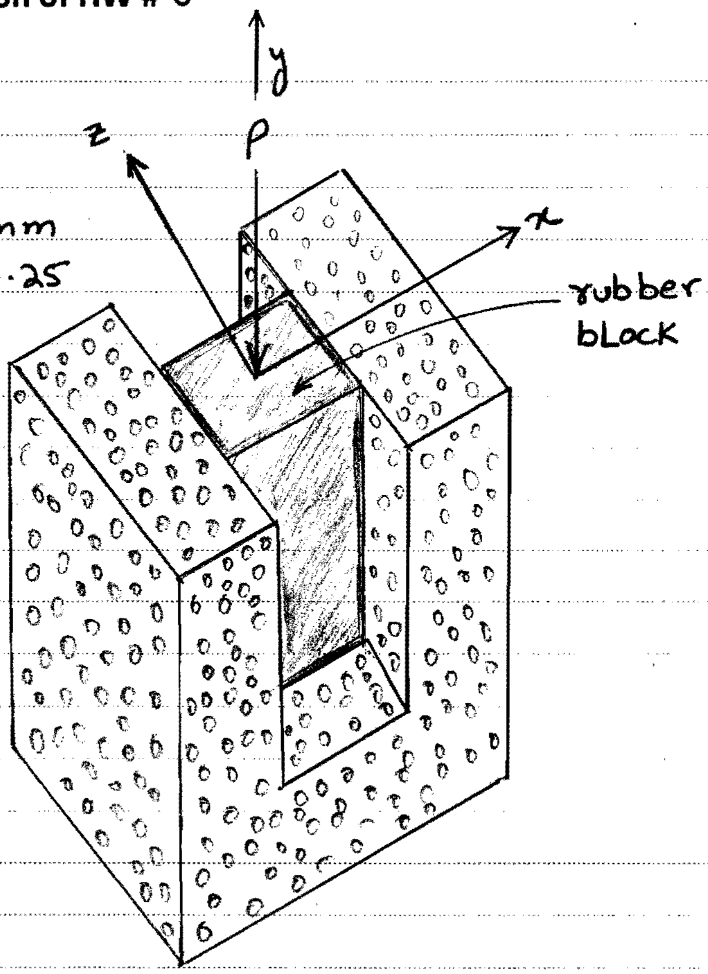
Problem #1:

Given:

The figure show
Rubber block: 100x100x100 mm
$P = 20$ kN, $E = 5$ MPa, $\nu = 0.25$



Required:

* $\sigma_x, \sigma_y, \sigma_z$
* V_f

Solution:

From the boundary conditions shown in the figure, we can conclude that

$$\boxed{\sigma_z = 0}$$
$$\varepsilon_x = 0$$

From the given data;

$$\sigma_y = -\frac{P}{A} = -\frac{20(10)^3}{0.1 \times 0.1} \Rightarrow \boxed{\sigma_y = -2\,\text{MPa} = 2\,\text{MPa "C"}}$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \Rightarrow$$

$$0 = \frac{1}{5(10)^6}\left[\sigma_x - 0.25\{(-2)(10)^6 + 0\}\right] \Rightarrow$$

$$\boxed{\sigma_x = -0.5\,\text{MPa} = 0.5\,\text{MPa "C"}}$$

To determine the final volume, we need to calculate the the final dimensions using the strains.

$\varepsilon_x = 0 \Rightarrow \Delta L_x = 0 \Rightarrow L_{xf} = 100\ mm$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$
$$= \frac{1}{5(10)^6}\left[-2(10)^6 - 0.25(-0.5)(10)^6 + 0\right]$$
$$= -0.375\ mm/mm$$

$$\varepsilon_y = \frac{\Delta L_y}{L_{y_o}} \Rightarrow \Delta L_y = -0.375(100) = -37.5\ mm$$
$$L_f = L_o + \Delta L \Rightarrow L_{yf} = 100 - 37.5 = 62.5\ mm$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$
$$= \frac{1}{5(10)^6}\left[0 - 0.25(-0.5 - 2)(10)^6\right]$$
$$= +0.125\ mm/mm$$

$$\Delta L_z = \varepsilon_z (L_z)_o = 0.125(100) = 12.5\ mm \Rightarrow$$
$$L_{zf} = 100 + 12.5 = 112.5\ mm$$

$$V_f = 100(62.5)(112.5) \Rightarrow \boxed{V_f = 703,125\ mm^3}$$

Note the % change in $V = \frac{V_f - V_o}{V_o}(100) \approx -30\%$

[This "big change" is due to the material being "weak" (rubber)].

