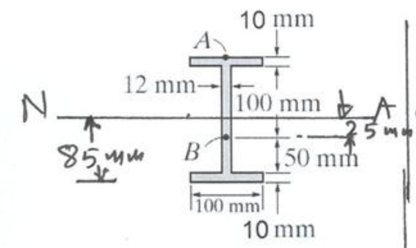
$\sigma_{max,t} = \frac{P}{A} + \frac{60P \times 100}{I} = \sigma_{allow,t}$

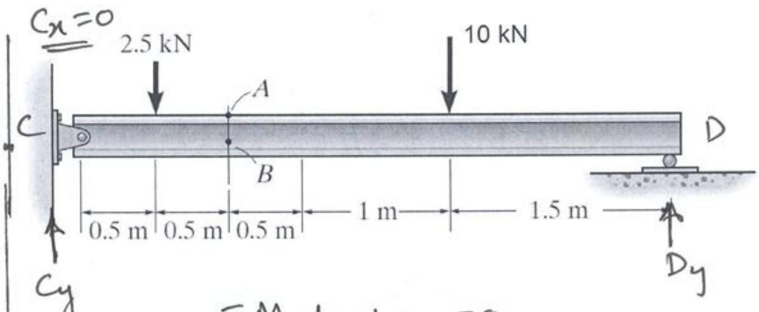
$\Rightarrow \frac{P}{2000} + \frac{6000P}{6.67 \times 10^6} = 20$

$\Rightarrow P = 14290\ N = 14.29\ KN$

$\therefore$ The largest value of $P = 14.29\ KN$ Ans.

2 - The given I-beam is subjected to the loading shown. Determine the stress components (normal & shear) at points *A* and *B* and show the results on a differential element at each of these points. Use the shear formula (*VQ/Ib*) to compute the shear stress.





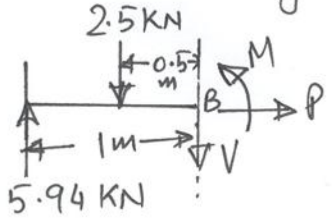
$$I_{NA} = \frac{100 \times 170^3}{12} - \frac{88 \times 150^3}{12} = 1.619 \times 10^7 \text{ mm}^4$$

$$Q_B = 100 \times 10 \times (85-5) + 50 \times 12 \times (25+25) = 11 \times 10^4 \text{ mm}^3$$

ΣM about D $= 0$

$$\Rightarrow -4C_y + 2.5 \times 3.5 + 10 \times 1.5 = 0$$

$$\Rightarrow C_y = 5.94 \text{ KN}$$



$$\Sigma F_x = 0 \Rightarrow P = 0$$

$$\Sigma F_y = 0 \Rightarrow 5.94 - 2.5 - V = 0$$

$$\Rightarrow V = 3.44 \text{ kN}$$

$$\Sigma M = 0$$

$$\Rightarrow -5.94 \times 1 + 2.5 \times 0.5 + M = 0$$

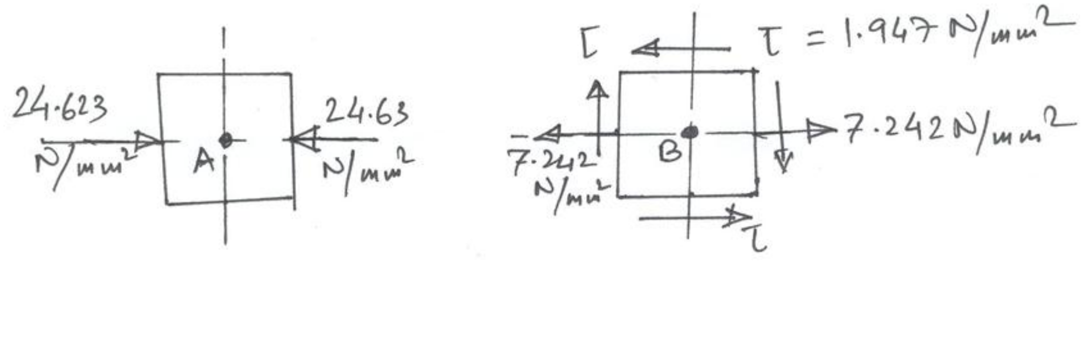
$$\Rightarrow M = 4.69 \text{ kN-m}$$

A
$$\sigma_A = \frac{4.69 \times 10^6 \times 85}{1.619 \times 10^7} = 24.623 \text{ N/mm}^2 \text{ (Compressive)}$$

