King Fahd University of Petroleum & Minerals
DEPARTMENT OF CIVIL ENGINEERING

Advanced Structural Analysis
(CE 511)

Final Exam

Closed Book & Notes
(3 Hours)

Student's Name ____________________

Student's No. ______ EXAM ~~+~~

Key Solution

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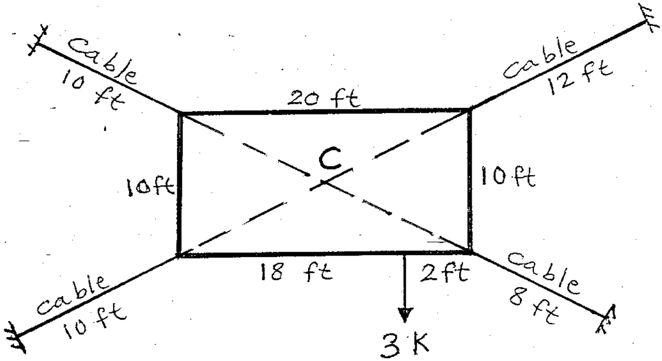
Problem # 1

The rectangular frame shown below is supported by four cables. Can you analyze it by the Stiffness Method? Why? Do not analyze.



	Frame	Cable
E	29000 ksi	29000 ksi
I	2 ft^4	0.8 ft^4
A	0.6 ft^2	0.2 ft^2

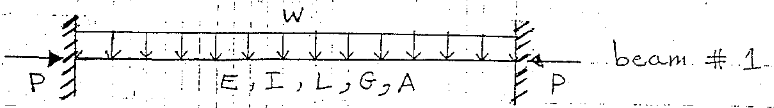
No

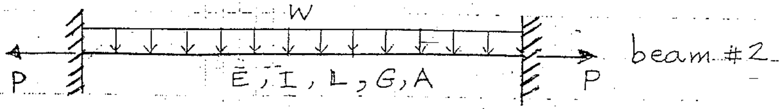
Because the frame is unstable.

$\Sigma M_c \neq 0$

Problem # 2

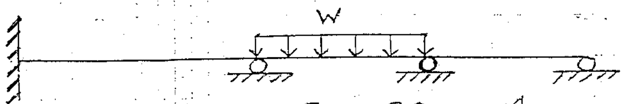
(a) Beam # 1 is subjected to an axial force P applied in compression while beam # 2 is subjected to the same force P applied in tension. Which beam will have higher fixed end moments? Why?



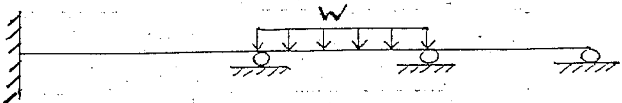


Beam # 1

(b) The two beams shown are identical in everything except the moment of inertia. Which beam will have higher reactions? Why?



beam (a) : $I = 300\ IN^4$



beam (b) : $I = 100\ IN^4$

The two beams will have the same reactions

Problem # 3

Circle the correct statements in the following:

(1) The member stiffness matrix is always invertible.

(2) The total structural stiffness matrix is always invertible.

(3) Maxwell's Theorem is a special case of Betti's Law.

(4) All stable structures made of linearly elastic materials may be analyzed by the stiffness method.

(5) Every structure stiffness matrix is the inverse of its flexibility matrix.

(6) A column subjected to an axial compressive load becomes unstable at the time of buckling.

(7) The two-dimensional member stiffness matrix (neglecting axial deformations) has a rank of 2.

