

HW

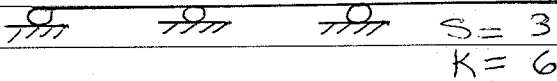
Solution

# CE 511

## HW # 1

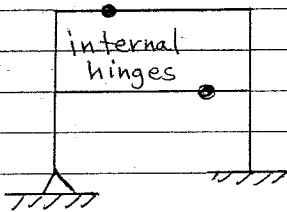
Determine the degree of static indeterminacy and the degree of kinematic indeterminacy for the structures shown:

①



②

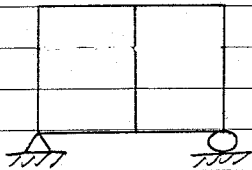
$S = 3$   
 $K = 21$



Plane  
Frame

③

$S = 6$   
 $K = 15$



Plane  
Frame

④

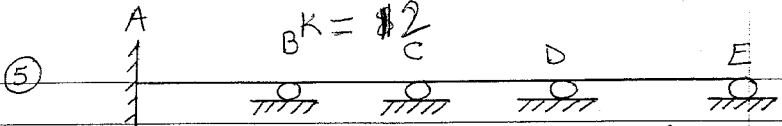


beam on elastic foundations

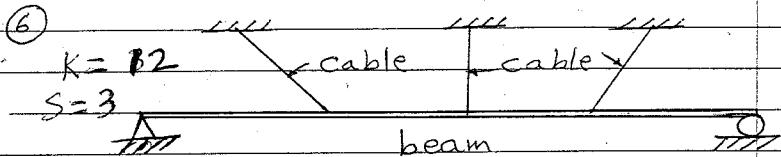
$S = 4$   
 $K = 12$

$$S = 4$$

$$K = \frac{2}{C}$$



beam with settlements at B, C, D and E.

