Structural Mechanics I

KEY OF HOME WORK : 4

by

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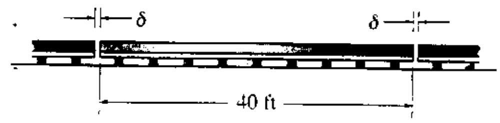
KEY TO HOMEWORK #4.

PROBLEM # ① (4-77).:-

SOLUTION:-

GIVEN:-

4-77. The 40-ft-long A-36 steel rails on a train track are laid with a small gap between them to allow for thermal expansion. Determine the required gap δ so that the rails just touch one another when the temperature is increased from $T_1 = -20°F$ to $T_2 = 90°F$. Using this gap, what would be the axial force in the rails if the temperature were to rise to $T_3 = 110°F$? The cross-sectional area of each rail is 5.10 in^2.



Prob. 4-77

$T_1 = -20° F$

$T_2 = 90° F$

$A = 5.10 \text{ in}^2$

$L = 40 \text{ ft}.$

$\alpha_{st} = 6.6 \times 10^{-6} / °F.$

$E_{st} = 29 \times 10^3 \text{ ksi}.$

Required:-

$\delta = ?$

$$\delta = \alpha_{st} \times \Delta T \times L_{st}.$$

$$\Rightarrow \delta = 6.6 \times 10^{-6} \times [90 - (-20)] \times 40 \times 12$$

$$\Rightarrow \delta = 0.348 \text{ in}.$$

now finding

$F_{st} = ?$ if $T_3 = 110° F.$

compatibility:-

$$\delta = \delta_T - \delta_F$$

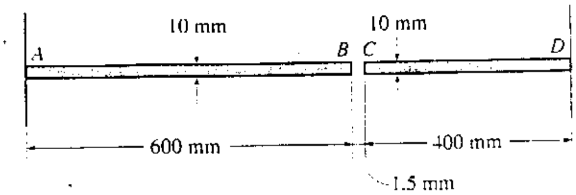
$$\Rightarrow \alpha_{st} \cdot \Delta T \cdot L_{st} - \frac{F_{st} \cdot L_{st}}{A_{st} \cdot E_{st}} = 0.348$$

$$\Rightarrow 6.6 \times 10^{-6} \times [110 - (-20)] \times 40 \times 12 - \frac{F_{st} \times 40 \times 12}{5.1 \times 29 \times 10^3} = 0.348$$

$$\Rightarrow F_{st} = 19.6 \text{ kip}.$$

PROBLEM # 2 (4-80):-

*4–80.** A thermo gate consists of a 6061-T6-aluminum plate AB and an Am-1004-T61-magnesium plate CD, each having a width of 15 mm and fixed supported at their ends. If the gap between them is 1.5 mm when the temperature is $T_1 = 25°C$, determine the temperature required to just close the gap. Also, what is the axial force in each plate if the temperature becomes $T_2 = 100°C$? Assume bending or buckling will not occur.



Prob. 4–80

Solution:-

Given:-

$T_1 = 25°C.$

$A = 10 \times 15 = 150\ mm^2$

$\alpha_{al} = 24 \times 10^{-6} / °C$

$E_{al} = 68.9 \times 10^3\ Mpa$

$\alpha_{am} = 26 \times 10^{-6} | °C$

$E_{am} = 44.7 \times 10^3\ Mpa.$

Compatibility:-

$\delta_{al} + \delta_{am} = 1.5\ mm.$

$\Rightarrow \alpha_{al} \cdot \Delta T \cdot L_{al} + \alpha_{am} \cdot \Delta T \cdot L_{am} = 1.5$

$\Rightarrow \Delta T = \dfrac{1.5}{(\alpha_{al} \cdot L_{al} + \alpha_{am} \cdot L_{am})}.$

$\Rightarrow \Delta T = \dfrac{1.5}{(24 \times 10^{-6} \times 600 + 26 \times 10^{-6} \times 400)} = 60.5°C$

$\Rightarrow \Delta T = 60.5°C$

$\Rightarrow T_2 = T_1 + \Delta T = 25 + 60.5$

$\Rightarrow T_2 = \underline{85.5°C}$

Now finding.

$F_{al} = ?$ & $F_{am} = ?$ if $T_2 = 100°C.$

Equilibrium:-

P.T.O.