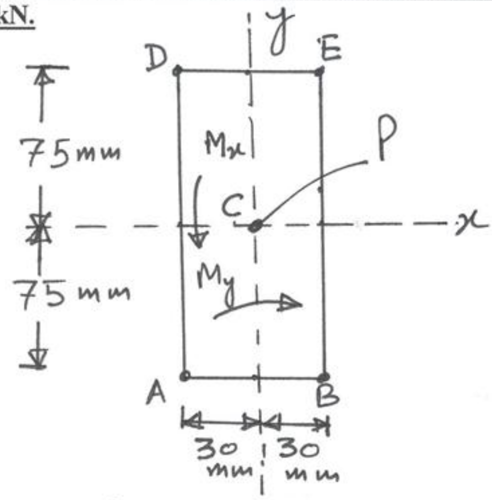
$$\tau_A = 0$$

B
$$\sigma_B = \frac{4.69 \times 10^6 \times 25}{1.619 \times 10^7} = 7.242 \text{ N/mm}^2 \text{ (Tensile)}$$

$$\tau_B = \frac{3.44 \times 10^3 \times 11 \times 10^4}{1.619 \times 10^7 \times 12} = 1.947 \text{ N/mm}^2$$



3 - Solve problem 8-45 (p. 427) in your textbook, but make the following change**: the block width should be 60 mm (instead of 75mm) , and the magnitude of the 30 kN force should be changed to 10 kN.**



$$P = 10 + 60 = 70\ \text{kN}\ (\downarrow)$$

$$M_x = 60 \times 75 - 10 \times 75 = 3750\ \text{kN-mm}$$

$$M_y = 60 \times 30 - 10 \times 30 = 1500\ \text{kN-mm}$$

$$I_x = \frac{60 \times 150^3}{12} = 16.875 \times 10^6\ \text{mm}^4$$

$$I_y = \frac{150 \times 60^3}{12} = 2.7 \times 10^6\ \text{mm}^4$$

$$A = 60 \times 150 = 9 \times 10^3\ \text{mm}^2$$

$$\sigma_A = \frac{-P}{A} - \frac{M_x \times 75}{I_x} + \frac{M_y \times 30}{I_y} = \frac{-70\times10^3}{9\times10^3} - \frac{3750\times10^3 \times 75}{16.875\times10^6} + \frac{1500\times10^3 \times 30}{2.7\times10^6}$$

$$= -7.78 - 16.67 + 16.67 = -7.78\ \text{N/mm}^2 = 7.78\ \text{N/mm}^2\ (\text{Comp.})$$

$$\sigma_B = \frac{-P}{A} - \frac{M_x \times 75}{I_x} - \frac{M_y \times 30}{I_y}$$

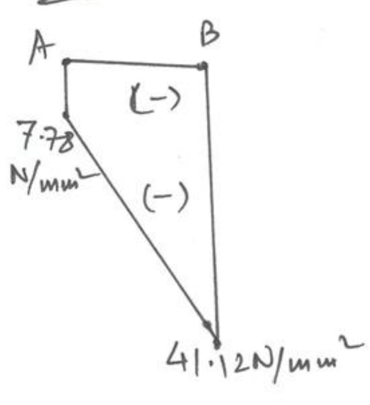
$$= -7.78 - 16.67 - 16.67 = -41.12\ \text{N/mm}^2 = 41.12\ \text{N/mm}^2\ (\text{Comp.})$$

$$\sigma_E = -\frac{P}{A} + \frac{M_x \times 75}{I_x} - \frac{M_y \times 30}{I_y} = -7.78 + 16.67 - 16.67 = -7.78\ \text{N/mm}^2 = 7.78\ \text{N/mm}^2\ (\text{Comp.})$$

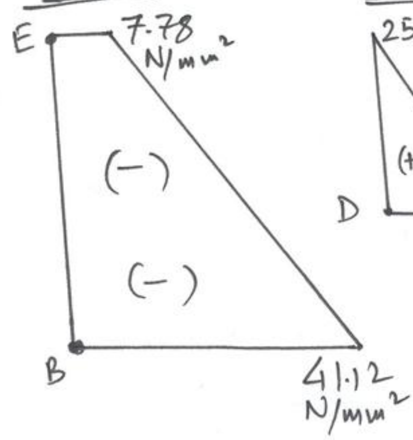
$$\sigma_D = -\frac{P}{A} + \frac{M_x \times 75}{I_x} + \frac{M_y \times 30}{I_y} = -7.78 + 16.67 + 16.67 = 25.56\ \text{N/mm}^2\ (\text{Tensile})$$

σ-Diagram :-

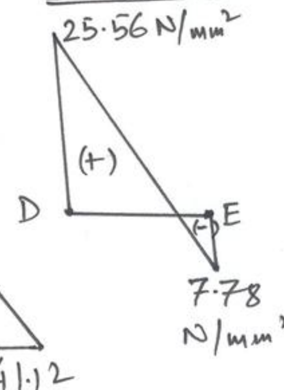
Along "A B"