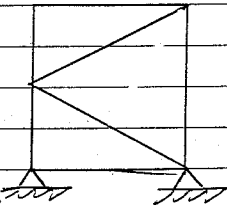


⑦  $S = 1$

$$K = 20$$

$$K = 6$$

neglecting rotations

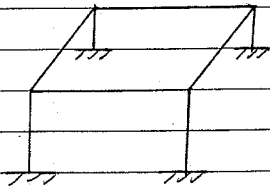


Plane  
Truss

⑧

$$S = 24$$

$$K = 24$$



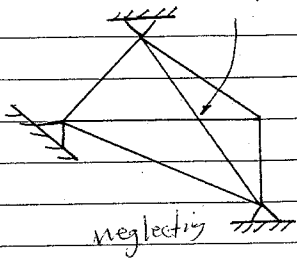
space  
frame

9

$$S = 3$$

$$K = 39$$

$$K = 3$$



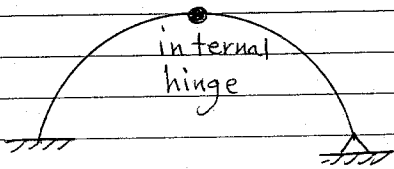
not a joint

Space  
Truss

10

$$S = 1$$

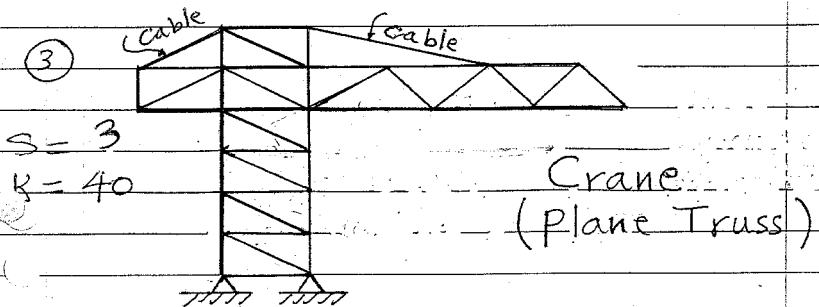
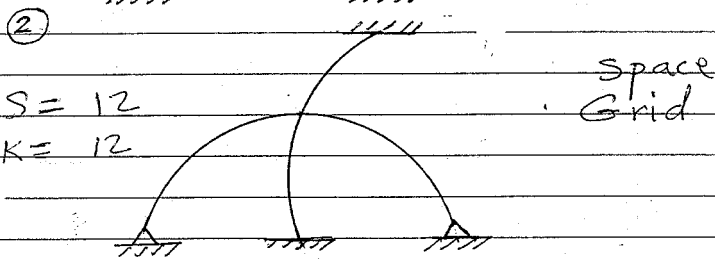
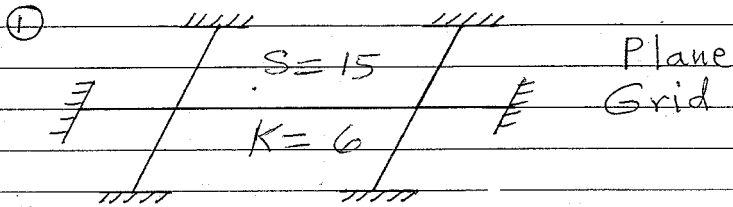
$$K = 5$$



arch

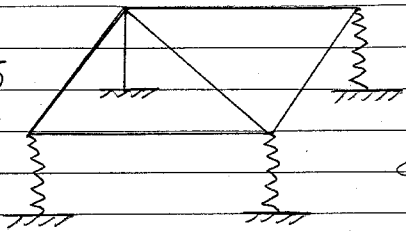
## HW #2

State the degree of static indeterminacy and the degree of kinematic indeterminacy for the structures shown:



④

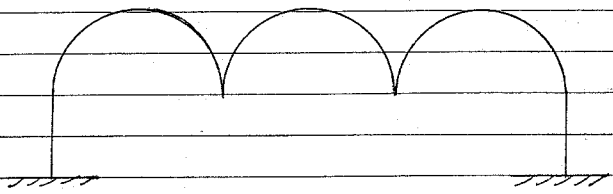
$S = 15$   
 $K = 24$



Space Frame  
on Elastic  
Foundations

⑤

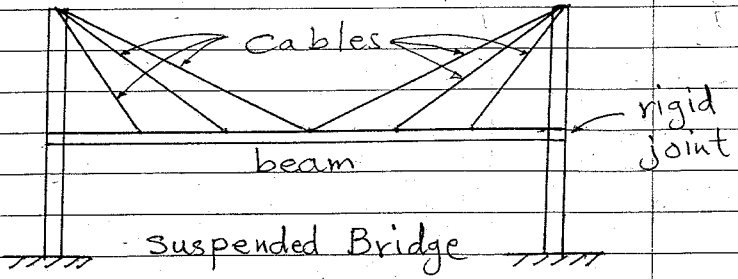
$S = 3$   
 $K = 6$



Plane Arch

⑥

$S = 9$   
 $K = 27$

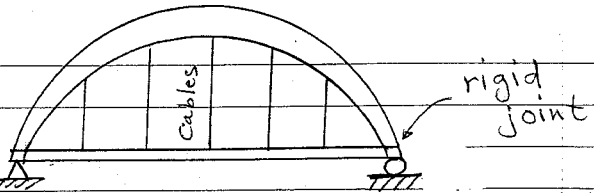


Suspended Bridge

⑦

$$S = 8$$

$$K = 18$$

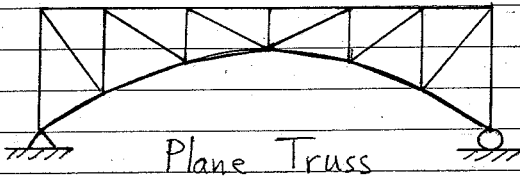


Suspended Bridge

⑧

$$S = 0$$

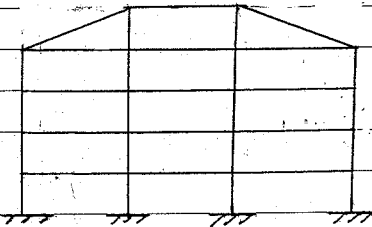
$$K = 25$$



⑨

$$S = 45$$

$$K = 54$$

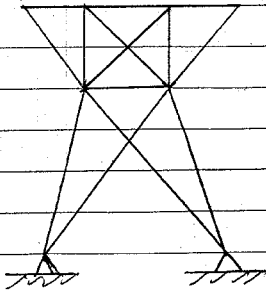


Planar  
Frame

⑩

$$S = 2$$

$$K = 16$$



Power  
Transmission  
Tower  
(Plane Truss)

