Structural Mechanics I

KEY OF HOME WORK : 3

by

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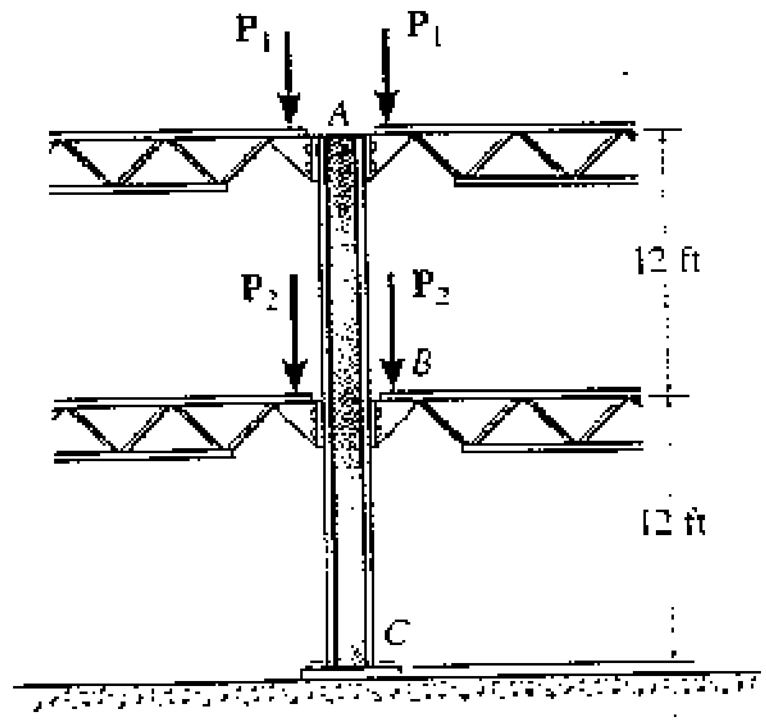
KEY TO HOMEWORK #3:-

PROBLEM #①(4-3):-

4–3. The A-36 steel column is used to support the symmetric loads from the two floors of a building. Determine the loads P_1 and P_2 if A moves downward 0.12 in. and B moves downward 0.09 in. when the loads are applied. The column has a cross-sectional area of 23.4 in^2.



Probs. 4–2/4–3

Solution:-

Given:-

$\delta_{A/C} = -0.12\,in.$

$\delta_{B/C} = -0.09\,in.$

Required:-

$P_1 = ?$ & $P_2 = ?$

Internal Forces:-

From fig. ①

$+\uparrow \Sigma y = 0$

$P_{AB} - 2P_1 = 0$

$\Rightarrow P_{AB} = 2P_1$ (comp).

From fig. ②

$+\uparrow \Sigma y = 0$

$P_{BC} - 2P_1 - 2P_2 = 0$

$\Rightarrow P_{BC} = 2P_1 + 2P_2.$

fig. ①

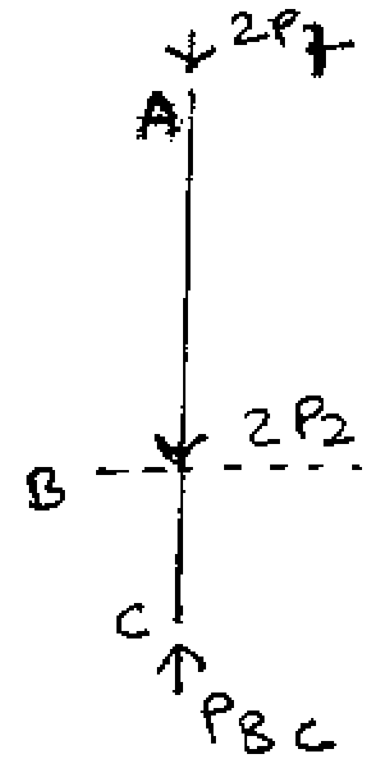


fig. ②

Displacements:-

$$\delta_{A/C} = \frac{P_{AB} \cdot L_{AB}}{A \cdot E} + \frac{P_{BC} \cdot L_{BC}}{A \cdot E} = -0.12\,in$$

$$= \frac{-2P_1 \times 12 \times 12}{23.4 \times 29 \times 10^3} - \frac{(2P_1 + 2P_2) \times 12 \times 12}{23.4 \times 29 \times 10^3} = -0.12$$

