Key Solution

HOME WORK # 10

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KEY TO HOMEWORK # 10

PROBLEM # 1 (8-2).

8–2. The open-ended polyvinyl chloride pipe has an inner diameter of 4 in. and thickness of 0.2 in. If it carries flowing water at 60 psi pressure, determine the state of stress in the walls of the pipe.

8–3. If the flow of water within the pipe in Prob. 8–2 is stopped due to the closing of a valve, determine the state of stress in the walls of the pipe. Neglect the weight of the water. Assume the supports only exert vertical forces on the pipe.

Probs. 8–2/8–3

SOLUTION:-

Given:-

$t = 0.2$ in

$r = \frac{4}{2} = 2$ in.

$p = 60$ psi

REQUIRED:-

$\sigma_1 = ?$ & $\sigma_2 = ?$

The pipe is a cylindrical pressure vessel.

Since, the pipe is open ended. It doesn't have the longitudinal stresses (i.e)

$$\sigma_l = \sigma_2 = 0.$$

This pipe has only hoop stress (i.e)

$$\sigma_h = \sigma_1 = \frac{pr}{t}$$

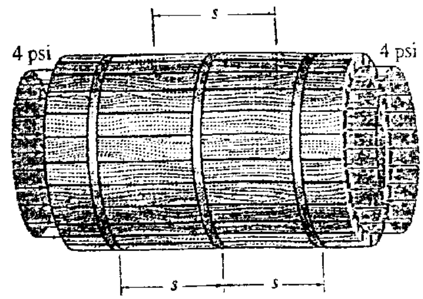
$$\Rightarrow \sigma_1 = \frac{60 \times 2}{0.2} = 600 \text{ psi}$$

$$\therefore \sigma_1 = 600 \text{ psi}$$

$$\& \ \sigma_2 = 0.$$

PROBLEM #② (8-9):-

8–9. A wood pipe having an inner diameter of 3 ft is bound together using steel hoops having a cross-sectional area of 0.2 in^2. If the allowable stress for the hoops is $\sigma_{allow} = 12$ ksi, determine their maximum spacing s along the section of pipe so that the pipe can resist an internal gauge pressure of 4 psi. Assume each hoop supports the pressure loading acting along the length s of the pipe.



Prob. 8–9

SOLUTION:-

Given:-

Inner dia. = 3'

$r = \frac{3}{2} = 1.5' = 18''$.

c/s area of steel hoops = 0.2 in^2 = A_{st}.

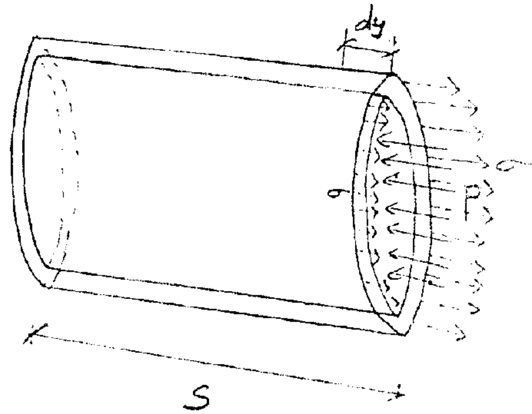
$\sigma_{all} = 12$ ksi.

$P = 4$ psi.

REQUIRED:-

$S = ?$

From fig.



fig

$\Sigma F_x = 0$

$2\sigma(dy \times t) - P(2r)S = 0$

$\Rightarrow 2\sigma(A_{st}) - 2PrS = 0$

$\Rightarrow S = \frac{\sigma A_{st}}{rP}$

$\Rightarrow S = \frac{12 \times 10^3 \times 0.2}{18 \times 4} = 33.3$ in.

$\therefore S = 33.3$ in.

PROBLEM #③ (8-19):-

SOLUTION:-

Given:-

$P = 30 KN$

$\sigma_{all} = 73 MPa.$

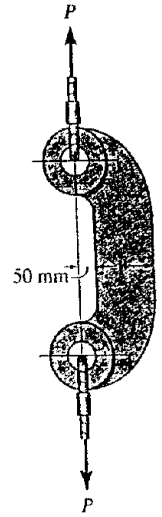
$t = 40 mm.$

REQUIRED:-

$w = ?$

8–19. The offset link supports the loading of $P = 30$ kN. Determine its required width w if the allowable normal stress is $\sigma_{\text{allow}} = 73$ MPa. The link has a thickness of 40 mm.

