Solution of HW #12

Prob #1:

Given:

The beam shown

\[ V = 250 \text{ kN} \]

\[ R_{beyond} = 75 \text{ kN} \]

Required:

Max spacing of bolts (s)

Solution:

First, we need to locate \( y \) and calculate \( ar{I} \).

Due to the double symmetry, the centroid is in the "middle".

Thus,

\[ y = 14 + 75 + 25 = 114 \text{ mm} \]

\[ \bar{I} = \frac{1}{12} \left[ 75(114+114)^3 - (75-14)(2(75+25))^3 - 14(25+25)^3 + 2(14)(150)^3 \right] \]

\[ = 4 \cdot 11397 \times 10^7 \text{ mm}^4 \]

Try to find \( I \) by "another way"!

\[ q = \frac{VQ}{I} \]

For the shaded area,

\[ Q = \iota \cdot Q_i = \iota (A \bar{y}) \cdot \iota \]

\[ Q = Q_0 + Q_2 \]

\[ = 75(14)(114 - 14) + 14(75)(\frac{25}{2} + 25) \]

\[ = 177,975 \text{ mm}^3 \]

\[ \Rightarrow q = \frac{250 \left(10^3\right)(177,975)}{4 \cdot 11397 \left(10^7\right)} = 1081.53 \text{ N/mm} \]

\[ \frac{R_{beyond}}{5} = q \]
The bolts act in "double shear" ⇒

\[
\frac{2(75)(10^3)}{S} = 1081.53 \Rightarrow \\
S = 138.7 \text{ mm}
\]

Note that due to symmetry, the spacing of the bolts in the lower row is the same as the type, or...
Problem #2

Given:
The beam with the cross-section shown

Required:
The min. strength of the glue connecting the lower flange to the web

Solution:
First, we need to find \( y \) and \( I \).
We will use a table. We need to first locate \( y \),
then calculate \( I \) (why?!!)

<table>
<thead>
<tr>
<th>Segment part</th>
<th>( A_i (\text{mm}^2) )</th>
<th>( y_i (\text{mm}) )</th>
<th>( A_i y_i (\text{mm}^3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>700</td>
<td>10 + 150/5 = 30</td>
<td>115,500</td>
</tr>
<tr>
<td>2</td>
<td>1500</td>
<td>10 + 75 = 85</td>
<td>127,500</td>
</tr>
<tr>
<td>3</td>
<td>1500</td>
<td>5</td>
<td>7,500</td>
</tr>
<tr>
<td>( \sum )</td>
<td>3700</td>
<td>5</td>
<td>250,500</td>
</tr>
</tbody>
</table>

\[ \bar{y} = \frac{\sum (A_i y_i)}{\sum A_i} = \frac{250,500}{3,700} = 67.7027 \text{ mm} \]
(Reasonable?!!)

\( \bar{y} \) is 'd'?! Review statics!!!
Solution of HW #12

| Segment part | $A_i$ (mm$^2$) | $d_i$ (mm) | $A_i d_i^2$ (mm$^4$) | $I_i$ (mm$^4$) | $I_i + A_i d_i^2$ (mm$^4$) |
|--------------|--------------|-----------|-----------------|-------------|----------------|}
| (i)          | 700          | 100       | $10^2 + 5 - y = 97.2973$ | 6626735     | 58333.333 | 6632569 |
| (ii)         | 1500         | 100       | $10 + 75 - y = 17.2973$ | 448795      | 2812500    | 3261295 |
| (iii)        | 1500         | 100       | $y - 5 = 62.7027$      | 5897443     | 12500      | 5909943 |
| <=           | 3700         |           |                 |             |             |

4. $I = \frac{1}{12} b h^3$

$$T_{\text{gmax}} = \frac{V_{\text{max}} Q}{I t}$$

To determine $V_{\text{max}}$, we need to draw the SFD. But first, we need to determine the reactions.