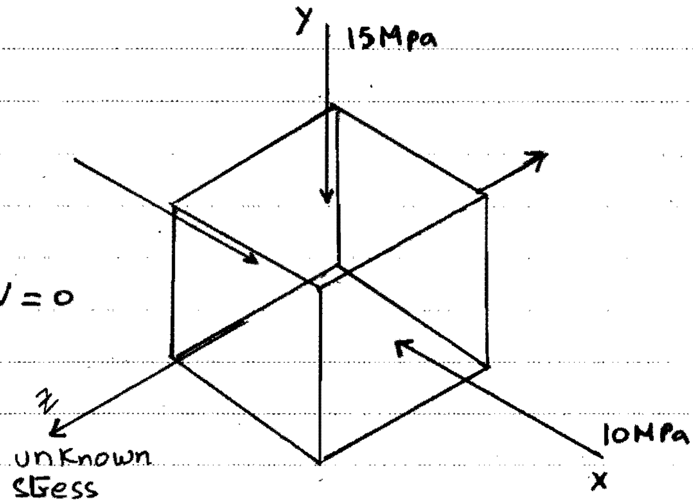
Problem # 2:

Given:

The figure shown

$\Delta V = 0$

$E = 50$ Mpa ; $\nu = 0.2$



Required:

σ_z to satisfy the condition $\Delta V = 0$

Solution:

I) We can use eq. 10-23 (P.511) in your text book directly!!

Since $\Delta V = 0$, the dilatation $= e = \frac{\Delta V}{V} = 0 \Rightarrow$

$$e \approx \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) \Rightarrow$$

$$0 = \frac{1-2(0.2)}{50(10)^6}(-10-15+\sigma_z) \Rightarrow \boxed{\sigma_z = 25 \text{ MPa "T"}}$$

Note that E and ν are not really needed in this problem!

II) Alternate Solution II

If we use eq. 10-22 (P.510), then

$$e = \frac{\delta V}{dV} \approx \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$= \frac{1}{E}[-10 - 0.2(-15 + \sigma_z)] = \frac{1}{E}(-7 - 0.2\sigma_z)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$= \frac{1}{E}\left[-15 - 0.2(-10 + \sigma_z)\right] = \frac{1}{E}(-13 - 0.2\sigma_z)$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$= \frac{1}{E}\left[\sigma_z - 0.2(-10 - 15)\right] = \frac{1}{E}(5 + \sigma_z)$$

Adding, $e = \frac{1}{E}\left[-7 - 13 + 5 + (-0.2 - 0.2 + 1)\sigma_z\right] \equiv 0$

$\Rightarrow \sigma_z = \frac{15}{0.6} \Rightarrow \sigma_z = 25\,MPa$ "T"

III Alternate Solution III

Since the e formulas used above are not "exact" as higher order terms in strains are neglected, we can use the exact formula. ⇒

$$e = \frac{\Delta V}{V_o}$$

$$= \frac{V_f - V_o}{V_o} = \left[\frac{(L_x + \Delta L_x)(L_y + \Delta L_y)(L_z + \Delta L_z) - L_x L_y L_z}{L_x L_y L_z}\right]$$

Since $\varepsilon = \frac{\Delta L}{L}$, $\Delta L = \varepsilon L \Rightarrow$

$$e = \frac{(L_x + \varepsilon_x L_x)(L_y + \varepsilon_y L_y)(L_z + \varepsilon_z L_z) - L_x L_y L_z}{L_x L_y L_z}$$

$$= \frac{L_x(1+\varepsilon_x) L_y(1+\varepsilon_y) L_z(1+\varepsilon_z) - L_x L_y L_z}{L_x L_y L_z}$$

$$e = \frac{L_x L_y L_z (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z) - L_x L_y L_z}{L_x L_y L_z}$$

$$= \frac{\cancel{L_x L_y L_z}\left[(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z) - 1\right]}{\cancel{L_x L_y L_z}}$$

$$= (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z) - 1 \qquad ①$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \qquad ②$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \qquad ③$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] \qquad ④$$

Then, we replace eqs ②,③,④, into eq ① after putting the σ_x and σ_y values => set $e = 0$ =>