$$\Rightarrow -288P_1 - 288P_1 - 288P_2 = -81432$$

$$\Rightarrow P_2 = 282.75 - 2P_1 \rightarrow ①$$

$$\& \quad \delta_{B/C} = \frac{P_{BC} \cdot L_{BC}}{A \cdot E} = -0.09\,in$$

P.T.O

$$\Rightarrow \delta_{B/C} = \frac{-(2P_1 + 2P_2) \times 12 \times 12}{23.4 \times 29 \times 10^3} = -0.09$$

$$\Rightarrow -288P_1 - 288P_2 = -61074$$

$$\Rightarrow P_1 + P_2 = 212 \rightarrow ②.$$

By solving ① & ② we get

$$+P_1 + (282.75 - 2P_1) = +212$$

$$\Rightarrow P_1 = \underline{70.75 \text{ kip}}$$

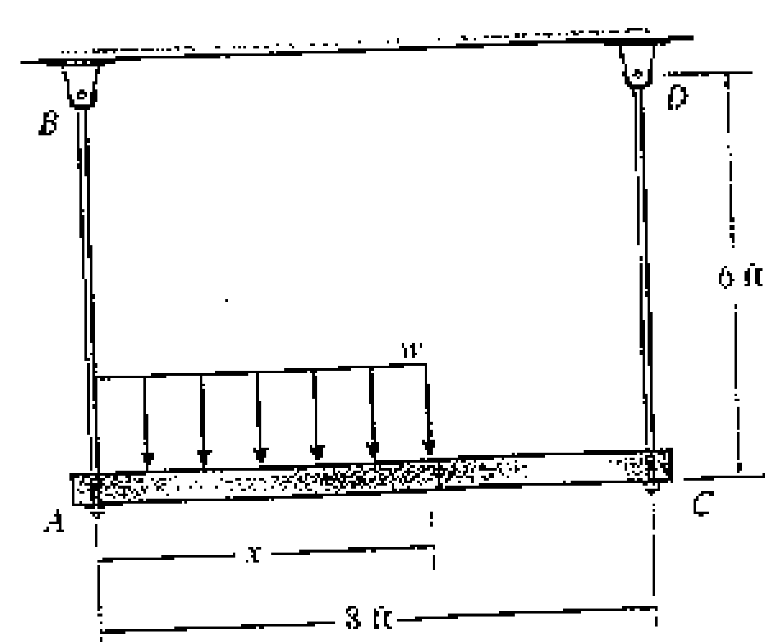
$$\& \; P_2 = 282.75 - 2 \times 70.75$$

$$\Rightarrow P_2 = \underline{141.25 \text{ kip}}.$$

PROBLEM # ② (4-21):-

Solution:-

4–21. The rigid beam is supported at its ends by two A-36 steel tie rods. The rods have diameters $d_{AB} = 0.5$ in. and $d_{CD} = 0.3$ in. If the allowable stress for the steel is $\sigma_{allow} = 16.2$ ksi, determine the intensity of the distributed load w and its length x on the beam so that the beam remains in the horizontal position when it is loaded.



Probs. 4–20/4–21

Given:-

$E = 29 \times 10^3$ ksi

$A_{AB} = \frac{\pi}{4}(0.5)^2 = 0.1964 \text{ in}^2.$

$A_{CD} = \frac{\pi}{4}(0.3)^2 = 0.0707 \text{ in}^2.$

$\sigma_{all} = 16.2$ ksi.

