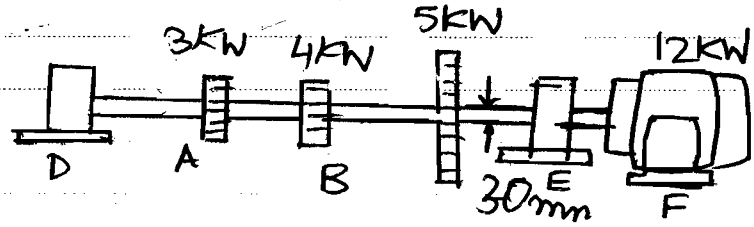
## Problem #1

Given:-

The figure shown

$D_{shaft} = 30 mm$

$Speed = 50\ rev/s$




Required:

$\tau_{max}$

Solution:-

To find $\tau_{max}$, we need to locate $T_{max}$ as we have only one D. We have "power" ⇒ we need to get T

$$\Rightarrow P = T\omega$$

$$\omega = \text{angular velocity} = 50\frac{rev}{s}\left(\frac{2\pi\ rad}{1\ rev.}\right) = 1000\pi\ rad/s$$

Clearly, $T_{max}$ is @ $P_{max}$ which is between C and F

$$\Rightarrow P_{max} = 12\ KW$$

$$\Rightarrow 12(10^3) = T_{max}(100\pi)$$

$$\Rightarrow T_{max} = 38.1972\ N\cdot m$$

$$\Rightarrow \tau_{max} = \frac{T_{max}\, r_{max}}{J} = \frac{T_{max}\, r_{out}}{J} = \frac{T_{max}\, C}{J}$$

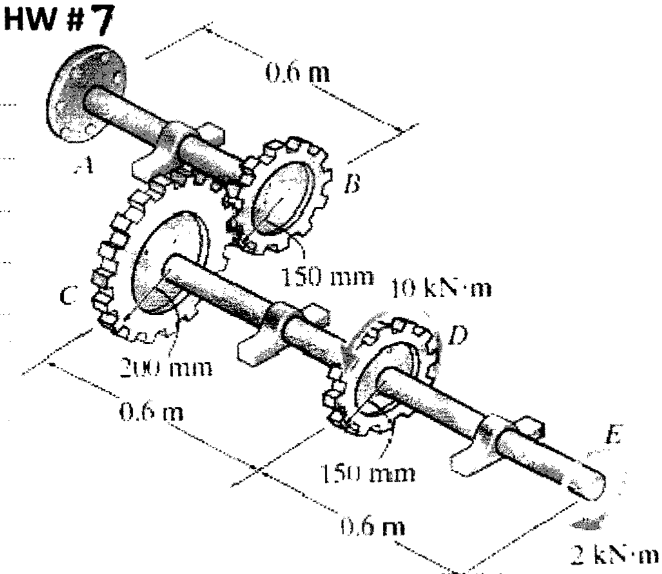
$$\tau_{max} = \frac{39.1972(15)(10)^{-3}}{\pi/2\left[(15)(10)^{-3}\right]^4}$$

$$\Rightarrow \boxed{\tau_{max} = 7.205\ MPa}$$

<u>Problem #2:-</u>

Given:-

The figure shown




A-36 steel shaft; $D = 80\,mm$

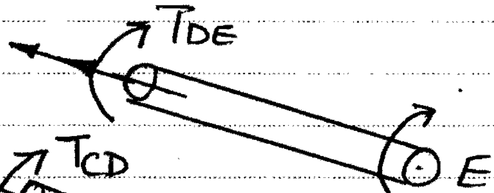
Required:-

$\tau_{max}$ ; $\phi_{E/B}$ ; $\phi_{E/A}$

Solution:

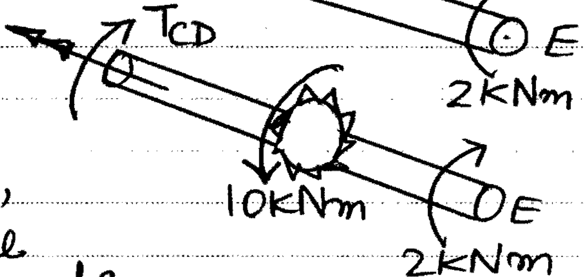
Since all shafts in the system have the same diameter, $\tau_{max}$ will be at $T_{max}$. T is the internal torque in each shaft. Thus, we need to draw FBD's for all "segments" to determine $T_{internal}$ in each, as shown below. Note that all internal T's are assumed ⊕ (i.e. on the "right" part).