Problem # 4

A structure has the matrix equation shown below. Can you obtain the values of the reactions R_5 and R_6? Why? Do not calculate the reactions.

$$\begin{Bmatrix} 12\ \text{k}-\text{ft} \\ 13\ \text{k}-\text{ft} \\ 11\ \text{k} \\ 14\ \text{k} \\ R_5 \\ R_6 \end{Bmatrix} = \begin{bmatrix} 4 & 2 & 6 & -6 & 3 & 2 \\ 2 & 4 & 6 & -6 & 2 & 2 \\ 6 & 6 & 12 & -12 & 1 & 3 \\ -6 & -6 & -12 & 12 & 2 & 1 \\ 3 & 2 & 1 & 2 & 8 & 2 \\ 2 & 2 & 3 & 1 & 2 & 8 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \delta_3 \\ \delta_4 \\ 0 \\ 0 \end{Bmatrix}$$

No, because the values of the reactions depend on the displacements, which cannot be determined because the Suu matrix is not invertible

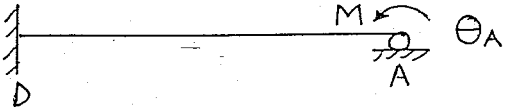
Problem # 5

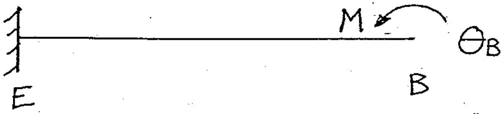
The three beams shown below are identical. The same moment M is applied at the ends shown producing rotations θ_A, θ_B and θ_C. Arrange the rotations in a descending order:

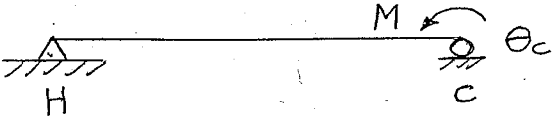
(1) largest rotation = $\theta_B = \dfrac{ML}{EI}$

(2) intermediate rotation = $\theta_C = \dfrac{ML}{3EI}$

(3) smallest rotation = $\theta_A = \dfrac{ML}{4EI}$







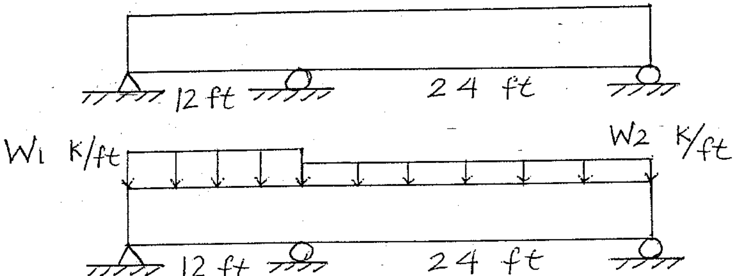
Problem # 6

The two beams shown below have the same EI and depth. The first beam is subjected to a linear temperature change from 30°F on the top of the beam to 130°F at the bottom. The second beam is subjected to uniform loads. What is the value of each load to make the displacements of the second beam equal to the corresponding displacements of the first beam.

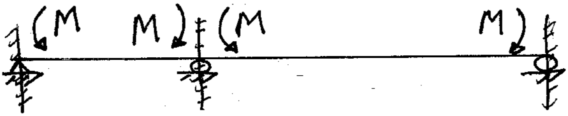
$EI = 8\times10^4$ k-ft^2

$\alpha = 6\times10^{-6}$ ft/ft°F

$d = 1$ ft (beam depth)



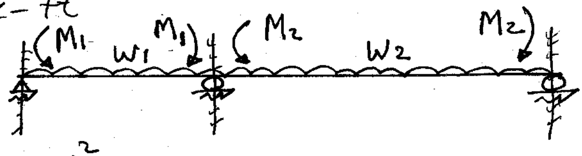
beam 1



$$M = \frac{\alpha \Delta T EI}{d} = \frac{6\times10^{-6}\times100\times8\times10^4}{1}$$