***8–20.** The offset link has a width of $w = 200$ mm and a thickness of 40 mm. If the allowable normal stress is $\sigma_{\text{allow}} = 75$ MPa, determine the maximum load P that can be applied to the cables.



Probs. 8–19/ 8–20

from fig:-

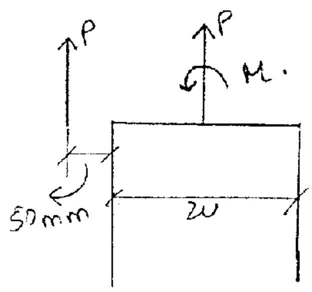


fig.

$$M = P \times \left(50 + \frac{w}{2}\right)$$

$$\Rightarrow M = 30\times10^3 \left(50 + \frac{w}{2}\right) N\text{-}mm.$$

∵ we know.

$$\sigma = \frac{P}{A} \pm \frac{My}{I}.$$

$$A = w \times 40 = 40\,w \ mm^2$$

$$y = \frac{w}{2}.$$

$$I = \frac{1}{12}(w)^3 (40).$$

$$\therefore \ 73 = \frac{30\times10^3}{40\times w} + \frac{30\times10^3 \times \left(50 + \frac{w}{2}\right)\frac{w}{2}}{\frac{1}{12}\times w^3 \times 40}$$

$$\Rightarrow 73 = \frac{750}{w} + \frac{225\times10^3}{w^2} + \frac{2250}{w}$$

$$\Rightarrow 73w^2 = 3000\,w + 225\times10^3.$$

$$\Rightarrow 73w^2 - 3000\,w - 225\times10^3 = 0$$

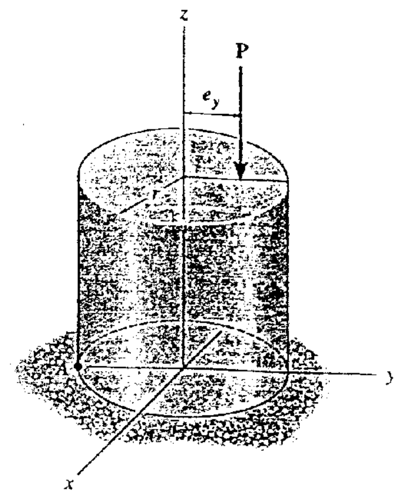
$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3000 \pm \sqrt{(3000)^2 + 4\times73\times225\times10^3}}{2\times73}$$

$\Rightarrow w = 79.74$ mm (taking +ve value).

$\Rightarrow$ $w = \underline{79.74 mm.}$

PROBLEM # 4 (8-39) :-

8–39. The cylinder of negligible weight rests on a smooth floor. Determine the eccentric distance e_y at which the load can be placed so that the normal stress at point A is zero.



Prob. 8–39

Solution:-

REQUIRED:-

$e_y = ?$

Given:-

$$\sigma_A = 0 = -\frac{P}{A} + \frac{Mc}{I}.$$

$$M = P \cdot e_y$$

$$\Rightarrow \frac{\cancel{P}}{A} = \frac{\cancel{P} \cdot e_y \times c}{I}$$

$$\Rightarrow e_y = \frac{I}{Ac}$$

$$I = \frac{1}{64} \times \pi \times (2r)^4$$

$$A = \pi r^2$$

$$c = r$$

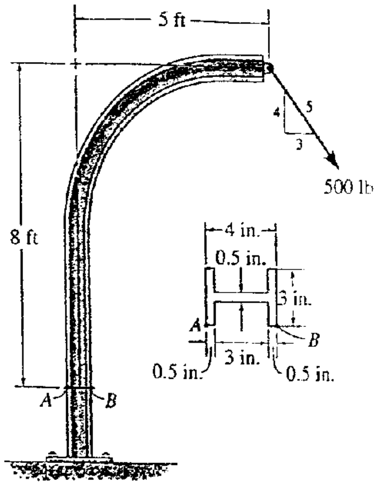
$$\therefore e_y = \frac{\frac{1}{64} \times \cancel{\pi} \times 16r^4}{\cancel{\pi} r^2 \times r}$$

$$\Rightarrow e_y = \frac{16r^4}{64r^3} = \frac{r}{4}.$$

$$\therefore e_y = \frac{r}{4}.$$

PROBLEM # 5 (8-50):-

8–50. The crane boom is subjected to the load of 500 lb. Determine the state of stress at points A and B. Show the results on a differential volume element located at each of these points.