$\Sigma T = 0 \Rightarrow T_{DE} = -2\,kN\cdot m$

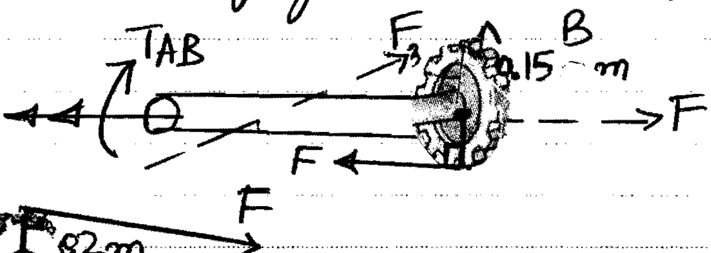


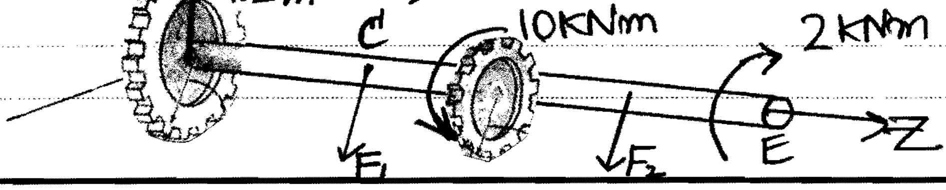

$-T_{CD} + 10 - 2 = 0$

$\rightarrow T_{CD} = 8\,kN\cdot m$




To find the torque in AB, we have to find the force between the two gears B and C (as explained in class by your instructor). Thus,



$\Sigma M_Z = 0$ in CDE $\Rightarrow 10 - 2 - 0.2(F) = 0 \Rightarrow F = 40\,kN$

$\Sigma M_Z = 0$ in AB $\Rightarrow -T_{AB} - 40(0.15) = 0 \Rightarrow T_{AB} = -6\,kNm$

From the values of $|T_{AB}|$, $|T_{CD}|$ and $|T_{DE}|$,

$T_{max} = |T_{CD}| = 8\,kNm$

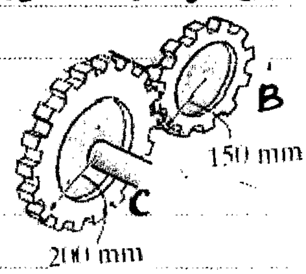
$$\Rightarrow \tau_{max} = \frac{T_{max} r_{max}}{J} = \frac{T_{CD} r_{out}}{J} = \frac{8(10)^3 (0.08/2)}{\pi/2\,(0.08/2)^4}$$

$$\Rightarrow \boxed{\tau_{max} = 79.58\ \text{Mpa @} r_{out} \text{ in segment CD}}$$

$\phi_{E/B} = \Sigma \phi_{E\,to\,B}$

$\phi_{E/A} = \Sigma \phi_{E\,to\,A}$

Since we have gears, we need to calculate the rotation of gear C relative to B.
As they are connected together, they must "travel" the same distance; thus,




$$\theta_B r_B = \theta_C r_C$$

In our case, $\theta$ = angle of twist $= \phi$

$$\Rightarrow \phi_B r_B = \phi_C r_C$$

$$\Rightarrow \phi_C = \frac{r_B}{r_C}\phi_B = \frac{150}{200}\phi_B = 0.75\,\phi_B$$

Therefore, $\phi_{E/A} = \phi_{E/C} + \phi_C$

$\phi_{E/A} = \phi_{CD} + \phi_{DE} + \phi_C = \phi_E$ as A is fixed

**Solution of HW #7**

$$\phi_{CD} = \left(\frac{TL}{JG}\right)_{CD} = \frac{8(10)^3(0.6)}{\pi/2(0.04)^4\underbrace{(75)(10)^9}_{\text{from the table at the end of the text book.}}} = 0.0159155 \text{ rad}$$

$$\phi_{DE} = \left(\frac{TL}{JG}\right)_{ED} = \frac{-2(1)^3(0.6)}{\pi/2(0.04)^4(75)(10)^9} = -0.00397887 \text{ rad}$$

To find $\phi_C$, we first need to determine $\phi_B$

$\phi_{B/A} = \phi_B = \phi_{AB}$ as A is fixed

$$\Rightarrow \phi_B = \left(\frac{TL}{JG}\right)_{AB} = \frac{-6(10)^3(0.6)}{\pi/2(0.04)^4(75)(10)^9} = -0.0119366 \text{ rad}$$

$\Rightarrow \phi_C = -0.75(-0.0119366) = 0.00895247$ rad [Be careful about signs!]