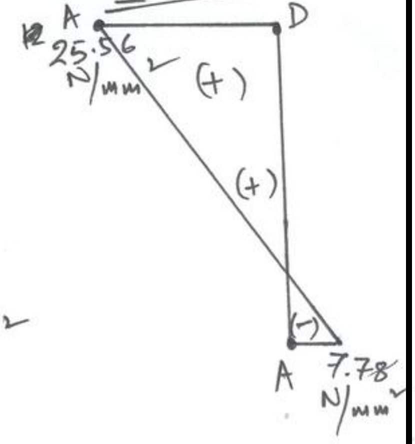
Along "BE"



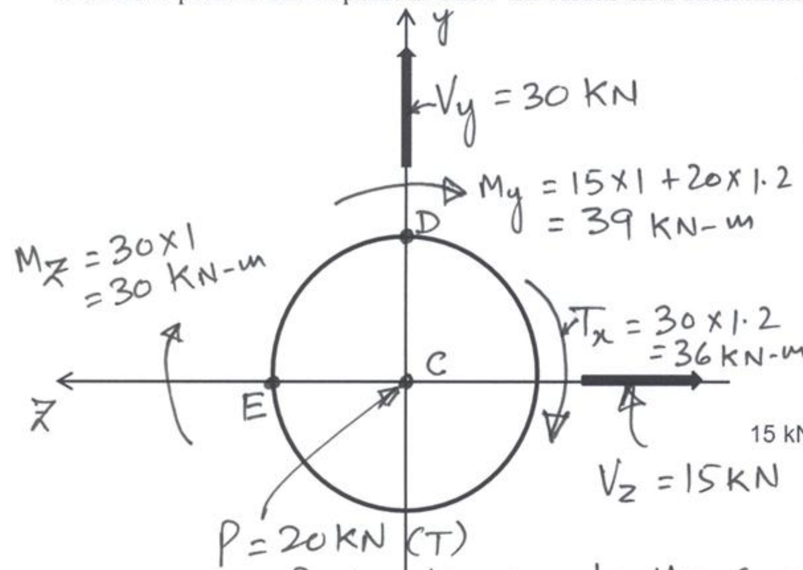
Along "ED"

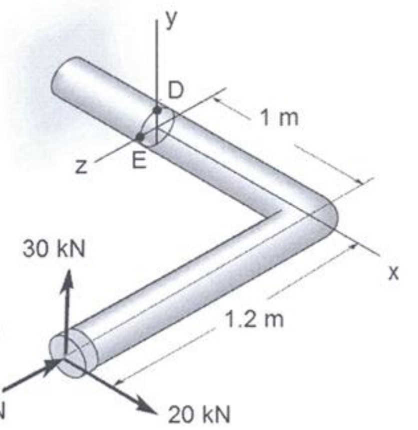


Along "DA"



4 – The 60 mm-diameter rod is fixed to the wall and is subjected to the loads shown. Determine the state of stress at point D and at point E. Show the results on a differential element located at each point.





Perpendicular to the Cross-section.

$$\sigma_D = \frac{P}{A} - \frac{M_z \times r}{I} = \frac{20\times10^3}{\pi\times30^2} - \frac{30\times10^6\times30}{\frac{\pi}{4}\times30^4} = 7.07 - 1414.71$$

$$= -1407.64\ N/mm^2 = 1407.64\ N/mm^2\ (\text{Comp.})$$

$$\sigma_E = \frac{P}{A} + \frac{M_y \times r}{I} = \frac{20\times10^3}{\pi\times30^2} + \frac{39\times10^6\times30}{\frac{\pi}{4}\times30^4} = 7.07 + 1839.12$$

$$= 1846.19\ N/mm^2\ (\text{Tensile})$$

$$\tau_D = \overrightarrow{\frac{V_z Q}{It}} + \overrightarrow{\frac{T_x \times r}{J}} = \frac{15\times10^3\times\frac{\pi}{2}(30)^2\times\frac{4\times30}{3\pi}}{\frac{\pi}{4}\times30^4\times60} + \frac{36\times10^6\times30}{\frac{\pi}{2}\times30^4}$$

$$= 7.07 + 848.82$$

$$= 855.89\ N/mm^2\ (\longrightarrow)$$

$$\tau_E = \frac{V_y Q}{It}(\uparrow) + \frac{T_x \times r}{J}(\uparrow) = \frac{30\times10^3\times\frac{\pi}{2}(30)^2\times\frac{4\times30}{3\pi}}{\frac{\pi}{4}\times30^4\times60} + \frac{36\times10^6\times30}{\frac{\pi}{2}\times30^4}$$

$$= 14.14 + 848.82$$

$$= 862.96\ N/mm^2\ (\uparrow)$$