Prob. 8–50

SOLUTION:-

REQUIRED:-

$\sigma_A = ?$ & $\sigma_B = ?$

$\tau_A = ?$ & $\tau_B = ?$

Resolving load in to two components in x & y directions.

$\therefore F_x = V = \frac{3}{5} \times 500 = 300 \, lb$

$F_y = N = \frac{4}{5} \times 500 = 400 \, lb.$

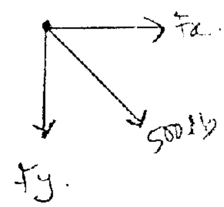


Fig ①

Moment at A & B.

$M = N \times 5 + V \times 8$

$\Rightarrow M = 400 \times 5 + 300 \times 8$

$\Rightarrow M = 4400 \, lb\,ft = 52800 \, lb.in.$

$\because \sigma = -\frac{P}{A} \pm \frac{My}{I}$

$A = 3 \times 4 - \{2 \times (1.25 \times 3.0)\} = 4.5 \, in^2$

$I_x = \frac{0.5 \times 3^3}{12} + 2\left\{\frac{3 \times 0.5^3}{12} + 3 \times 0.5 \times (2 - \frac{0.5}{2})^2\right\}$

$\Rightarrow I_x = 10.375 \, in^4.$

$\sigma_A = -\frac{N}{A} + \frac{Mc}{I_x} = \frac{-400}{4.5} + \frac{52800 \times 2}{10.375} = 10089.42 \, Psi$

$\Rightarrow \sigma_A = 10.1 \, ksi \, (T)$

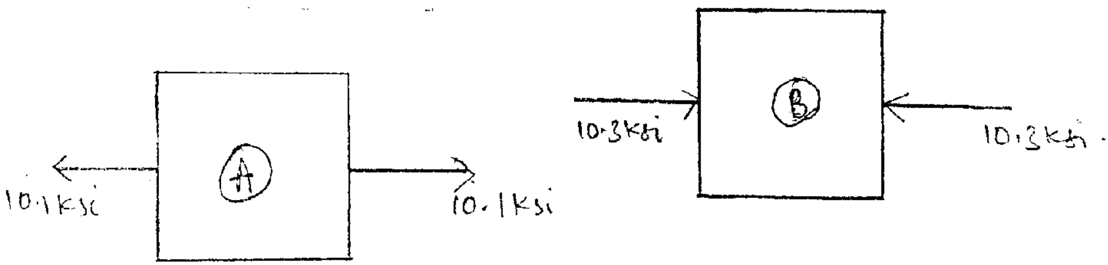
$$\sigma_B = -\frac{N}{A} - \frac{M \times c}{I_x} = \frac{-400}{4.5} - \frac{52800 \times 2}{10.375}$$

$$\Rightarrow \sigma_B = -10267.20 \text{ psi}$$

$$\Rightarrow \sigma_B = -10.3 \text{ ksi}$$

$$\Rightarrow \sigma_B = 10.3 \text{ ksi (C)}.$$

$\tau_A = \tau_B = 0$ ($\because$ Points A & B are at corners).



TWO DIMENSIONAL INFINITESIMAL ELEMENTS

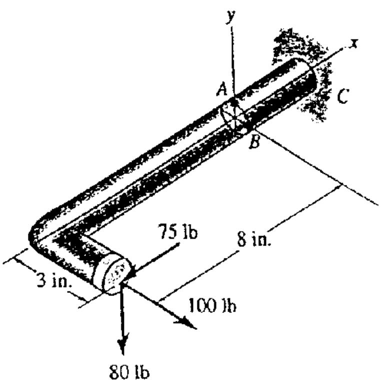
PROBLEM #6 (8-55):-

8–55. The 1-in.-diameter rod is subjected to the loads shown. Determine the state of stress at point B, and show the results on a differential element located at this point.

SOLUTION:-

REQUIRED:-

State of Stress at B.