Thus, $\phi_{E/A} = 0.0159155 - 0.00397887 + 0.00895247$

$$\boxed{\phi_{E/A} = 0.02088 \uparrow \text{rad} = 1.197^\circ}$$

$\phi_{E/B} = \phi_{E/A} - \phi_B$ (Be careful about signs!)

$\Rightarrow \phi_{E/B} = 0.020889 - (-0.0119366)$

$$\Rightarrow \boxed{\phi_{E/B} = 0.03283 \text{ rad} = 1.881^\circ}$$

Problem # 3:

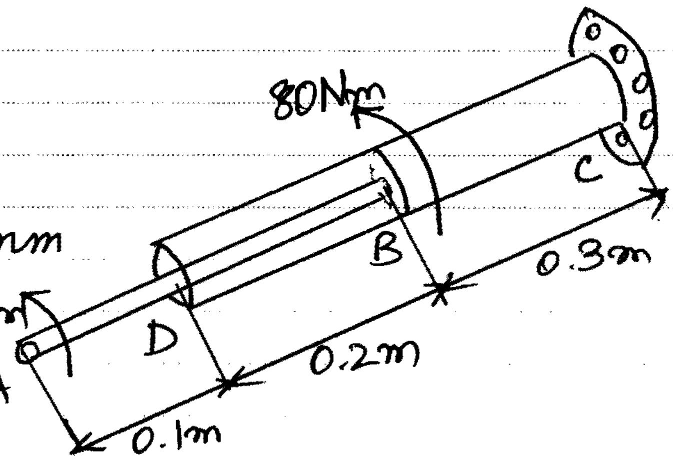
Given:

The figure shown

$D_{AB} = 20\ mm$

DC: $D_{out} = 55\ mm$; $t = 5\ mm$

$G = 100\ GPa$



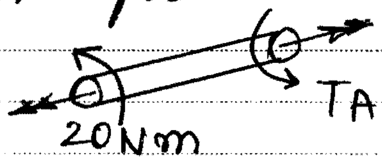

Required:

$\tau_{max}$; $\phi_A$; $\phi_D$

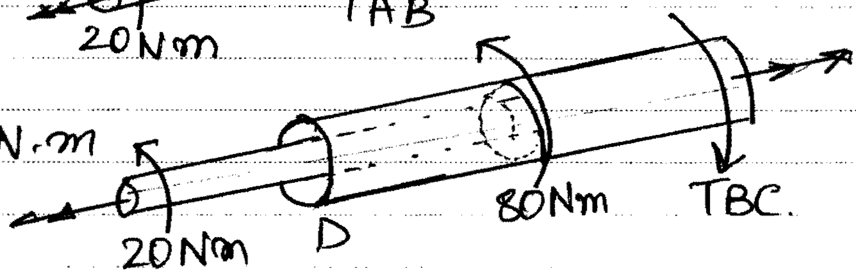
Solution:

We know that $\tau = \frac{Tr}{J}$ in each shaft; r is given for each, and we need to determine T (which is internal) for each segment from FBD's as shown below

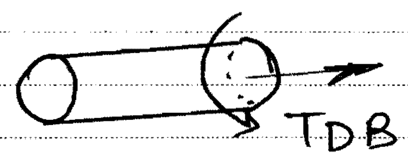
$T_{AB} = 20\ N.m$




$T_{BC} = 20 + 80 = 100\ N.m$




$T_{DB} = 0$




$$\tau_{AB}^{max} = \frac{T_{AB}\, r_{max}}{J} = \frac{20(10)(10)^{-3}}{\pi/2\left[(10)(10)^{-3}\right]^4} = 12.732\ MPa.$$

$$\tau_{BC}^{max} = \frac{100\ (55/2)(10)^{-3}}{\pi/2\left[\left\{\frac{55}{2}(10)^{-3}\right\}^4 - \left\{\frac{55-10}{2}(10)^{-3}\right\}^4\right]} = 5.5468\ MPa$$