Required:-

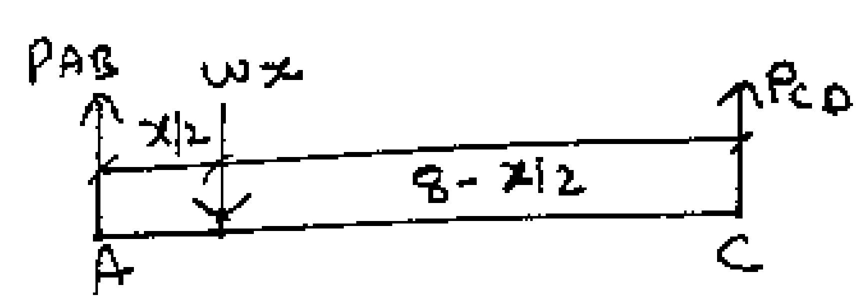
$w = ?$ & $x = ?$

INTERNAL FORCES:-

$\circlearrowleft + \Sigma M_A = 0$

$$-wx \cdot \frac{x}{2} + 8P_{CD} = 0$$

$$\Rightarrow P_{CD} = \frac{wx^2}{16}.$$



$+\uparrow \Sigma F_y = 0$

$$P_{AB} - wx + P_{CD} = 0$$

$$\Rightarrow P_{AB} = wx - P_{CD} = wx - \frac{wx^2}{16}$$

$$\Rightarrow P_{AB} = w\left(x - \frac{x^2}{16}\right).$$

P-7.0

Displacements:-

$\because \delta_{A/B} = \delta_{C/D}$.

$$\Rightarrow \frac{P_{AB} \cdot L_{AB}}{A_{AB} \cdot E} = \frac{P_{CD}\, L_{CD}}{A_{CD} \cdot E}$$

$$\Rightarrow \frac{\cancel{w}\left(x - \frac{x^2}{16}\right) \times 6 \cancel{\times 12}}{0.1964 \times 29 \cancel{\times 10^3}} = \frac{\frac{\cancel{w} x^2}{16} \times 6 \cancel{\times 12}}{0.0707 \times 29 \cancel{\times 10^3}}$$

$$x - \frac{x^2}{16} = \frac{2.78\, x^2}{16}$$

$$\Rightarrow \cancel{x} = \frac{3.78\, x^{\cancel{2}}}{16}$$

$$\Rightarrow x = \frac{16}{3.78} = 4.235 \text{ ft}$$

$$\therefore \underline{x = 4.235 \text{ ft}}$$

Now $P_{AB} = w\left(4.235 - \frac{4.235^2}{16}\right) = 3.114\, w$ kip

& $P_{CD} = w\left(\frac{4.235^2}{16}\right) = 1.121\, w$ kip.

$$\therefore \sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{3.114\, w}{0.19364} = 16.2$$

$$\Rightarrow w = 1.021 \text{ kip/ft}$$

$$\sigma_{CD} = \frac{P_{CD}}{A_{CD}} = \frac{1.121\, w}{0.0707} = 16.2$$

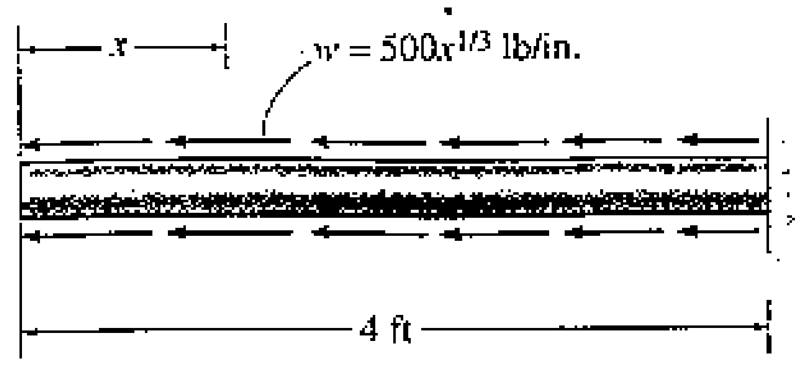
$$\Rightarrow w = 1.021 \text{ kip/ft}$$

$$\therefore w = 1.02 \text{ kip/ft}$$

PROBLEM # 3 (4.31) :-

SOLUTION :-

4–31. The bar has a cross-sectional area of $A = 3\ \text{in}^2$, and $E = 35(10^3)$ ksi. Determine the displacement of its end A when it is subjected to the distributed loading.