# HW # 3

$$\delta_{11} = \frac{7751.11}{EI} = \delta_{22}$$

$$\delta_{21} = \frac{7360}{EI} = \delta_{12}$$

$$\Delta_1 = -\frac{1.67 \times 10^6}{EI} \text{ ft}$$

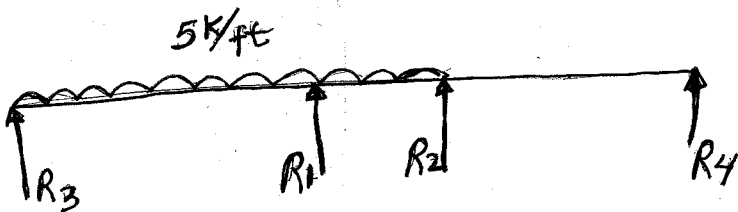
$$\Delta_2 = -1.596 \times 10^6 \text{ ft}$$

$$R_3 = 82.8 \text{ K}$$

$$R_1 = 203.5$$

$$R_2 = 12.7 \text{ K}$$

$$R_4 = 1 \text{ K}$$





# HW # 4

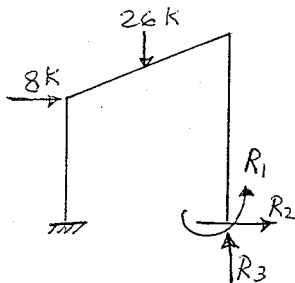
## Solution

inverse =

1.1319	-0.0500	0.1845	0.1440	-3.2159	-0.7610	-1.0240
-0.0500	1.4809	-0.0748	-0.1443	-5.0583	0.6191	0.9300
0.1845	-0.0748	1.0364	0.5028	-6.3158	-3.8223	-2.3463
0.1440	-0.1443	0.5028	0.6871	-4.5322	-2.9593	-3.3195
-3.2159	-5.0583	-6.3158	-4.5322	111.6470	26.5507	28.0134
-0.7610	0.6191	-3.8223	-2.9593	26.5507	18.5859	14.0864
-1.0240	0.9300	-2.3463	-3.3195	28.0134	14.0864	24.2645

# HW #5

## Solution



$$\{D_L\} + [d_{xx}]\{X\} = \{D_x^o\}$$

$$\{D_L\} = \begin{bmatrix} \delta_{1L} \\ \delta_{2L} \\ \delta_{3L} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} -280,507 \\ -2,215,440 \\ -6,642,930 \end{bmatrix}$$

$$[d_{xx}] = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1,566.8 & 18,375.6 & 20,184.3 \\ 18,375.6 & 294,376.0 & 190,411.0 \\ 20,184.3 & 190,411.0 & 455,969.0 \end{bmatrix}$$

$$\{X\} = \begin{bmatrix} R_1^o \\ R_2^o \\ R_3^o \end{bmatrix}$$

$$\{D_x^o\} = \begin{bmatrix} \delta_{x1}^o \\ \delta_{x2}^o \\ \delta_{x3}^o \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{1}{E} \begin{bmatrix} -280,507 \\ -2,215,440 \\ -6,642,930 \end{bmatrix} + \frac{1}{E} \begin{bmatrix} 1,566.8 & 18,375.6 & 20,184.3 \\ 18,375.6 & 294,376.0 & 190,411.0 \\ 20,184.3 & 190,411.0 & 455,969.0 \end{bmatrix} \begin{bmatrix} R_1^o \\ R_2^o \\ R_3^o \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving,

