CE 203 STRUCTURAL MECHANICS I

First Semester 2012 / 2013 (**121**)

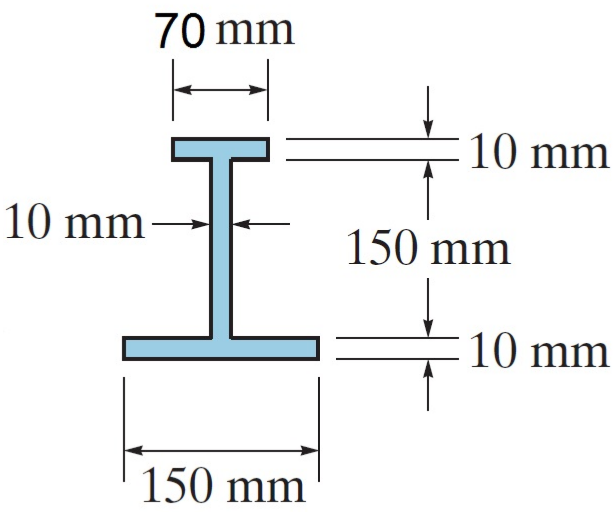
HOMEWORK NO. 11 (Key Solution)

- **Textbook Sections Covered:** 6.4 (Beam Bending) & 7.1-7.2 (Shear Stress in Beams)
- **DUE DATE**: **Monday, 3 December 2012**

Problem # 1:-

Given Data:

- The given cross section.
- M= +4 kN.m.



Required:

- The resultant force (due to bending) that act on each of the following parts :

  a) on the top flange,

  b) on the bottom flange , and

  c) on the web.

Solution:

From the data of the given cross section:-

$$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{0.15*0.01*0.005 + 0.15*0.01*0.085 + 0.07*0.01*0.165}{0.15*0.01 + 0.15*0.01 + 0.07*0.01} = 0.0677\ m$$

$$I = \sum(\bar{I} + Ad^2)$$

$$= \frac{0.15*0.01^3}{12} + 0.15*.01*(0.0677-0.005)^2 + \frac{0.01*0.15^3}{12} + 0.15*.01*(0.0677-0.085)^2 + \frac{0.07*0.01^3}{12} + 0.07*.01*(0.0677-0.165)^2$$

$$I = 1.58*10^{-5}\ m^4$$

$$\sigma = \frac{M * c}{I} \quad \Rightarrow$$

$$\sigma_{top} = \frac{M * (0.17 - .0677)}{I} = \frac{4 * (0.17 - 0.0677)}{1.58 * 10^{-5}} = 25.90\ MPa.$$

$$\sigma_{bottom} = \frac{M * .0677}{I} = \frac{4 * 0.0677}{1.58 * 10^{-5}} = 17.14\ MPa.$$

Resultant force at the top flange (compression) is

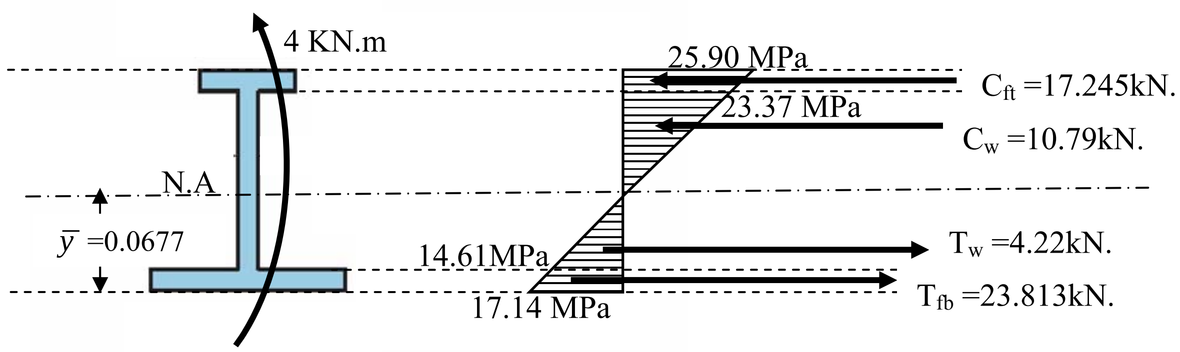
$\frac{(25.9+23.37)}{2} * 70 * 10 = 17244.5\ N = 17.245\ KN.$ (compression) *Ans*