Prob. 4–31

GIVEN :-

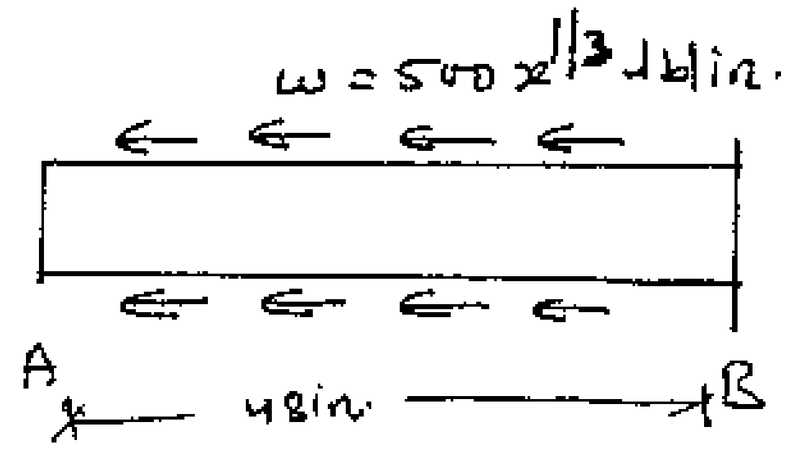
$A = 3\ in^2.$

$E = 35 \times 10^6$ psi.

Required :- δ_A.

$P_A = wx$. (At any section).

& $\delta = \int \frac{P\,dx}{AE}$



$\therefore\ \delta_A = \int_0^{48} \frac{P_A\,dx}{AE}$

$= \int_0^{48} \frac{wx\,.dx}{AE}$

$= \int_0^{48} \frac{500 \times x^{1/3} \times x\ dx}{AE}$

$= \frac{500}{AE} \int_0^{48} x^{4/3}\,dx$

$= \frac{500}{AE} \left[\frac{x^{7/3}}{7/3} \right]_0^{48}$

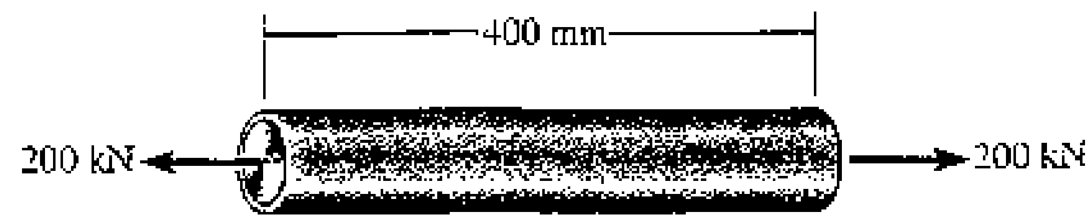
$= \frac{500 \times (48)^{7/3} \times 3}{3 \times 35 \times 10^6 \times 7}$

$\therefore\ \delta_A = 0.017$ in.

PROBLEM # 4 (4-39).

Solution:-

4–39. The A-36 steel pipe has a 6061-T6 aluminum core. It is subjected to a tensile force of 200 kN. Determine the average normal stress in the aluminum and the steel due to this loading. The pipe has an outer diameter of 80 mm and an inner diameter of 70 mm.



Prob. 4–39

Given:-

$E_{st} = 200 \times 10^9\,Pa$

$E_{Al} = 68.9 \times 10^9\,Pa$

Internal forces:-

$\xrightarrow{+} \Sigma F_x = 0$

$-200 + F_{st} + F_{AL} = 0 \rightarrow ①$

Displacement compatibility:-

$\delta_{st} = \delta_{al}$.

$\therefore \dfrac{F_{st} \cdot L_{st}}{A_{st} \cdot E_{st}} = \dfrac{F_{AL} \cdot L}{A_{AL} \cdot E_{AL}}$

$\Rightarrow F_{st} = F_{AL} \left(\dfrac{A_{st}}{A_{AL}} \right) \left(\dfrac{E_{st}}{E_{AL}} \right)$

$= F_{AL} \left[\dfrac{\pi (40^2 - 35^2)}{\pi \times 35^2} \right] \left[\dfrac{200 \times 10^9}{68.9 \times 10^9} \right]$

$\Rightarrow F_{st} = 0.89\, F_{AL} \rightarrow ②$

By solving ① & ② we get

$F_{AL} = 105.82\,kN$

& $F_{st} = 94.18\,kN$.

$\therefore \sigma_{st} = \dfrac{F_{st}}{A_{st}} = \dfrac{94.18 \times 10^3}{\pi (40^2 - 35^2)} = 79.95\,MPa$.