From the FBD

4. $2M_D = 0 \Rightarrow -4.6 A y + 5(3.6) + 3(\frac{y}{3}) = 0$  
$\Rightarrow A y = 4.78261$ kN

1. $SE F_y = 0 \Rightarrow$  
$4.78261 - 5 - 3 + D_y = 0$  
$\Rightarrow D_y = -3.21739$ kN

Clearly, $V_{\text{max}} = 4.78261$ kN

$$T_{\text{gmax}} = \frac{V_{\text{max}} Q}{I t}$$

$$y = 67.7027 - 5 = 62.7027$$

$$Q = A y = 10(150)(62.7027)$$

$$Q = 94054.1 \text{ mm}^3$$

$$T_{\text{gmax}} = \frac{4782.61(94054.1)}{(15.8038x10^6)(10)} = 2.8463 \text{ MPa}$$

$\Rightarrow$ Required glue strength = 2.85 MPa
Problem 3

Given:
The cross-section of the beam shown

- \( T_{\text{allow}} = 2 \text{ MPa} \)
- \( R_{\text{load}} = 600 \text{ N} \)
- \( S = 100 \text{ mm}, S' = 120 \text{ mm} \)

Required:

- \( V_{\text{max}} \)

Solution:

\[ T_{\text{max}} = \frac{V_{\text{max}} Q_{\text{max}}}{It} \]

First, we need to locate the centroid (\( \bar{y} \)), then calculate \( I \).

\[ \bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} \]

\[
\bar{y} = \frac{40(250)(\frac{3}{2} + 25) + 25(250)(\frac{35}{2}) + 2(25)(75)(\frac{75}{2})}{40(250) + 25(250) + 2(25)(75)} = 85.9375 \text{ mm}
\]

\[
I_{CA} = \left[ \frac{1}{12}(40)(250^3) + 40(250)(125 + 25 - 85.9375)^2 \right] + \left[ \frac{1}{12}(250)(25^3) + 25(25)(85.9375 - 12.5)^2 \right] + \left[ \frac{1}{12}(2)(25)(75^3) + 2(25)(75)(85.9375 - 75)^3 \right]
\]

\[ = 1.3771 \times 10^8 \text{ mm}^4 \]
Solution of HW #12

Criterion 1: \( \tau_{\text{allow}} = 2 \text{ MPa} \)

Clearly, \( \tau_{\text{max}} @ \text{C.A.} \).

It is easier to take the lower (shaded) area to calculate \( Q \to \phi \to \tau \)
(Why? !)

Try the upper part!

\[ Q_{Gn} = Ay = 40(250 + 25 - 85.9375)^2 / 2 \]

\[ = 714.3893 \text{ mm}^3 \]

\[ \tau_{\text{max}} = \frac{V_{\text{max}}^0 Q_{Gn}}{I_t G_{n}} = \frac{V_{\text{max}}^0 714.3893}{1.3771 \times 10^8} = 1.2978 \times 10^{-4} \]

Set \( \tau_{\text{max}} = \tau_{\text{allow}} = 2 \text{ MPa} \Rightarrow \)

\[ V_{\text{max}} = 2 \sqrt{1.2978 \times 10^{-4}} = 1.5411 \text{ KN} \]

Criterion 2: \( R_{\text{nail}} = 600 \text{ KN} ; S = 100 \text{ mm} \)

(side nails)

\[ R_n = qS = \frac{VQ}{I} S \]

\[ Q = Ay = 25(75)(85.9375 - 25) \]

\[ = 90 \times 820 \text{ mm}^3 \]

\[ 600 = \frac{V_{\text{max}}^2 (90,820)}{1.3771 \times 10^8} \]

\[ \Rightarrow V_{\text{max}}^2 = 9.0978 \text{ KN} \]

Criterion 3

\( R_{\text{nail}} = 600 \text{ KN} ; S' = 120 \text{ mm} \)

(top nails)

\[ R_n = qS' = \frac{VQ}{I} S' \]

\[ Q = Ay = 40(250)(\frac{250}{2} + 25 - 85.9375) = 640.625 \]
Note that it is easier to take the lower area; you may choose the upper part. (Try!)

\[ 600 = \frac{V_{\text{max}}}{1.3771 \times 10^8} \]

\[ V_{\text{max}}^{(3)} = 1.0748 \text{ kN} \]

For \( V_{\text{max}} \) allow, we choose \( \min(V_{\text{max}}^{(1)}, V_{\text{max}}^{(2)}, V_{\text{max}}^{(3)}) \)

\[ \Rightarrow V_{\text{max allow}} = 1.0748 \text{ kN} \]
Prob # 4

Given:
The two figures shown
\[ P = 0.5 \text{ MPa}, \ t = 6\text{ mm} \]
\[ D = 200 \text{ mm} \]

Required:
The state of stress in the cylinder wall in cases (a) and (b)

Solution:

Case (a)

Note that in case (a), there is a circumferential (hoop) stress, but no longitudinal (axial) stress, so it is “free to expand” in the axial (vertical) direction (up).