Probs. 8–54/8–55

EQUILIBRIUM EQUATIONS:-

$\Sigma F_x = 0$

$F_x = 75\,lb = P$

$\Sigma F_y = 0$

$F_y = 80\,lb = V$

$\Sigma F_z = 0$

$F_z = 100\,lb.$

$\Sigma M_x = 0$

$M_x = 80 \times 3 = 240\,lb\,in.$

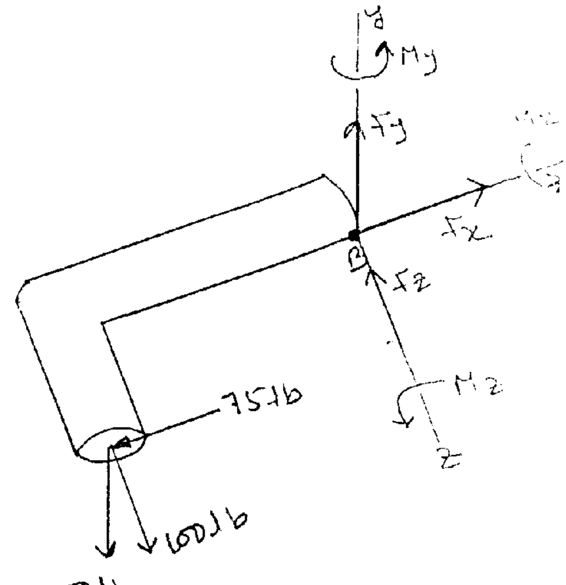
$\Sigma M_y = 0$

$M_y = 75 \times 3 - 100 \times 8$

$\Rightarrow M_y = -575\,lb\,in.$

$\Sigma M_z = 0$

$M_z = 80 \times 8 = 640\,lb\,in.$



F.B.D

STRESS COMPONENTS:-

Normal Force:- $\sigma_B = \frac{P}{A} = \frac{75}{\pi(0.5)^2} = 95.5\ psi = 0.0955\ ksi$

Shear Force:-

$Q = \bar{y}'A' = \frac{4(0.5)}{3\pi}\left[\frac{1}{2}\pi(0.5)^2\right] = 0.08333\,in^2.$

$\tau_B = \frac{VQ}{It} = \frac{80 \times 0.0833}{\frac{1}{64} \times \pi (1)^4 \times 2 \times 0.5} = 135.75\ psi.$

Bending Moment:-

$$\sigma_B = \frac{Mc}{I} = \frac{-575 \times 0.5}{\frac{1}{64} \times \pi (1)^2} = -5856.9 \text{ psi} = -5.857 \text{ ksi}$$

TORSIONAL MOMENT:-

$$\tau_{xy} = \frac{Tc}{J} = \frac{240 \times 0.5}{\frac{1}{2} \times \pi \times (0.5)^4} = 1222.3 \text{ psi} = 1.22 \text{ ksi}.$$

$$\tau_{xz} = 0$$

Superposition:-

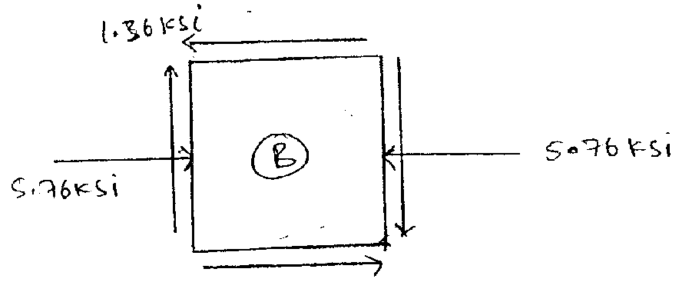
$$\sigma_B = 95.5 - 5857 = -5761.5 = -5.76 \text{ ksi}.$$

$$\Rightarrow \sigma_B = 5.76 \text{ ksi (C)}$$

$$\tau_{(xy)B} = 135.75 + 1222.3 = 1358 \text{ psi} = 1.36 \text{ ksi}$$

$$\Rightarrow \tau_{(xy)B} = 1.36 \text{ ksi}.$$

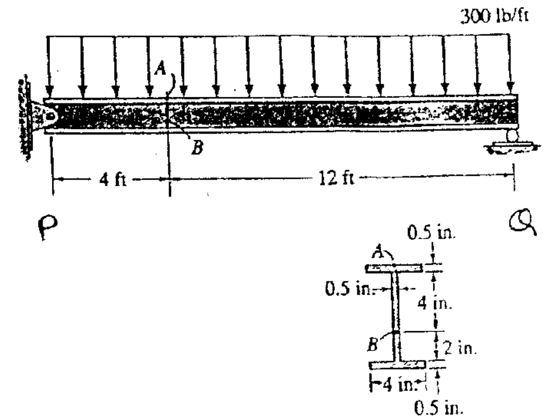
$$\tau_{(xz)B} = 0.$$



TWO DIMENSIONAL INFINITESIMAL ELEMENT:-

PROBLEM # 7 (8-77):-

8–77. The wide-flange beam is subjected to the loading shown. Determine the state of stress at points A and B, and show the results on a differential volume element located at each of these points.