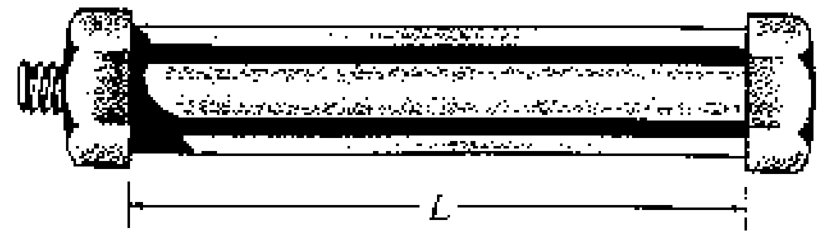
& $\sigma_{Al} = \dfrac{F_{AL}}{A_{AL}} = \dfrac{105.82 \times 10^3}{\pi \times 35^2} = 27.5\,MPa$.

$\therefore \sigma_{st} = 79.95\,MPa$

& $\sigma_{AL} = 27.5\,MPa$.

PROBLEM #5(4-55)

4–55. The assembly consists of an A-36 steel bolt and a C83400 red brass tube. The nut is drawn up snug against the tube so that $L = 75$ mm. Determine the maximum additional amount of advance of the nut on the bolt so that none of the material will yield. The bolt has a diameter of 7 mm and the tube has a cross-sectional area of 100 mm².



Probs. 4–54/4–55

SOLUTION:-

Given:-

$E_b = 200$ GPa
$E_t = 101$ GPa
$\sigma_b = 250$ MPa
$\sigma_t = 70$ MPa

From table. (Text book last page)

$A_b = \frac{\pi}{4}(7)^2 = 38.5\,mm^2$

$A_t = 100\,mm^2$

Required:-

a = ? (amount of advances of nut on bolt).

Equilibrium:-

$\xrightarrow{+} \Sigma F_x = 0$

$F_b - F_t = 0$

$\therefore F_b = F_t$.

∵ it is given that none of the material will yield, and the yield strength of tube is less than bolt.

∴ we assume that tube will yield first.

$\therefore F_b = F_t = \sigma_t \times A_t = F$.

$\Rightarrow F = \sigma_t \times A_t = 70 \times 100 = 7000\,N = 7\,KN$

$\therefore F = 7\,KN$

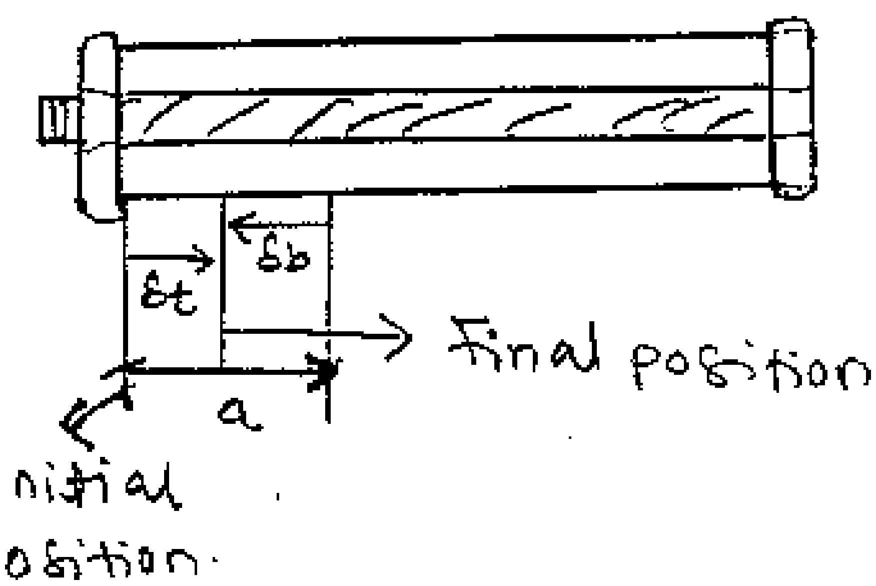
P.T.O.

Compatibility:-

When nut is tightened on the bolt, the tube will shorten δt and the bolt will elongate δb.

And as the nut is tightened on the bolt it will advances a distance 'a' as shown in figure below.