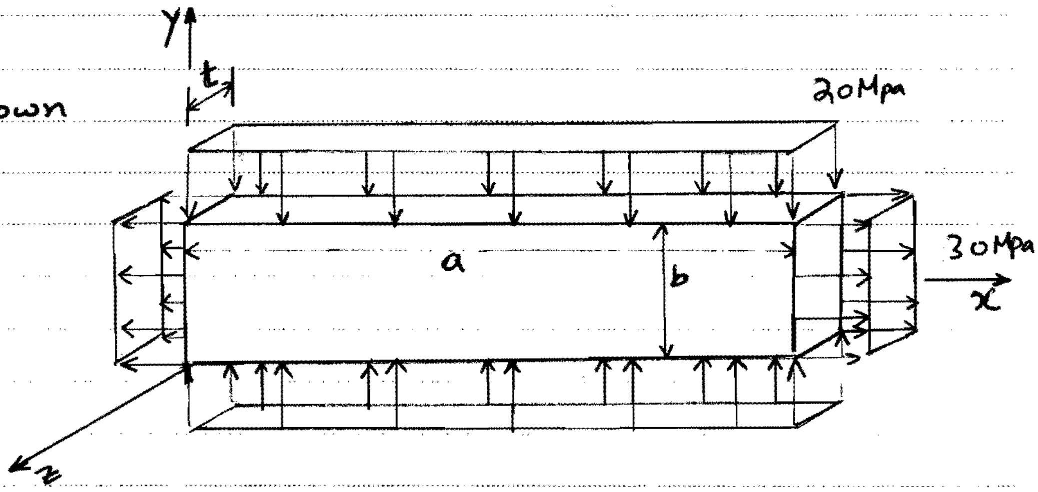
Find σ_z.

See how lengthy this method is compared with the one-line solution in method I. This "exact" method is not worth doing as the approximate formula is good enough for small strains.

Problem # 3:

Given:

The figure shown
Free in Z
$a = 500mm$,
$b = 100 mm$,
$t = 80mm$
$E = 100$ Mpa
$\nu = 0.2$



Required:
final dimensions and final volume.

Solution:

$$a_f = a_o + \Delta a$$

$$\varepsilon_x = \frac{\Delta a}{a_o} \Rightarrow \Delta a = a_o \varepsilon_x$$

Similerly, $\Delta b = b_o \varepsilon_y$, $\Delta t = t_o \varepsilon_z$

Thus, $a_f = a_o + a_o \varepsilon_x = a_o(1+\varepsilon_x)$
$b_f = b_o(1+\varepsilon_y)$
$t_f = t_o(1+\varepsilon_z)$

Therefore we need to find $\varepsilon_x, \varepsilon_y, \varepsilon_z \Rightarrow$

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$\sigma_x = 30$ Mpa ; $\sigma_y = -20$ Mpa ; $\sigma_z = 0$

$$\Rightarrow \varepsilon_x = \frac{1}{100(10)^6}\left[30(10)^6 - 0.2(-20)(10)^6\right]$$
$$= 0.34 \text{ mm/mm}$$

$$\varepsilon_y = \frac{1}{100(10)^6}\left[-20 - 0.2(30)(10)^6\right]$$
$$= -0.26 \text{ mm/mm}$$

$$\varepsilon_z = \frac{1}{100(10)^6}\left[-0.2(30-20)(10)^6\right]$$
$$= 0.02 \text{ mm/mm}$$

Thus,
$$a_f = 500(1+0.34) \Rightarrow \boxed{a_f = 670 \text{ mm}}$$
$$b_f = 100(1-0.26) \Rightarrow \boxed{b_f = 74 \text{ mm}}$$
$$t_f = 80(1-0.02) \Rightarrow \boxed{t_f = 78.4 \text{ mm}}$$

$$V_f = a_f\, b_f\, t_f = 670(74)(78.4) \Rightarrow$$

$$\boxed{V_f = 3{,}887{,}072 \text{ mm}^3}$$

Extra Note:

$$\% \text{ change in Volume} = \frac{V_f - V_o}{V_o} \times 100 = \frac{3{,}887{,}072 - (500)(100)(80)}{500(100)(80)}$$
$$= -2.82\,\%$$

Note that the % change in volume is relatively "big" as the material is weak "due to E value in MPa"