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$F_{AL} = F_{AM}. \quad \text{---} (1)$

Compatability:-

$$(\delta_{al})_T - (\delta_{al})_F = -(\delta_{am})_T + (\delta_{am})_F$$

$$\Rightarrow \alpha_{al} \cdot \Delta T \cdot L_{al} - \frac{F_{al} \times L_{al}}{A_{al} \times E_{al}} = -\alpha_{am} \cdot \Delta T \cdot L_{am} + \frac{F_{am} \cdot L_{am}}{A_{am} \cdot E_{a}}$$

$$\Rightarrow 24 \times 10^{-6} \times (100 - 85.5) \times 600 - \frac{F_{al} \times 600}{150 \times 68.9 \times 10^{3}} =$$

$$-26 \times 10^{-6} (100 - 85.5) \times 400 + \frac{F_{am} \times 400}{150 \times 44.7 \times 10^{3}}$$

$$0.209 - 5.806 \times 10^{-5} F_{al} = -0.1508 + 5.966 \times 10^{-5} F_{a}$$

$\quad$ ---- (2)

solving (1) & (2) we get.

$$5.966 \times 10^{-5} F_{am} + 5.806 \times 10^{-5} F_{am} = 0.209 + 0.1508$$

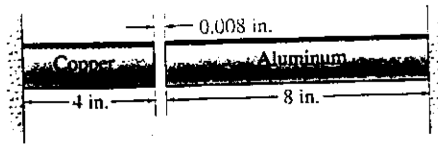
$$\Rightarrow F_{am} = 3056\,N.$$

$$\therefore F_{am} = 3.05\,kN.$$

$$\Rightarrow F_{am} = F_{al} = 3.05\,kN.$$

PROBLEM # 3 (4-85):-

4–85. The two circular rod segments, one of aluminum and the other of copper, are fixed to the rigid walls such that there is a gap of 0.008 in. between them when $T_1 = 60°F$. What larger temperature T_2 is required in order to just close the gap? Each rod has a diameter of 1.25 in., $\alpha_{al} = 13(10^{-6})/°F$, $E_{al} = 10(10^3)$ ksi, $\alpha_{cu} = 9.4(10^{-6})/°F$, $E_{cu} = 18(10^3)$ ksi. Determine the average normal stress in each rod if $T_2 = 200°F$.



Probs. 4–85/4–86

Solution:-

Given:-

$T_1 = 60°F$.

$A = \frac{\pi}{4}(1.25) = 1.227\ in^2$.

$\alpha_{al} = 13\times10^{-6}\ /°F$; $E_{al} = 10\times10^3$ Ksi.

$\alpha_{cu} = 9.4\times10^{-6}\ /°F$; $E_{cu} = 18\times10^3$ Ksi.

compatibility:-

$\delta_{al} + \delta_{cu} = 0.008$

$\Rightarrow \alpha_{al}\cdot\Delta T\cdot L_{AL} + \alpha_{cu}\cdot\Delta T\cdot L_{cu} = 0.008$

$$\Delta T = \frac{0.008}{13\times10^{-6}\times8 + 9.4\times10^{-6}\times4}$$

$\Rightarrow \Delta T = 56.5°F$

$\therefore T_2 = T_1 + \Delta T = 60 + 56.5 = 116.5°F$

$\therefore T_2 = 116.5°F$.

Now finding

$\sigma_{AL} = ?$ & $\sigma_{cu} = ?$

Equilibrium:-

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$F_{AL} = F_{cu}$. —①

Compatibility:-

$(\delta_{cu})_T - (\delta_{cu})_F = -(\delta_{AL})_T + (\delta_{AL})_F$

$$\Rightarrow \alpha_{cu}\cdot\Delta T\cdot L_{cu} - \frac{F_{cu}\cdot L_{cu}}{A_{cu}\cdot E_{cu}} = -\alpha_{AL}\cdot\Delta T\cdot L_{AL} + \frac{F_{AL}\cdot L_{AL}}{A_{AL}\cdot E_{AL}}$$

$$9.4\times10^{-6}\times(200-116.5)\times4-\frac{F_{cu}\times4}{1.227\times19\times10^{3}}=$$

$$-13\times10^{-6}\times(200-116.5)\times8+\frac{F_{AL}\times8}{1.227\times10\times10^{3}}.$$

$$3.1396\times10^{-3}-1.812\times10^{-4}F_{cu}=-8.684\times10^{-3}+6.52\times10^{-4}F_{AL}.$$

$$6.52\times10^{-4}F_{AL}+1.812\times10^{-4}F_{cu}-0.012=0 \quad -②$$

By solving ① & ② we get.