Resultant force at the bottom flange (tension) is

$\frac{(17.14+14.61)}{2} * 150 * 10 = 23813\ N = 23.813\ KN.$ (tension) *Ans*

Resultant force on the web is

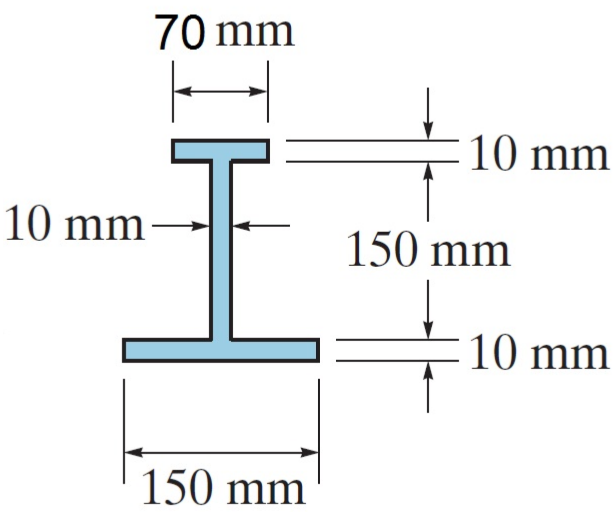
$\frac{(23.37)}{2} * (150 - 57.7) * 10 - \frac{(14.61)}{2} * (57.7) * 10 = 6570\ N =$
$6.57\ KN.$ (compression) *Ans*



Problem # 2:-

Given Data:

- The given cross section.
- A downward vertical shear force V = 10 kN.



Required:

- The value of the shear stress at the following locations:
  - a) top of cross section ,
  - b) bottom of cross section ,
  - c) at the Neutral Axis,
  - d) at just above and just below the junction of web and top flange (*2 values are needed here),*
  - *e)* At just above and just below the junction of web and bottom flange (*2 values are needed here)* .
- The distribution of shear stress along the vertical axis using the calculated shear stresses.

Solution:

From the problem # 1: $\bar{y} = 0.0677\ m$ **;** **and** $I = 1.58 * 10^{-5}\ m^4$

$$\tau = \frac{VQ}{It} \quad ; \quad Q = \bar{y}' A'$$

$$\tau_1 = 0$$

$$\tau_2 = \frac{10 * 10^3[0.070 * 0.010 * (0.165 - 0.0677)]}{1.58 * 10^{-5} * 0.07} = 0.616\ MPa.$$

$$\tau_3 = \frac{10 * 10^3[0.070 * 0.010 * (0.165 - 0.0677)]}{1.58 * 10^{-5} * 0.01} = 4.311\ MPa.$$

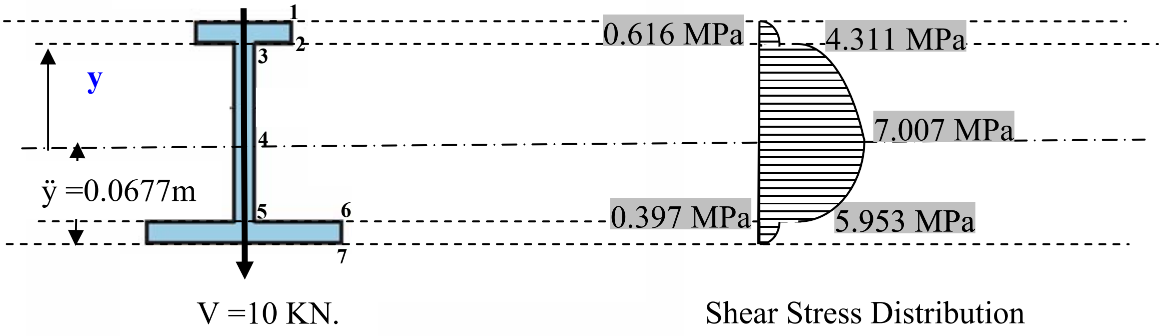
$$\tau_4 = \frac{10 * 10^3[0.070 * 0.010 * (0.165 - 0.0677) + \frac{0.01 * (0.15 - 0.0577)^2}{2}]}{1.58 * 10^{-5} * 0.01} = 7.007\ MPa$$