$$\begin{bmatrix} R_1^o \\ R_2^o \\ R_3^o \end{bmatrix} = - \begin{bmatrix} 4.69802 \times 10^{-3} & -2.17488 \times 10^{-4} & -1.17144 \times 10^{-4} \\ -2.17488 \times 10^{-4} & 1.47225 \times 10^{-5} & 3.47946 \times 10^{-6} \\ -1.17144 \times 10^{-4} & 3.47946 \times 10^{-6} & 5.92574 \times 10^{-6} \end{bmatrix} \begin{bmatrix} -280,507 \\ -2,215,440 \\ -6,642,930 \end{bmatrix}$$

$$\begin{bmatrix} R_1^o \\ R_2^o \\ R_3^o \end{bmatrix} = \begin{bmatrix} 57.82 \text{ ft-k} \\ -5.28 \text{ k} \\ 14.21 \text{ k} \end{bmatrix}$$

# HW # 6

$$\Delta_1 = -0.026 \text{ ft}$$

$$\Delta_2 = -0.013 \text{ ft}$$

$$\Delta_3 = 0$$

$$\delta_{11} = 3243/EI$$

$$\delta_{12} = -8000/EI$$

$$\delta_{13} = -200/EI$$

$$\delta_{21} = -8000/EI$$

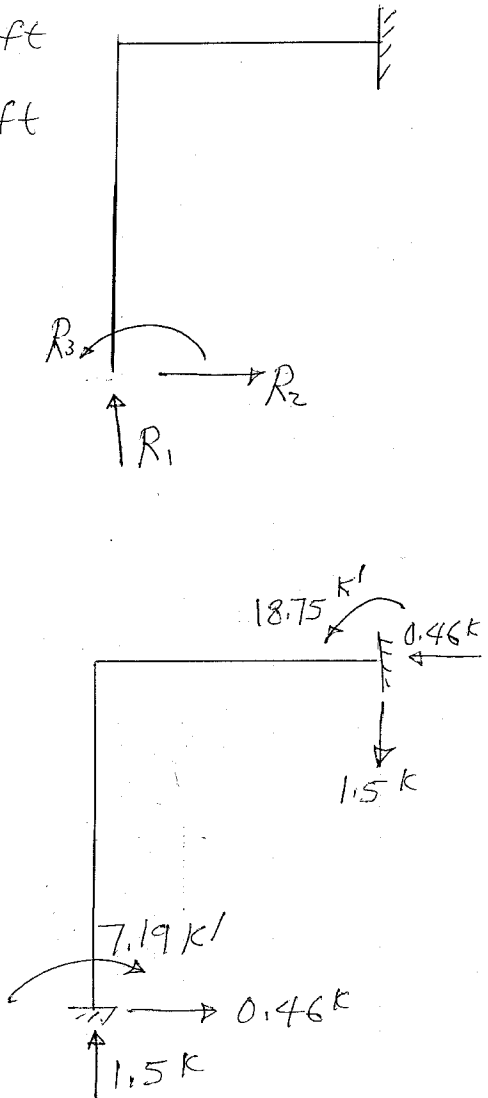
$$\delta_{22} = 53621.5/EI$$

$$\delta_{23} = 1600/EI$$

$$\delta_{31} = -200/EI$$

$$\delta_{32} = 1600/EI$$

$$\delta_{33} = 60/EI$$



PROGRAM TO FIND THE STRUCTURAL STIFFNESS MATRIX OF PLANAR ORTHOGONAL STRUCTURES

HW #8  
Solution

SMOIN = 1.00000 YOUNG = 1.00000 ELENG = 10.00000

MEMBER STIFFNESS MATRIX

.40000	.20000	.06000	-.06000
.20000	.40000	.06000	-.06000
.06000	.06000	.01200	-.01200
-.06000	-.06000	-.01200	.01200

MEMBER STIFFNESS MATRIX

.40000	.20000	.06000	-.06000
.20000	.40000	.06000	-.06000
.06000	.06000	.01200	-.01200
-.06000	-.06000	-.01200	.01200