$\tau_{DB} = 0$

$\tau_{max} = \tau_{AB}^{max} \Rightarrow \boxed{\tau_{max} = 12.73 \text{ Mpa at } r_{out} \text{ in } AB}$

$\phi_A = \phi_{A/C}$ since C is fixed

$= \phi_{AB} + \phi_{BC}$

$= \left(\frac{TL}{JG}\right)_{AB} + \left(\frac{TL}{JG}\right)_{BC}$

$= \frac{0.3}{100(10)^9 \pi/2}\left[\frac{20}{\{10(10)^{-3}\}^4} + \frac{100}{\{\frac{55}{2}(10)^{-3}\}^4 - \{\frac{55-10}{2}(10)^{-3}\}^4}\right]$

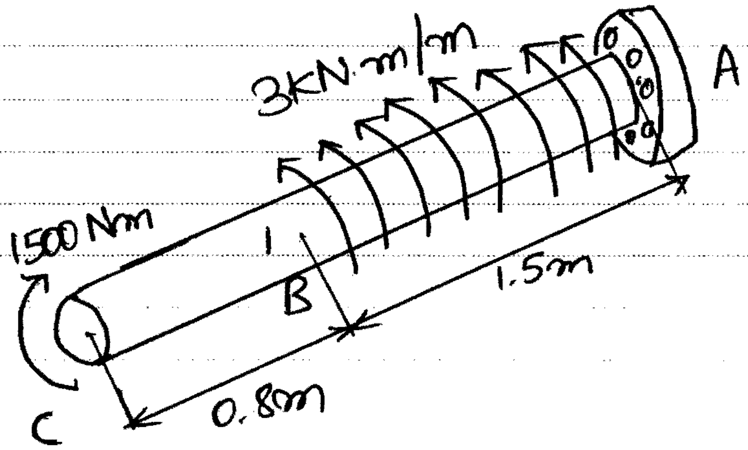
$\boxed{\phi_A = 9.8708(10)^{-4}_{rad} = 0.05656^\circ}$

$\phi_D = \phi_{DB} + \phi_{BC} = \underset{as\, T_{DB}=0}{0} + \frac{0.3(100)}{\frac{\pi}{2}\left[\left\{\frac{55}{2}(10)^{-3}\right\}^4 - \left\{\frac{55-10}{2}(10)^{-3}\right\}^4\right]100 \times 10^9}$

$\boxed{\phi_D = 6.051(10)^{-4} rad = 0.03467^\circ}$

Problem #4:-

Given:
The figure shown
$\phi_C^{max} = 1^\circ$
$\tau_{allow} = 60 MPa$




Required:-
Dmin

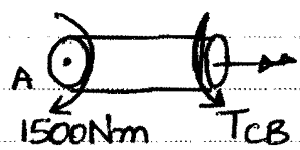
Solution :-
Here, we have two criteria we need to satisfy: the angle of twist and the shear stress. There are two ways to solve the problem. The first one is to determine Dmin for each case, then take the bigger one. Or, assume one criterion controls (i.e. its max/allow will be reached before the other one, and from that determine Dmin. After that we need to check our assumption by calculating the other criterion using Dmin found. We will follow the first method, as it is easier for the student to comprehend.

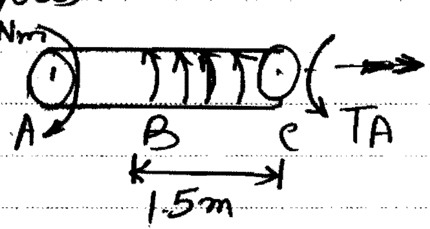
Start with $\tau_{max}$ criterion:

$$\tau = \frac{Tr}{J}$$

Since the shaft has one diameter, and the two applied forces are in different directions, $\tau_{max}$ will be in segment CB or at end A, depending on the values of the internal torques.