$$\tau_5 = \frac{10 * 10^3 [0.150 * 0.010 * (0.0677 - 0.005)]}{1.58 * 10^{-6} * 0.01} = 5.953 \text{ MPa}$$

$$\tau_6 = \frac{10 * 10^3 [0.150 * 0.010 * (0.0677 - 0.005)]}{1.58 * 10^{-6} * 0.15} = 0.397 \text{ MPa.}$$

$$\tau_7 = 0$$

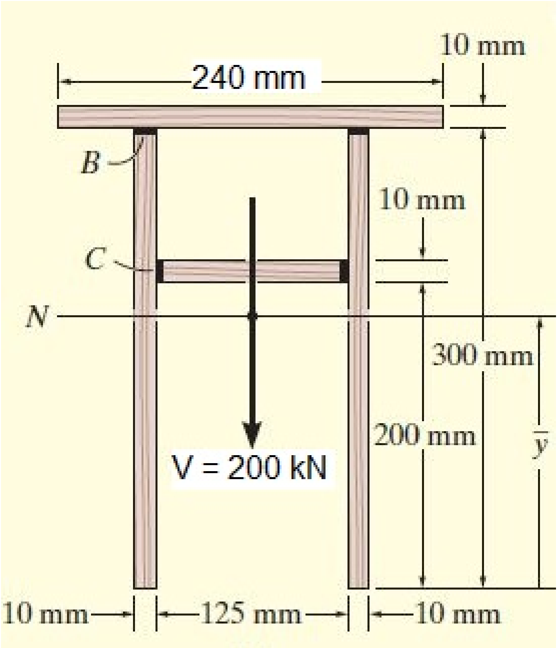
Shear stress $\tau_{(y)}$ distribution along c/s depth:



Problem # 3:-

Given Data:

- The given shear force and cross section.



Required:

- The value of the maximum shear stress due to the given shear force.

Solution:

From the data of the given cross section:-

$$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{0.01*0.3*0.15*2+0.01*0.125*0.205+0.24*0.01*0.305}{0.01*0.3*2+0.01*0.125+0.24*0.01} = 0.1957\,m$$

$$I = \sum(\bar{I}+Ad^2)$$

$$= [\left\{\frac{0.01*0.3^3}{12}+0.01*0.3*(0.15-0.1957)^2\right\}*2+\frac{0.125*0.01^3}{12}+0.01*0.125*(0.205-0.1957)^2+\frac{0.24*0.01^3}{12}+0.24*0.01*(0.305-0.1957)^2]$$

$$I = 86.34*10^{-6}\ m^4$$

$\because$ **Max shear stress is at the neutral axis,** $\rightarrow$

$$Q_{at\ the\ neutral\ axis} = yA = \left[0.1957-\frac{0.1957}{2}\right]*0.1957*0.01*2 = 0.383*10^{-3}m^3$$

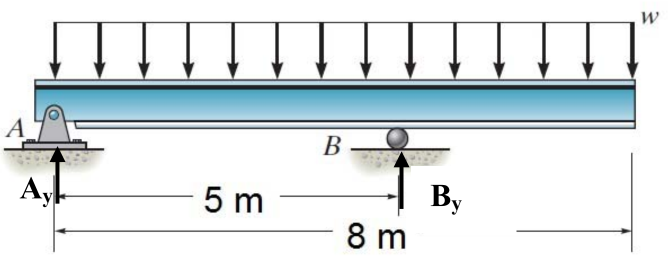
$$\tau = \frac{VQ}{It} = \frac{200*10^3*0.383*10^{-3}}{86.34*10^{-6}*0.02} = 44.36\,MPa.\ Ans$$