MEMBER STIFFNESS MATRIX

.40000	.20000	.06000	-.06000
.20000	.40000	.06000	-.06000
.06000	.06000	.01200	-.01200
-.06000	-.06000	-.01200	.01200

MEMBER STIFFNESS MATRIX

.40000	.20000	.06000	-.06000
.20000	.40000	.06000	-.06000
.06000	.06000	.01200	-.01200
-.06000	-.06000	-.01200	.01200

MEMBER STIFFNESS MATRIX

.40000	.20000	.06000	-.06000
.20000	.40000	.06000	-.06000
.06000	.06000	.01200	-.01200
-.06000	-.06000	-.01200	.01200

MEMBER STIFFNESS MATRIX

.40000	.20000	.06000	-.06000
.20000	.40000	.06000	-.06000
.06000	.06000	.01200	-.01200
-.06000	-.06000	-.01200	.01200

CORRESPONDING DISPLACEMENT NUMBERS FOR ELEMENTS

1 14 5 11

CORRESPONDING DISPLACEMENT NUMBERS FOR ELEMENTS

1 2 10 9

CORRESPONDING DISPLACEMENT NUMBERS FOR ELEMENTS

2 15 5 12

CORRESPONDING DISPLACEMENT NUMBERS FOR ELEMENTS

2 3 9 8

CORRESPONDING DISPLACEMENT NUMBERS FOR ELEMENTS

3 6 5 13

CORRESPONDING DISPLACEMENT NUMBERS FOR ELEMENTS

3 4 8 7

STRUCTURE STIFFNESS MATRIX

.800	.200	.000	.000	.060	.000	.000	.000	.000	-.060	.060	-.060	.000	.000	.200	.000
.200	1.200	.200	.000	.060	.000	.000	-.060	.000	.060	.000	.000	-.060	.000	.000	.200
.000	.200	1.200	.200	.060	.200	-.060	.000	.060	.000	.000	.000	.000	-.060	.000	.000
.000	.000	.200	.400	.000	.000	-.060	.060	.000	.000	.000	.000	.000	.000	.000	.000
.060	.060	.060	.000	.036	.060	.000	.000	.000	.000	.000	-.012	-.012	-.012	.060	.060
.000	.000	.200	.200	.060	.400	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	-.060	-.060	.000	.000	.012	-.012	.000	.000	.000	.000	.000	.000	.000	.000
.000	-.060	.000	.060	.000	.000	-.012	.024	-.012	.000	.000	.000	.000	.000	.000	.000
-.060	.000	.060	.000	.000	.000	.000	-.012	.024	-.012	.000	.000	.000	.000	.000	.000
.060	.060	.000	.000	.000	.000	.000	.000	.000	.000	.012	.000	.000	.000	-.060	.000
-.060	.000	.000	.000	-.012	.000	.000	.000	.000	.000	.000	.000	.012	.000	.000	-.060
.000	-.060	.000	.000	-.012	.000	.000	.000	.000	.000	.000	.000	.000	.012	.000	.000
.000	.000	-.060	.000	-.012	-.060	.000	.000	.000	.000	.000	.000	.000	.000	.400	.000
.200	.000	.000	.000	.060	.000	.000	.000	.000	.000	.000	-.060	.000	.000	.400	.000
.000	.200	.000	.000	.060	.000	.000	.000	.000	.000	.000	.000	-.060	.000	.000	.400