Now



$$\delta t = a - \delta b.$$

$$\therefore a = \delta t + \delta b$$

$$\Rightarrow a = \frac{F_t \times L_t}{A_t \times E_t} + \frac{F_b \times L_b}{A_b \times E_b}$$

$$= \frac{7 \times 75}{100 \times 101 \times 10^3} + \frac{7 \times 75}{38.5 \times 200 \times 10^3}$$

$$\Rightarrow a = \frac{7 \times 10^3 \times 75}{100 \times 101 \times 10^3} + \frac{7 \times 10^3 \times 75}{38.5 \times 200 \times 10^3}$$

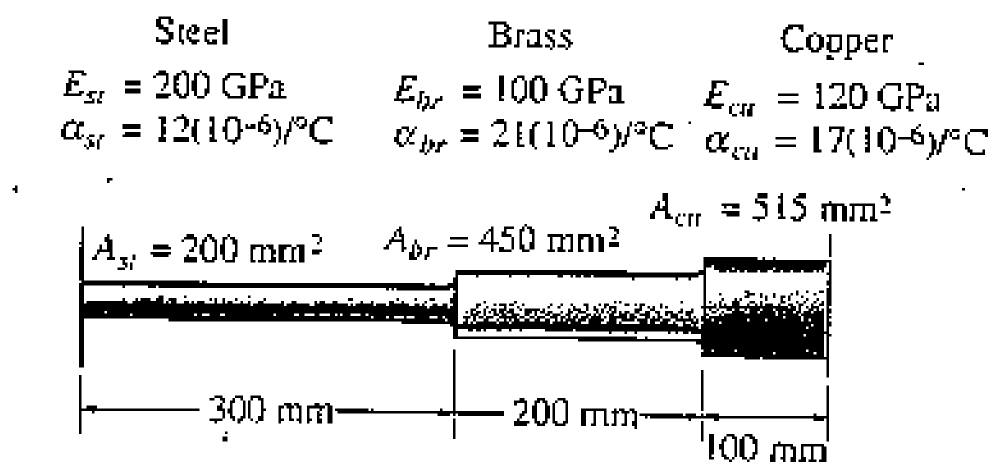
$$\Rightarrow a = \underline{\underline{0.12}}\ mm$$

$$\therefore a = \underline{\underline{0.120}}\ mm$$

PROBLEM # 6 (4-74)!-

Solution!-

4–74. Three bars each made of different materials are connected together and placed between two walls when the temperature is $T_1 = 12°C$. Determine the force exerted on the (rigid) supports when the temperature becomes $T_2 = 18°C$. The material properties and cross-sectional area of each bar are given in the figure.



Prob. 4–74

Given! -

$T_1 = 12°C$

$T_2 = 18°C$

Required!-

$F = ?$

Equilibrium!-

$\xrightarrow{+} \Sigma F_x = 0$

$F_A = F_B$.

Compatibility!-

$\delta = 0$

& $\Delta T = T_2 - T_1$

$\Rightarrow \Delta T = 18 - 12 = 6°C$.

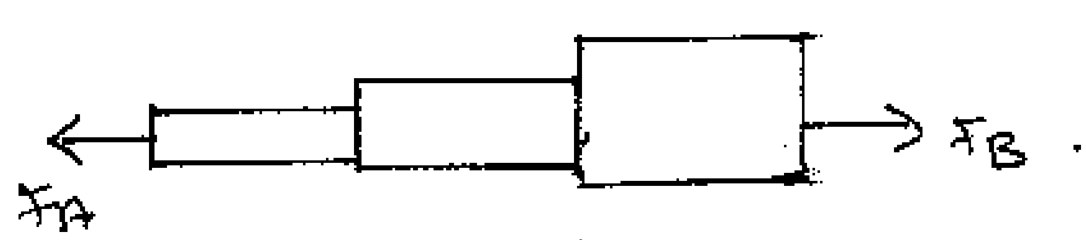


Fig. ①

$\therefore \quad (\delta_{st})_T + (\delta_{br})_T + (\delta_{cu})_T + (\delta_{st})_F + (\delta_{br})_F + (\delta_{cu})_F = 0$.