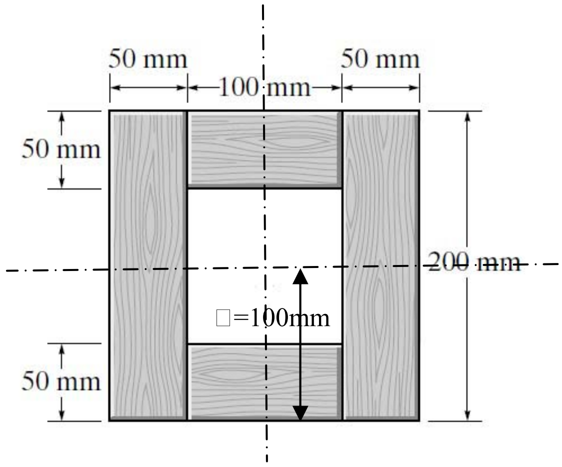
Problem # 4:-

Given Data:

- The shown loaded beam and its cross section.
- Allowable normal stress is 30 MPa.
- Allowable shear stress = 10 MPa.

Hint : First, you need to draw the shear and moment diagrams to determine the maximum values of V and M).





Required:

❖ The largest value of the distributed load ***w*** that can be safely applied**.**

Solution:

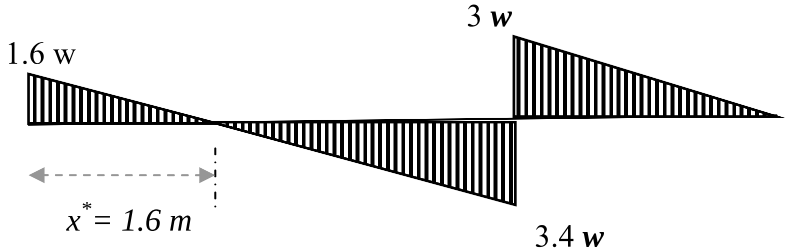
First we have to find the reactions and then draw the shear and bending moment diagrams as follows:-

$$\sum M_B = 0; \quad A_y * 5 - w * 5 * 2.5 + w * 3 * 1.5 = 0 \Rightarrow A_y = 1.6\,w$$

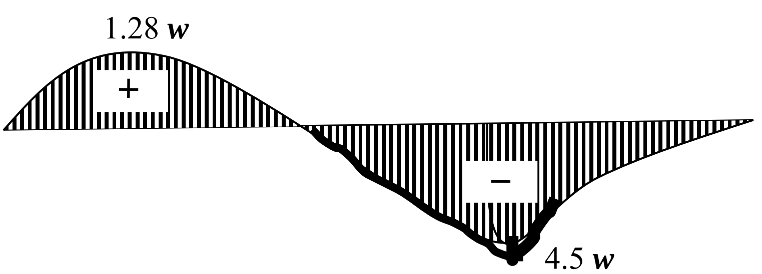
$$\sum M_A = 0; \quad B_y * 5 - w * 5 * 2.5 - w * 3 * 6.5 = 0 \Rightarrow B_y = 6.4\,w$$

Now we can draw the shear and bending moment diagrams as follows:-

SFD:



BMD:



From the shear diagram, the maximum shear force is **3.4 w**.
And from the bending moment diagram, the maximum bending moment is - **4.5w.**

For the given cross section, □ = 0.1 m as shown by inspection

$$I = \frac{0.2 * 0.2^3}{12} - \frac{0.1 * 0.1^3}{12} = 1.25 * 10^{-4}\ m^4$$

From the bending moment,

$$\sigma_{max} = \frac{M * c}{I} = 30\frac{N}{mm^2} = 30 * 10^6\frac{N}{m^2} = \frac{4.5W * 0.1}{1.25 * 10^{-4}}$$

w = 8330 N/m = 8.33 kN/m

From the Shear,

$$\tau_{max} = \frac{V * Q}{It} = 10\frac{N}{mm^2} = 10 * 10^6\frac{N}{m^2} = \frac{3.4W * [0.075 * 0.1 * 0.05 + 2 * 0.05 * 0.1 * 0.05]}{1.25 * 10^{-4} * 0.1}$$

w = 42017 N/m = 42.02 kN/m

Then, the largest value of the distributed load ***w*** that can be safely applied is the minimum value of [8.33 kN/m and 42.02 kN/m] = **8.33 *k*N/m**. **Ans.**