THE STRUCTURAL STIFFNESS MATRIX

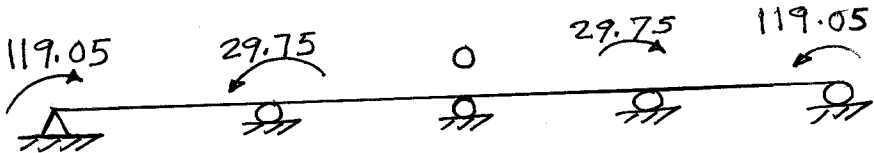
.800	.200	.000	.000	.060	.000	.000	.000	.000	-.060	.060	-.060	.000	.000	.200	.000
.200	1.200	.200	.000	.060	.000	.000	.000	-.060	.000	.060	.000	-.060	.000	.000	.200
.000	.200	1.200	.200	.060	.200	-.060	.000	.060	.000	.000	.000	.000	-.060	.000	.000
.000	.000	.200	.400	.000	.000	-.060	.060	.000	.000	.000	.000	.000	.000	.000	.000
.060	.060	.060	.000	.036	.060	.000	.000	.000	.000	.000	-.012	-.012	-.012	.060	.060
.000	.000	.200	.000	.060	.400	.000	.000	.000	.000	.000	.000	.000	-.060	.000	.000
.000	.000	-.060	-.060	.000	.000	.012	-.012	.000	.000	.000	.000	.000	.000	.000	.000
.000	-.060	.000	.060	.000	.000	-.012	.024	-.012	.000	.000	.000	.000	.000	.000	.000
-.060	.000	.060	.000	.000	.000	.000	-.012	.024	-.012	.000	.000	.000	.000	.000	.000
.060	.060	.000	.000	.000	.000	.000	.000	-.012	.012	.000	.000	.000	.000	.000	.000
-.060	.000	.000	.000	-.012	.000	.000	.000	.000	.000	.012	.000	.000	.000	-.060	.000
.000	-.060	.000	.000	-.012	.000	.000	.000	.000	.000	.000	.012	.000	.000	.000	-.060
.000	.000	-.060	.000	-.012	-.060	.000	.000	.000	.000	.000	.000	.012	.000	.000	.000
.200	.000	.000	.000	.060	.000	.000	.000	.000	.000	-.060	.000	.000	.000	.400	.000
.000	.200	.000	.000	.060	.000	.000	.000	.000	.000	.000	.000	-.060	.000	.000	.400

Stop - Program terminated.

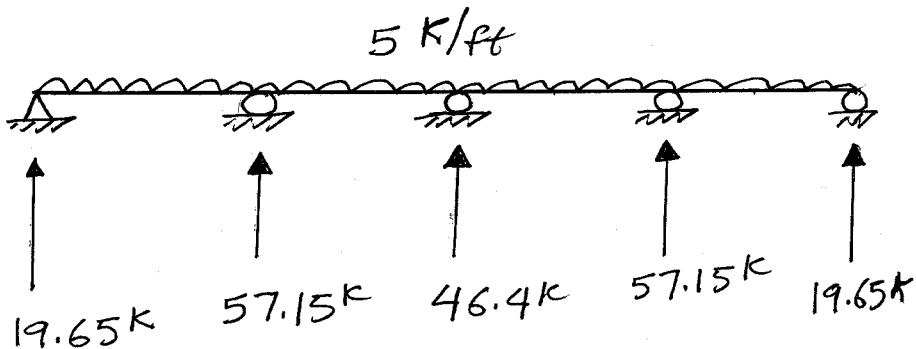
Press any key to continue

O/K

# HW # 9



displacements

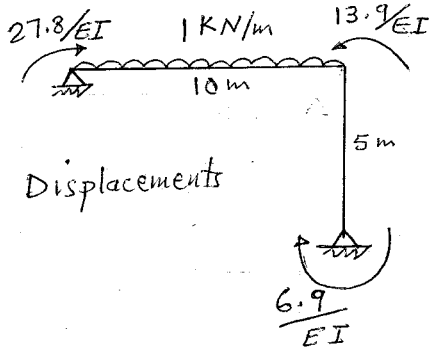
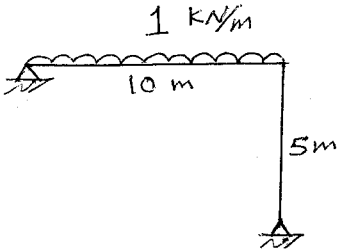