$= 48$ k-ft

beam 2



$$M_1 = \frac{W_1 L_1^2}{12} = 12W_1$$

$$M_2 = \frac{W_2 L_2^2}{12} = 48W_2$$

$M_1 = M \Rightarrow W_1 = 4$ k/ft

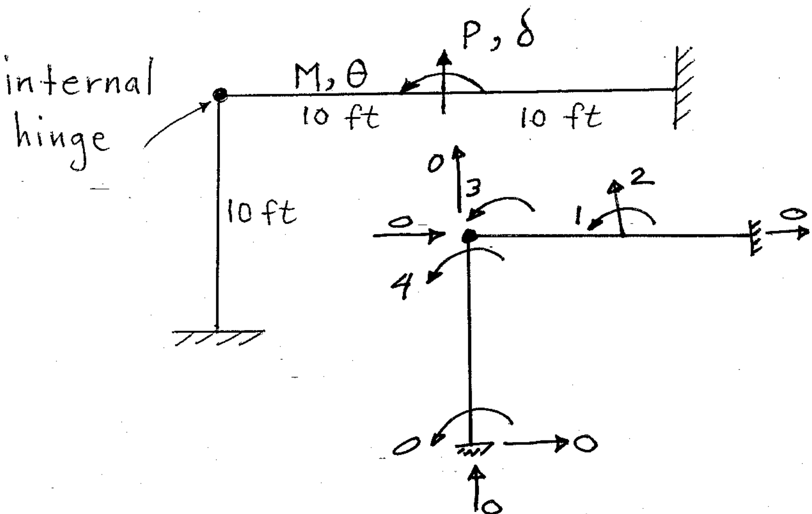
$M_2 = M \Rightarrow W_2 = 1$ k/ft

Problem # 7

Obtain the 2×2 stiffness matrix which appears in the equation:

$$\begin{Bmatrix} M \\ P \end{Bmatrix} = \begin{bmatrix} K11 & K12 \\ K21 & K22 \end{bmatrix} \begin{Bmatrix} \theta \\ \delta \end{Bmatrix}$$

E and I are the same for all members. Neglect axial deformations.



K_1

$$K_1 = \frac{EI}{10} \begin{array}{c} \left[\begin{array}{ccc} 4 & 2 & -\frac{6}{10} \\ 2 & 4 & -\frac{6}{10} \\ -\frac{6}{10} & -\frac{6}{10} & \frac{12}{100} \end{array}\right] \end{array} \begin{array}{c} 3 \\ 1 \\ 2 \end{array}$$

$$K_2 = \frac{EI}{10} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \left[\begin{array}{cc} 4 & \frac{6}{10} \\ \frac{6}{10} & \frac{12}{100} \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \end{array}$$

$$
S = \frac{EI}{10}\;
\begin{array}{c c}
\begin{array}{ccc} \quad 1 \quad & \quad 2 \quad & \quad 3 \quad \end{array} & \\
\begin{bmatrix} 8 & 0 & 2 \\ 0 & \frac{24}{100} & -\frac{6}{10} \\ 2 & -\frac{6}{10} & 4 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array}
\end{array}
$$

$$
\begin{array}{c}
F \\
\left\{ \begin{array}{c} M \\ P \\ \hline 0 \end{array} \right\}
\end{array}
= \frac{EI}{10}
\begin{array}{c}
\begin{array}{cc} S_{11} \qquad\qquad & \qquad S_{12} \end{array} \\
\left[\begin{array}{cc|c} 8 & 0 & 2 \\ 0 & \frac{24}{100} & -\frac{6}{10} \\ \hline 2 & -\frac{6}{10} & 4 \end{array} \right] \\
\begin{array}{cc} S_{21} \qquad\qquad & \qquad S_{22} \end{array}
\end{array}
\left\{ \begin{array}{c} \theta \\ \delta \\ \hline \theta_3 \end{array} \right\}
\leftarrow \Delta
$$

$$
\begin{Bmatrix} F \\ 0 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} \Delta \\ \theta_3 \end{Bmatrix}
$$

$$F = S_{11}\,\Delta + S_{12}\,\theta_3$$

$$0 = S_{21}\,\Delta + S_{22}\,\theta_3$$

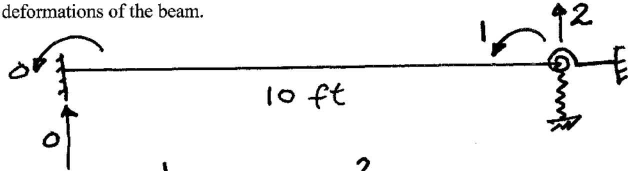
$$-S_{22}\,S_{21}\,\Delta = \theta_3$$

$$F = S_{11}\,\Delta - S_{12}\,S_{22}^{-1}\,S_{21}\,\Delta$$

$$F = \left(S_{11} - S_{12}\,S_{22}^{-1}\,S_{21}\right)\Delta$$

Problem # 8

Obtain the 4×4 stiffness matrix of the beam shown. Let E = 1 k/ft^2 and I = 1 ft^4. Neglect axial deformations of the beam.