Thus,
\[ \sigma_c = \sigma_1 = \frac{P r}{t} = \frac{0.5 \times 10^6 \times 200 \times 10^{-3}}{6 \times 10^{-3}} \]

\[ \Rightarrow \sigma_c = \sigma_1 = 8.333 \text{ MPa} \]

\[ \sigma_1 = \sigma_2 = 0 \]

Case (b)

In case (b), there is an axial stress on the cylinder, as we can see it from the boundary conditions. However, \( \sigma_c \) is as in case (a).

\[ \Rightarrow \sigma_c = \sigma_1 = 8.833 \text{ MPa} \]
\[ \sigma_1 = \sigma_2 = \frac{P r}{2 t} = \frac{0.5 \times 200}{2 \times 6} \]

\[ \sigma_2 = \sigma_c = 4.167 \text{ MPa} \]
We can also find $V_L$ as we did in the "axially-loaded member" chapter.

For case (a), from the FBD,

$$V_L = 0$$

For case (b), from the FBD,

$$N_{\text{total}} = R_{\text{total}}$$

and $R_{\text{total}} = F$, and $F = PA_{\text{pressure}}$

Thus, $N = R = F = \text{Pressure} \times \text{Area}$

$$\Rightarrow N = 0.5 \times 10^6 \left[ \pi \times \left( \frac{200}{2} \right)^2 \right] \times 10^{-6}$$

$$= 5000 \pi \ (N)$$

$$V_{\text{axial}} = V_L = V_2 = \frac{N}{A}$$

$A = \text{Area of material}$

$$\approx \pi \times \frac{4}{2} = 1200 \pi \ \text{mm}^2$$

$$\Rightarrow V_L = V_2 = \frac{5000 \pi}{1200 \pi \times 10^{-6}} = 4.167 \ \text{MPa} \ (A \text{ as above})$$
Prob #5

Given:
The figure shown, a steel tank filled with water

\( v_{\text{water}} = 10 \text{kN/m}^3 \), and

\( v_{\text{steel}} = 70 \text{kN/m}^3 \)

Required
State of Stress at A

Solution:
The FBD after making a section through A and taking the "upper" part (why?) is shown.
The stress at A is caused by the weight of the steel above A. "Steel carries itself" and "water carries itself" vertically.

In addition, the water causes "hydrostatic pressure" on the steel wall according to "Pascal's law", that is, \( P = (\gamma h)_{\text{water}} \). (Review it if you took it; read about it or ask your instructor if you did not).

This \( P \) causes \( V_h \) (or \( V_c \)) as studied in the pressure vessels section

\[
W_{\text{steel}} = v_{\text{steel}} V_{\text{steel}}
\]

\[
= 70 \left( 810^2 - 800^2 \right) \pi \times 10^{-6} \times 1.2
\]

\[
= 4.2487 \text{ kN}
\]
\[ V_{\text{vert}} = V_{\text{long}} = V_{\text{axial}} = \frac{W_{\text{steel}}}{A_{\text{steel}} \text{ at short section}} \]
\[ = \frac{-4.2487 \times 10^3}{(810^2 - 800^2) \pi \times 10^{-6}} \]

\[ V_{\text{vert}} = -84,000 \text{ kPa} = 84 \text{ kPa} \] "C"

\[ V_{\text{hoop}} = V_{\text{circumferential}} = \frac{Pr}{t} \]

\[ P = \gamma_{\text{water}} h_{\text{water}} = 10 \times 10^3 \times 1.2 = 12 \text{ kPa} \]

\[ \Rightarrow V_h = \frac{12 \times 800}{10} = 960 \text{ kPa} \] "\( T \)" = \( V_h \)

The state of stress is
as shown

Note that the internal pressure due to water does not cause longitudinal stress in the open tank (why?!)