From the FBD's

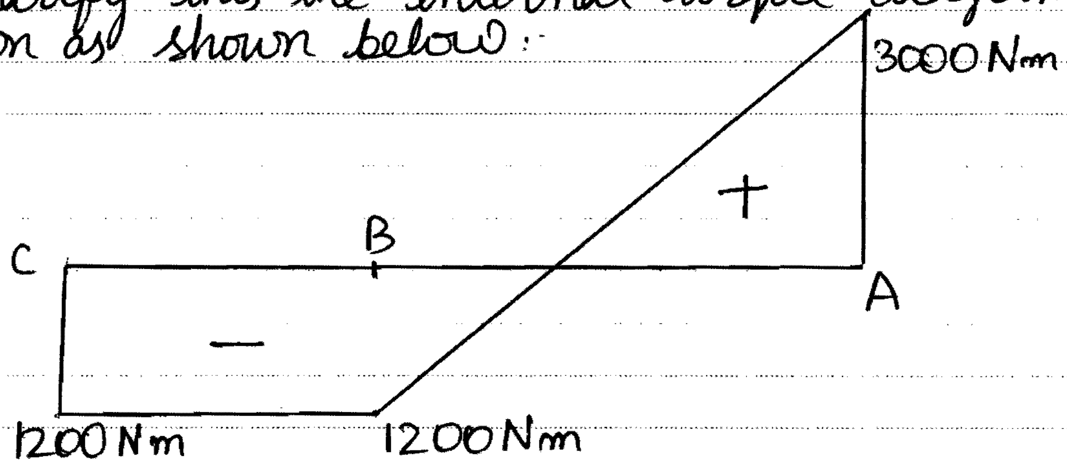



$T_{CB} = -1500\ Nm$

$T_A = 3000(1.5) - 1500 = 3000\,Nm$

To clarify this the internal torque diagram can be drawn as shown below:



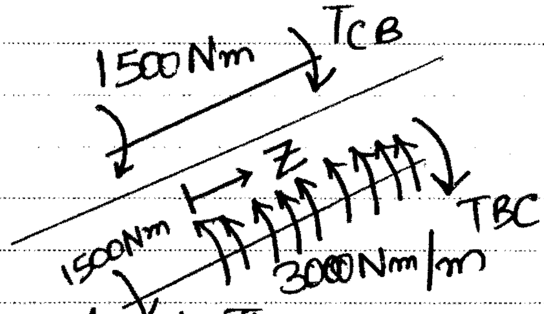

$\tau_{max} = \dfrac{T_{max}\, r_{max}}{J}$

Set $\tau_{max} = 60(10)^6 = \dfrac{3000\,(r_{out})}{\frac{\pi}{2}(r_{out})^4} \Rightarrow$

$r_{min}^{\tau} = 0.031692\ m$

Next, $\phi$ Criterion:-

$\phi_C = \phi_{CB} + \phi_{BA}$




For CB, T, r and G are constant. Thus, we can use the formula $\phi = \dfrac{TL}{JG}$; but for BA, T is <u>not</u> constant.

So we must integrate as $\phi = \int \dfrac{T}{JG}\, dz$.

$\phi_C = \left(\dfrac{TL}{JG}\right)_{CB} + \int_0^{1.5} \left(\dfrac{T}{JG}\right)_{BA} dz$

$T_{CB} = -1500\,Nm$ $\qquad T_{BC} = -1500 + 3000z$

$$= \frac{1}{JG}\left[-1500(0.8) + \int_0^{1.5}(-1500+3000z)\,dz\right]$$ (As JG are common & constant)

$$= \frac{1}{\pi/2\, r^4 (80)(10)^9}\left[-1500(0.8) - 1500(1.5) + \frac{3000}{2}(1.5)^2\right]$$

$$= \frac{-1.875(10)^{-9}}{\pi r^4}$$ (Do not worry about the − sign! why?)

Now, set $\phi = \phi_{min} = 1^\circ = \frac{\pi}{180^\circ} = \frac{1.875(10)^{-9}}{\pi r^4_{mm}}$

$\Rightarrow \underline{r^{\phi}_{min} = 0.013599\ m}$

From $r^{\tau}_{min}$ & $r^{\phi}_{min}$, we pick the bigger one for the min $r$ (why?).

$\Rightarrow r_{min} = 0.031692\, m$

[Note: for "typical" values and materials, the stress usually controls; i.e. the diameter needed to satisfy the shear stress condition is usually bigger than that required for the angle of twist, as the case here].