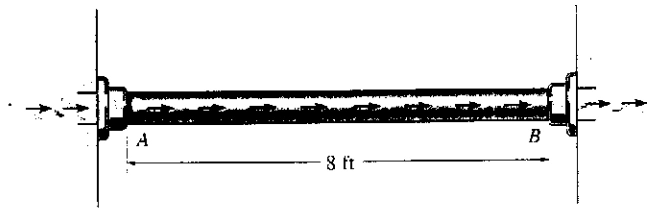
$F_{AL}=F_{cu}=14.2$ kip.

$$\therefore\ \sigma_{AL}=\sigma_{cu}=\frac{F_{AL}\ (\text{or})\ F_{cu}}{A_{AL}\ (\text{or})\ A_{cu}}=\frac{14.2}{1.227}$$

$$\Rightarrow \sigma_{AL}=\sigma_{cu}=11.6\ \text{ksi}$$

PROBLEM # 4 (4-87):-

4–87. The pipe is made of A-36 steel and is connected to the collars at A and B. When the temperature is 60°F, there is no axial load in the pipe. If hot gas traveling through the pipe causes its temperature to rise by $\Delta T = (40 + 15x)$°F, where x is in feet, determine the average normal stress in the pipe. The inner diameter is 2 in., the wall thickness is 0.15 in.



Probs. 4–87/4–88

SOLUTION:-

Given:-

A-36 Steel.

$E = 29 \times 10^3$ ksi.

$\alpha = 6.6 \times 10^{-6} / °F$.

$\Delta T = (40 + 15x)°F$.

$x \to$ in feet.

$\because$ Change in length $= \delta_T = \int_0^L \alpha \, \Delta T \, dx$

$\therefore \delta_T = \alpha \int_0^8 (40 + 15x) \, dx$

$= 6.6 \times 10^{-6} \left[40x + \frac{15x^2}{2} \right]_0^8$

$= 6.6 \times 10^{-6} \left[40 \times 8 + \frac{15 \times 8^2}{2} \right]$

$\Rightarrow \delta_T = 5.28 \times 10^{-3}$ ft.

$\because$ Average Normal Stress $= \sigma_{avg} = \frac{\delta_T \cdot E}{L}$

$\Rightarrow \sigma_{avg} = \frac{5.28 \times 10^{-3} \times 29 \times 10^3 \times 12}{8 \times 12}$

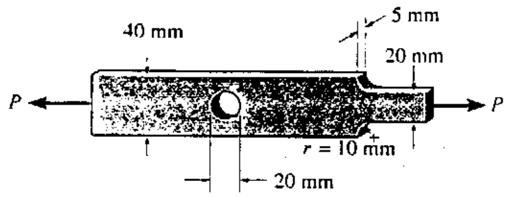
$= 19.14$ Ksi

$\Rightarrow \sigma_{avg} = 19.14$ Ksi.

PROBLEM # 5 (4-95) :-

SOLUTION :-

4–95. If the allowable normal stress for the bar is $\sigma_{allow} = 120$ MPa, determine the maximum axial force P that can be applied to the bar.



Prob. 4–95

Given :-

$\sigma_{all} = 120 \text{ MPa}$.

Required :-

$P_{max} = ?$

First considering the circular hole :-

Finding stress concentration factor 'k' from Figure 4.25 (text; page 159).

$\because r = \frac{20}{2} = 10\text{mm}$; $w = 40\text{mm}$.

$\therefore \frac{r}{w} = \frac{10}{40} = 0.25$

$\Rightarrow K = 2.375$ (from Fig. 4.25).

$\& \; K = \frac{\sigma_{max}}{\sigma_{avg}} \Rightarrow \sigma_{avg} = \frac{\sigma_{max}}{k} = \frac{120}{2.375} = 50.5$

$\Rightarrow \sigma_{avg} = \frac{P}{(w-2r)t} = 50.5 \text{ MPa}$

$\Rightarrow P = (w-2r)t \times 50.5 = (40-20) \times 5 \times 50.5$

$\Rightarrow P = \underline{5.05 \text{ KN}}$.

Now considering the fillet :-

Finding 'k' from Figure 4.24 (text; page 159)

$\because r = 10\text{mm}$; $h = 20\text{mm}$; $w = 40\text{mm}$.

$\therefore \frac{r}{h} = \frac{10}{20} = 0.5$; $\frac{w}{h} = \frac{40}{20} = 2.0$.

P.T.O.

$\therefore K = 1.4$ [from fig. 4.24).

& $K = \dfrac{\sigma_{max}}{\sigma_{avg}}$.

$\Rightarrow \sigma_{avg} = \dfrac{\sigma_{max}}{K} = \dfrac{120}{1.4} = 85.71\,Mpa$.

& $\sigma_{avg} = \dfrac{P}{ht} = 85.71\,Mpa$

$\Rightarrow P = ht \times 85.71$

$\Rightarrow P = 20 \times 5 \times 85.71$

$\Rightarrow P = 8.57\,KN$.