reactions

# HW # 10

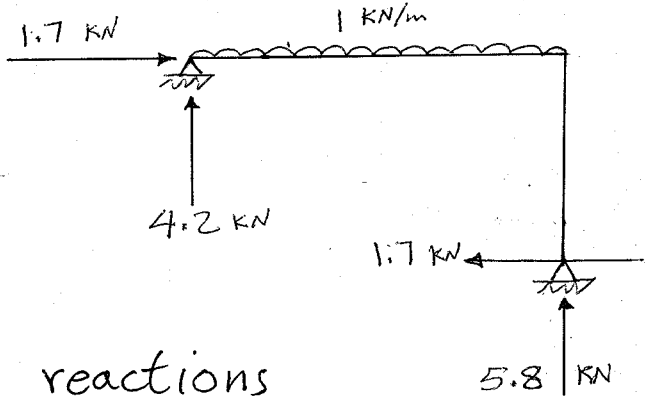
Constant  $E$  &  $I$

Stiffness Method



Not including axial deformations

Displacements

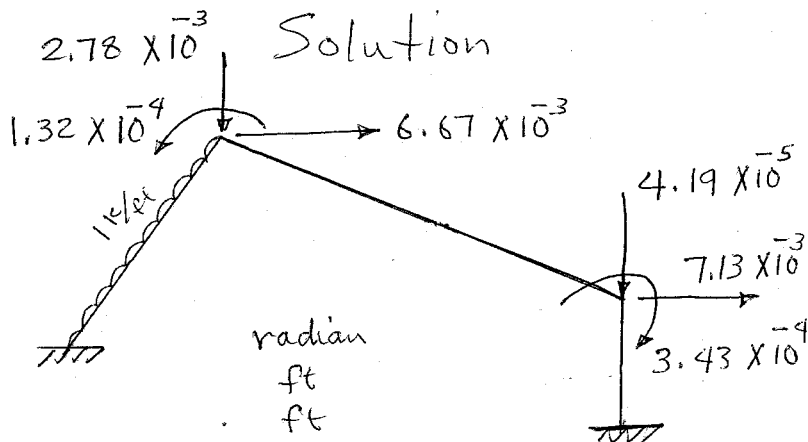


reactions

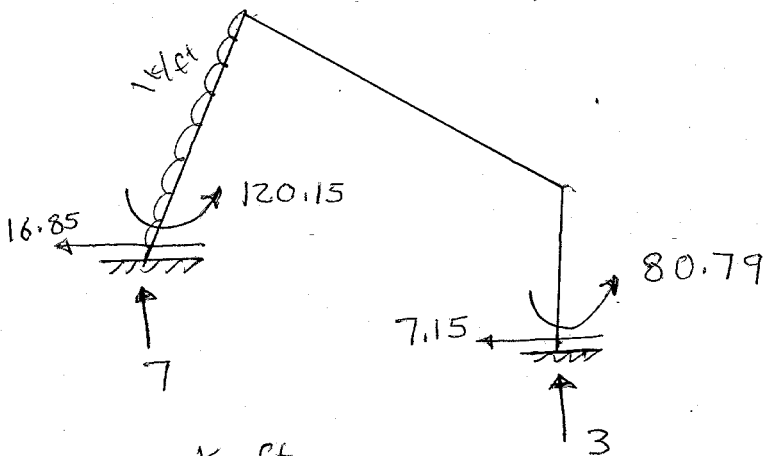


# HW # 11

## Solution

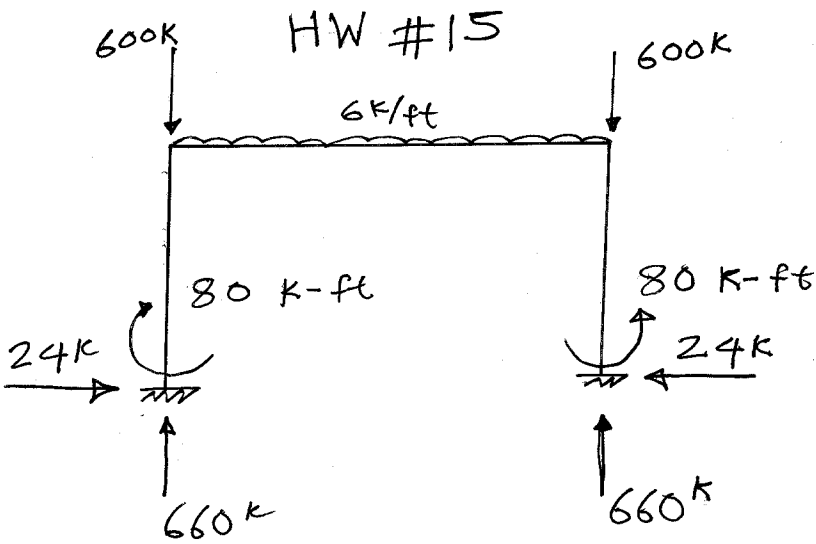


displacements

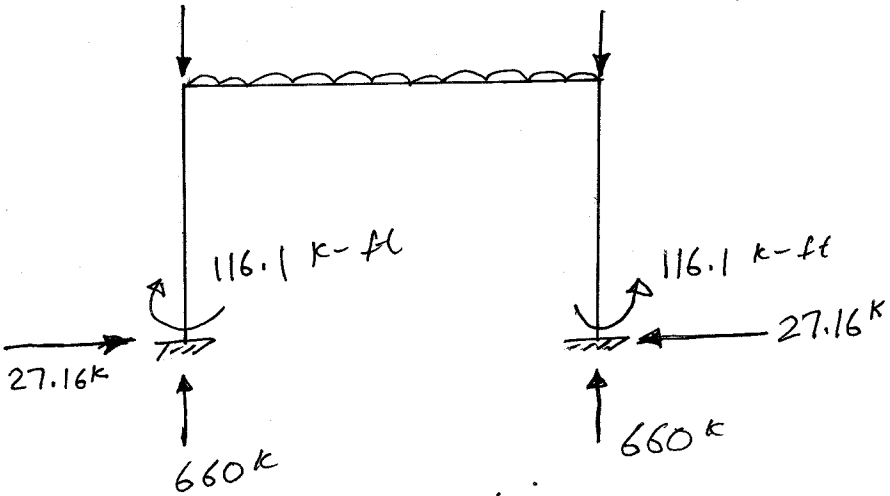


K-ft  
K  
K

reactions.

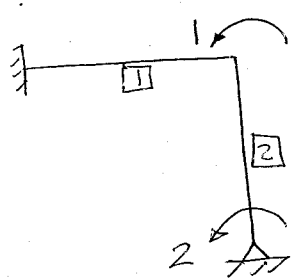


reactions  
without axial load effect



reactions  
with axial load effect

# Solution # 16



$$K_1 = \begin{bmatrix} 1 \\ \frac{r_1}{L_1} \end{bmatrix}_1$$

$$K_2 = \begin{bmatrix} 1 & 2 \\ \frac{r_2}{L_2} & \frac{C_2 r_2}{L_2} \\ \frac{C_2 r_2}{L_2} & \frac{r_2}{L_2} \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

$$S_{uu} = \begin{bmatrix} 1 & 2 \\ \frac{r_1}{10} + \frac{r_2}{10} & \frac{C_2 r_2}{10} \\ \frac{C_2 r_2}{10} & \frac{r_2}{10} \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

$$|S_{uu}| = r_2(r_1 + r_2) - C_2^2 r_2^2$$

$$F_1 = F \cos 45 = 0.707 F$$

$$F_2 = F \sin 45 = 0.707 F$$

$$\therefore F_1 = F_2 \quad \therefore r_1 = r_2 = r$$

$$\therefore 2r^2 - C_2^2 r^2 = 0 \Rightarrow r^2(2 - C_2^2) = 0$$

$$\therefore r = 0 \quad \text{or} \quad C = \sqrt{2} = \pm 1.4142$$

$$\therefore \phi = 2 \quad \text{or} \quad \phi = 1.3 \quad \text{or} \quad \phi = 3$$

$$\text{Smallest } \phi = 1.3$$

$$\therefore \phi = \frac{.707 F}{P_c} = \frac{.707 F \times 100}{\pi^2 EI} = 1.3$$

$$\therefore F = .18 EI = 1800 \text{ K}$$