$$\Rightarrow \alpha_{st}\,\Delta T\,L_{st} + \alpha_{br}\,\Delta T\,L_{br} + \alpha_{cu}\,\Delta T\,L_{cu} = -\left(\frac{F_{st} \cdot L_{st}}{A_{st} \cdot E_{st}} + \frac{F_{br}\,L_{br}}{A_{br} \cdot E_{br}} + \frac{F_{cu} \cdot L_{cu}}{A_{cu} \cdot E_{cu}}\right).$$

$$\Rightarrow (12 \times 10^{-6})(6)(300) + (21 \times 10^{-6})(6)(200) + (17 \times 10^{-6})(6)(100) = -\left(\frac{F \times 300}{200 \times 200 \times 10^3} + \frac{F \times 200}{450 \times 100 \times 10^3} + \frac{F \times 100}{515 \times 120 \times 10^3}\right)$$

$\Rightarrow \quad 0.057 = -1.356 \times 10^{-5} F$

$\Rightarrow F = -4202\ N$

$\Rightarrow F = -4.2\ kN$.

$\therefore F = \underline{4.2\ kN\ (Comp)}$

PROBLEM # 7 (4-85):-

4–85. The two circular rod segments, one of aluminum and the other of copper, are fixed to the rigid walls such that there is a gap of 0.008 in. between them when $T_1 = 60°F$. What larger temperature T_2 is required in order to just close the gap? Each rod has a diameter of 1.25 in., $\alpha_{al} = 13(10^{-6})/°F$, $E_{al} = 10(10^3)$ ksi, $\alpha_{cu} = 9.4(10^{-6})/°F$, $E_{cu} = 18(10^3)$ ksi. Determine the average normal stress in each rod if $T_2 = 200°F$.

Probs. 4–85/4–86

Solution:-

Given:-

$T_1 = 60°F$.

$A = \frac{\pi}{4}(1.25) = 1.227 \, in^2$.

$\delta_{al} = 13 \times 10^{-6} / °F$; $E_{al} = 16 \times 10^3$ Ksi.

$\delta_{cu} = 9.4 \times 10^{-6} / °F$; $E_{cu} = 18 \times 10^3$ Ksi.

compatibility:-

$\delta_{al} + \delta_{cu} = 0.008$

$\Rightarrow \delta_{AL} \cdot \Delta T \cdot L_{AL} + \alpha_{CU} \cdot \Delta T \cdot L_{CU} = 0.008$

$$\Delta T = \frac{0.008}{13 \times 10^{-6} \times 8 + 9.4 \times 10^{-6} \times 4}$$

$\Rightarrow \Delta T = 56.5°F$

$\therefore T_2 = T_1 + \Delta T = 60 + 56.5 = 116.5°F$

$\therefore T_2 = 116.5°F$.

Now finding

$\sigma_{AL} = ?$ & $\sigma_{CU} = ?$

Equilibrium:-

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$F_{AL} = F_{CU}$. —(1)

compatibility:-

$$(\delta_{cu})_T - (\delta_{cu})_F = -(\delta_{AL})_T + (\delta_{AL})_F$$

$$\Rightarrow \alpha_{CU} \cdot \Delta T \cdot L_{CU} - \frac{F_{CU} \cdot L_{CU}}{A_{CU} \cdot E_{CU}} = -\alpha_{AL} \cdot \Delta T \cdot L_{AL} + \frac{F_{AL} \cdot L_{AL}}{A_{AL} \cdot E_{AL}}$$

$$9.4\times10^{-6}\times(200-116.5)\times4-\frac{F_{cu}\times4}{1.227\times18\times10^{3}}=$$

$$-13\times10^{-6}\times(200-116.5)\times8+\frac{F_{AL}\times8}{1.227\times10\times10^{3}}.$$

$$3.1396\times10^{-3}-1.812\times10^{-4}F_{cu}=-8.684\times10^{-3}+6.52\times10^{-4}F_{AL}.$$

$$6.52\times10^{-4}F_{AL}+1.812\times10^{-4}F_{cu}-0.012=0 \quad -②$$

By solving ① & ② we get.

$$F_{AL}=F_{cu}=14.2\ \text{kip}.$$

$$\therefore\ \sigma_{AL}=\sigma_{cu}=\frac{F_{AL}\ (\text{or})\ F_{cu}}{A_{AL}\ (\text{or})\ A_{cu}}=\frac{14.2}{1.227}$$

$$\Rightarrow \sigma_{AL}=\sigma_{cu}=11.6\ \text{ksi}$$