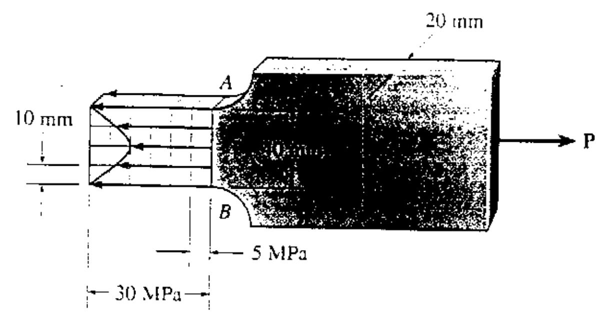
$\therefore$ Pmax is lowest value of above two P's.

$\therefore P_{max} = 5.05\,KN$

PROBLEM # 6 (4-100) :-

Solution :-

*4–100. The resulting stress distribution along the section AB of the bar is shown in the figure. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress-concentration factor for this geometry?



Prob. 4–100

$\because$

$P = \int \sigma \, dA$.

$\Rightarrow P =$ volume of the stress distribution diagram

Assuming distribution to be triangular.

$\Rightarrow P = [$whole Rectangular volume − triangular volume$]$

$\Rightarrow P = [30 \times 40 \times 20 - \frac{1}{2} \times 40 \times 10 \times 20]$

$\Rightarrow$ $P = 20$ KN.

$\because K =$ Stress concentration factor $= \dfrac{\sigma_{max}}{\sigma_{avg}}$.

$\therefore \sigma_{avg} = \dfrac{P}{ht} = \dfrac{20 \times 10^3}{40 \times 20}$

$\Rightarrow \sigma_{avg} = 25$ Mpa

& $\sigma_{max} = 30$ Mpa.

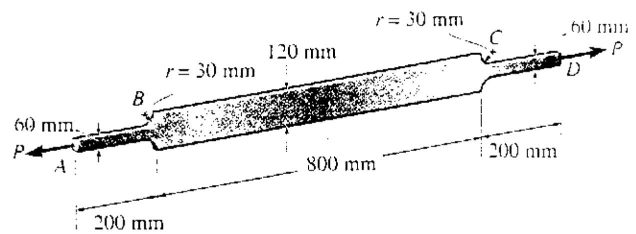
$\Rightarrow K = \dfrac{\sigma_{max}}{\sigma_{avg}} = \dfrac{30}{25} = 1.2$

$\Rightarrow K = 1.2$

PROBLEM # 7 (4-101):-

Solution:-

4–101. The A-36 steel plate has a thickness of 12 mm. If there are shoulder fillets at B and C, and $\sigma_{allow} = 150$ MPa, determine the maximum axial load P that it can support. Compute its elongation neglecting the effect of the fillets.



Prob. 4–101

Given:-

A-36 Steel

$E = 200 \times 10^3$ MPa.

$t = 12$ mm.

$w = 120$ mm.

$h = 60$ mm.

$r = 30$ mm.

$\sigma_{all} = 150$ Mpa.

Required:-

$P = ?$

$\delta_{A/D} = ?$

Computing Maximum axial Load 'P':-

First determining stress concentration factor 'K' from Figure 4-24 (text; page# 159).

$$\frac{w}{h} = \frac{120}{60} = 2 \quad \& \quad \frac{r}{h} = \frac{30}{60} = 0.5$$

$$\therefore K = 1.4.$$

$$\because \sigma_{avg} = \frac{P}{ht} = \frac{P}{60 \times 12} = \frac{P}{720}$$

and $K = \frac{\sigma_{max}}{\sigma_{avg}} \Rightarrow \sigma_{avg} = \frac{\sigma_{max}}{K}$

$$\therefore \frac{P}{720} = \frac{150}{1.4}$$

$$\Rightarrow P = \underline{77.14 \text{ KN}}$$

P.T.O.

Computing Elongation:-

$$\delta_{A/D} = \Sigma \frac{PL}{AE} = \left(\frac{PL}{AE}\right)_{A/B} + \left(\frac{PL}{AE}\right)_{B/C} + \left(\frac{PL}{AE}\right)_{C/D}.$$

$$= \left\{\frac{77.14 \times 10^{3} \times 200}{(60\times12)(200\times10^{3})}\right\} + \left\{\frac{77.14\times10^{3}\times 800}{(120\times12)(200\times10^{3})}\right\} +$$

$$\left\{\frac{77.14\times10^{3}\times 200}{(60\times12)(200\times10^{3})}\right\}$$

$$\Rightarrow \delta_{A/D} = 0.4286 \text{ mm}$$

$$\Rightarrow \delta_{A/D} = \underline{\underline{0.429 \text{ mm}}}.$$