Prob. 8–77

SOLUTION:-

EQUILIBRIUM EQUATIONS:-

From Fig (2)

$\Sigma F_x = 0$

$N_P = 0$.

$\Sigma F_y = 0$

$V_P + V_Q = 300 \times 16$

From Fig. (2)

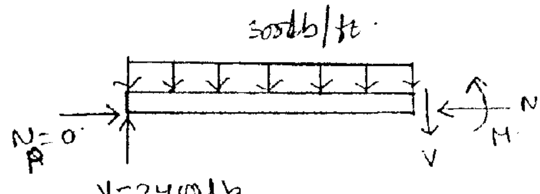
$\Sigma M_P = 0$.

$V_Q \times 16 = 300 \times 16 \times 8$

$\Rightarrow V_Q = 2400\ lb$.

& $V_P = 4800 - V_Q$

$\Rightarrow V_P = 2400\ lb$.



FBD. Fig (1)

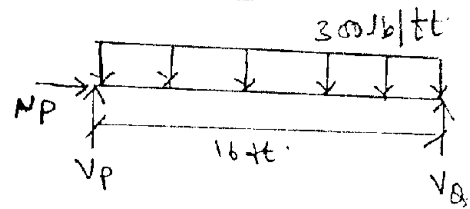


Fig. (2)

From Fig. (1) FBD

$\Sigma F_y = 0$.

$V = V_P - 300 \times 4$

$\Rightarrow V = 2400 - 1200$

$\Rightarrow V = 1200\ lb$.

$\Sigma M_{AB} = 0$.

$M + 300 \times 4 \times 2 - 2400 \times 4 = 0$

$\Rightarrow M = 7200\ lb\,ft$.

$I = \frac{1}{12}\{0.5 \times 6^3\} + 2\left[\frac{1}{12} \times 4 \times (0.5)^3 + 4 \times 0.5 \times (3.25)^2\right]$

$\Rightarrow I = 51.33\ in^4$.

P.T.O

Stresses:-

At A:-

$$\sigma_A = \frac{N_A}{A} - \frac{M_A C}{I}$$

$$= 0 - \frac{7200 \times 12 \times 3.5}{51.33} = -5891.94 \text{ psi}$$

$$\Rightarrow \sigma_A = 5.89 \text{ ksi (C)}.$$

$\tau_A = 0$ ($\because Q = 0$, as A is at corner).

At B:-

$$\sigma_B = \frac{N_B}{A} + \frac{M_B \cdot C}{I}$$

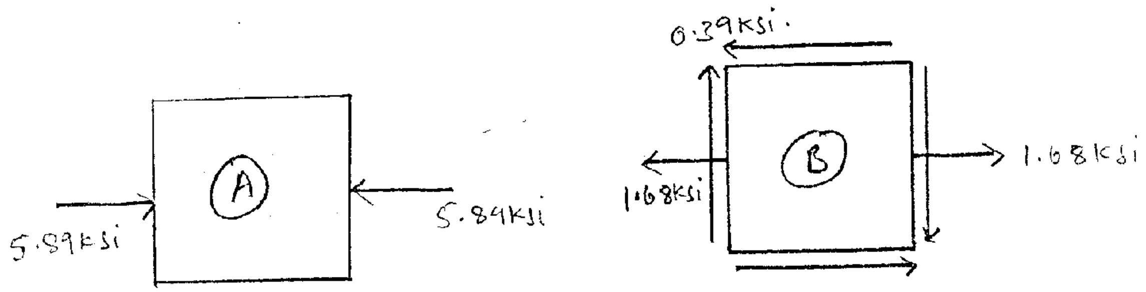
$$= 0 + \frac{7200 \times 12 \times 1}{51.33} = 1683.12 \text{ psi}$$

$$\Rightarrow \sigma_B = 1.683 \text{ ksi. (T)}$$

$$Q = \Sigma A'\bar{y}' = 4 \times 0.5 \times (3.5 - \tfrac{0.5}{2}) + 2 \times 0.5 \times (3.5 - 0.5 - 1) = 8.5 \text{ in}^3$$

$$\therefore \tau_B = \frac{VQ}{It} = \frac{1200 \times 8.5}{51.33 \times 0.5} = 397.4 \text{ psi}$$

$$\Rightarrow \tau_B = 0.397 \text{ ksi}.$$



TWO-DIMENSIONAL INFINITESIMAL ELEMENTS