Thus, $\boxed{D_{min} = 0.063384\, m = 63.4\, mm}$

## Problem #5

Given:

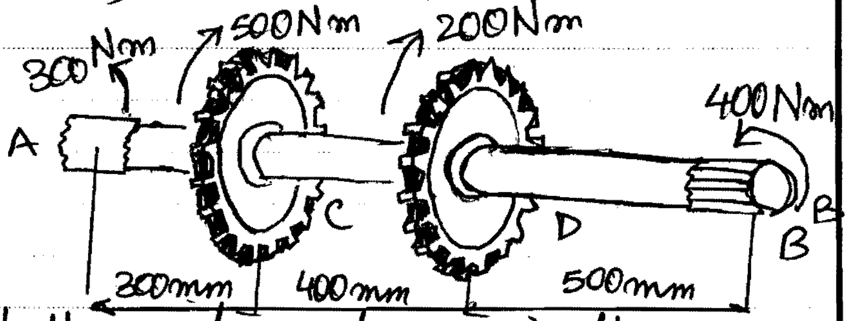
The figure shown,

$D_{out} = 50mm$ ; $D_{in} = 30mm$ ; $G = 100\ GPa$

Required:

$\tau$ in CD

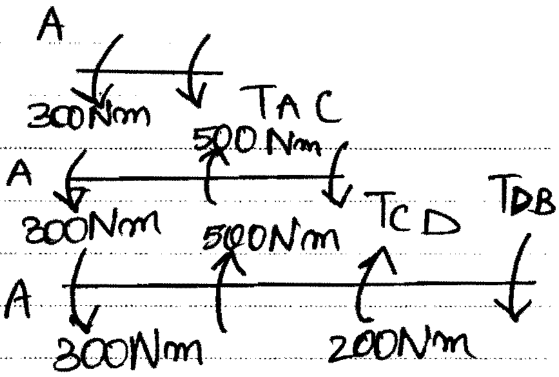
$\phi_{B/A}$




Solution:-

First, we need to find the internal T in the 3 segments AC, CD and DB. (why 3 segments?)

From the FBD's




$T_{AC} = -300Nm$

$T_{CD} = 500-300 = 200Nm$

$T_{DB} = -300+500+200 = 400Nm$

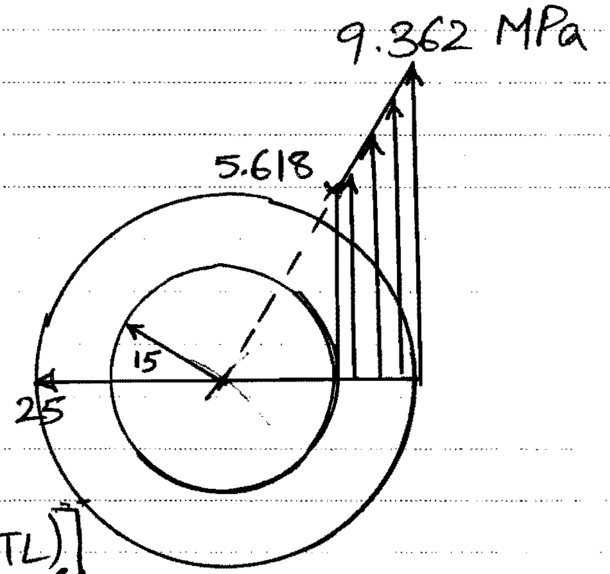
$$\tau = \frac{Tr}{J}$$

$$J = \pi/2 \left[\left(\frac{50}{2}\right)^4 - \left(\frac{30}{2}\right)^4\right]\left[(10)^{-3}\right]^4 = 1.7(10)^{-7}\pi\ m^4$$

$$\tau_{in}^{CD} = \frac{T_{CD} r_{in}}{J} = \frac{200(15)(10)^{-3}}{1.7(10)^{-7}\pi} = \boxed{\tau_{in}^{CD} = 5.617MPa}$$

$$\tau_{CD\ out} = \frac{T_{CD} r_{out}}{J} = \frac{200(25)}{1.7(10)^{-7}\pi} \Rightarrow \boxed{\tau_{out}^{CD} = 9.362MPa}$$

It has a linear distribution as shown in the figure.




$$\phi_{B/A} = \phi_{B/D} + \phi_{D/C} + \phi_{C/A}$$

Since T, J and G are constant in all 3 segments, we can use the formula $\phi = \frac{TL}{JG}$ directly.

$$\phi_{B/A} = \frac{1}{JG}\left[(TL)_{BD} + (TL)_{DC} + (TL)_{CA}\right]$$

$$= \frac{1}{1.7(10)^{-7}\pi(100)(10)^9}\left[400(0.5) + 200(0.4) + (-3000)(0.3)\right]$$

$$\boxed{\phi_{B/A} = 3.558(10)^{-3}\ \text{rad} = 0.2038^{0}}$$