$$\begin{array}{c} \begin{array}{cc} 1 & \quad\quad 2 \end{array} \\ K = EI \begin{bmatrix} \frac{4}{L} & -\frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{12}{L^3} \end{bmatrix} \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

$$\begin{array}{c} \begin{array}{cc} 1 & \quad\quad 2 \end{array} \\ K = \begin{bmatrix} 0.4 & -0.06 \\ -0.06 & 0.012 \end{bmatrix} \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

$$\begin{Bmatrix} M_1 + R_1 \\ F_1 + R_2 \end{Bmatrix} = \begin{bmatrix} .4 & -.06 \\ -.06 & .012 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \delta_1 \end{Bmatrix}$$

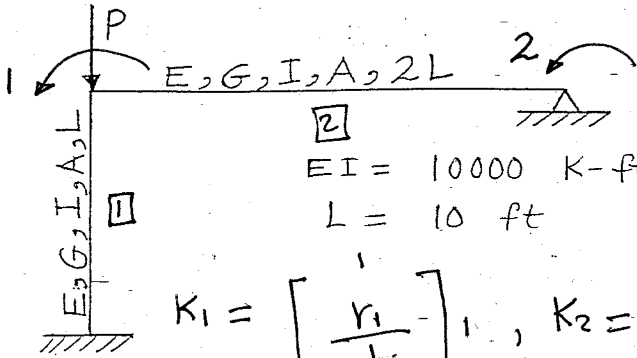
$$\begin{Bmatrix} M_1 \\ F_1 \end{Bmatrix} = \begin{bmatrix} .4 & -.06 \\ -.06 & .012 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \delta_1 \end{Bmatrix} - \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix}$$

$$\begin{Bmatrix} M_1 \\ F_1 \end{Bmatrix} = \begin{bmatrix} .4 & -.06 \\ -.06 & .012 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \delta_1 \end{Bmatrix} - \begin{Bmatrix} -8\theta_1 \\ -10\delta_1 \end{Bmatrix}$$

$$\begin{Bmatrix} M_1 \\ F_1 \end{Bmatrix} = \begin{bmatrix} .4+8 & -.06 \\ -.06 & .012+10 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \delta_1 \end{Bmatrix}$$

Problem # 9

Obtain the buckling value of the force P acting on the structure shown.



$EI = 10000 \ \text{K-ft}^2$

$L = 10 \ \text{ft}$

$$K_1 = \begin{array}{cc} 1 & \\ \left[\dfrac{r_1}{L} \right] & 1 \end{array} , \quad K_2 = \begin{array}{ccc} 1 & 2 & \\ \multicolumn{2}{c}{\left[\begin{array}{cc} \dfrac{r_2}{2L} & \dfrac{C_2 r_2}{2L} \\ \dfrac{C_2 r_2}{2L} & \dfrac{r_2}{2L} \end{array} \right]} & \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

$$S_{uu} = \begin{bmatrix} \dfrac{r_1}{L} + \dfrac{r_2}{2L} & \dfrac{C_2 r_2}{2L} \\ \dfrac{C_2 r_2}{2L} & \dfrac{r_2}{2L} \end{bmatrix}$$

$$|S_{uu}| = 0 = 2 r_1 r_2 + r_2^2 - C_2^2 r_2^2$$

$$\phi_1 = \phi \ , \ \phi_2 = 0 \Rightarrow \begin{array}{l} r_2 = 4 \\ C_2 = 0.5 \end{array}$$

$$\therefore \ 0 = 8 r_1 + 16 - 4 = 8 r_1 + 12$$

$$\therefore \ r_1 = -1.5$$

$$\therefore \ \phi = 2.45 \quad (\text{from tables})$$

$$\therefore \ P = \phi P_E = 2.45 \pi^2 \frac{EI}{L^2}$$