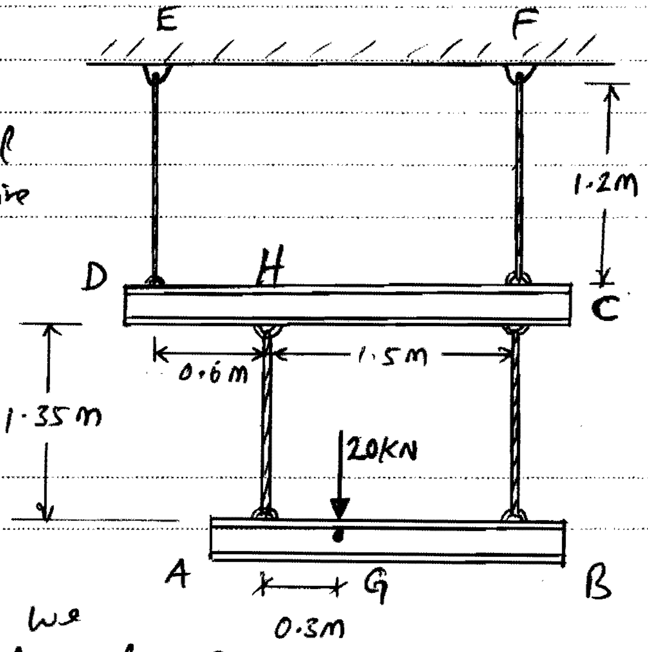
Problem #1

Given:

The figure shown

Material: 304 stainless steel

$A = 30 \, mm^2$ each wire



Required:

The displacement of G (at the load)

Solution:

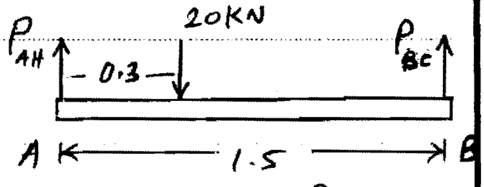
In order to determine the displacement of G, first we need to calculate the elongations (e) of the four wires. (Why?)

$$e_{wire} = \delta = \frac{PL}{AE}$$

L, A and E are known. $E = 193$ GPa (from the table at the end of the textbook)

We need to determine P for each. Since the problem is statically determinate. (Why and how?!), we can use "Statics" to find it. ⇒

In FBD ① (Why start with AB ?!),



FBD ①

$$+\circlearrowleft \sum M_B = 0 \Rightarrow$$

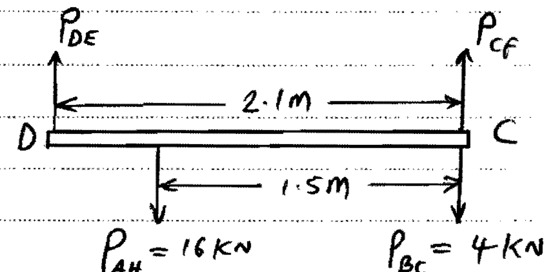
$$20(1.5 - 0.3) - P_{AH}(1.5) = 0$$

$$\Rightarrow P_{AH} = 16 \text{ KN}$$

$$+\uparrow \sum F_y = 0 \Rightarrow 16 - 20 + P_{BC} = 0$$

$$\Rightarrow P_{BC} = 4 \text{ KN}$$

In FBD (2) (Note the directions of P_{AH} and P_{BC}),



FBD (2)

$+\circlearrowleft\ \Sigma M_C = 0 \Rightarrow$

$16(1.5) - 2.1(P_{DE}) = 0$

$\Rightarrow P_{DE} = 11.429$ KN

$+\uparrow\ \Sigma F_y = 0 \Rightarrow P_{CF} + 11.429 - 16 - 4 = 0 \Rightarrow$

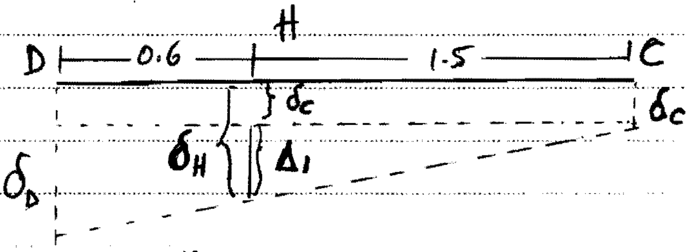
$P_{CF} = 8.5714$ KN

E and F are fixed. Thus, the displacement of D is equal to the elongation of DE $\Rightarrow$

$$\delta_D = \ell_{DE} = \left(\frac{PL}{AE}\right)_{DE} = \frac{11429\,(1.2)}{30(10^{-6})(193)(10)^9} = 2.3687\,(10)^{-3}\ \text{m}$$

$$\delta_C = \ell_{CF} = \frac{8571.4\,(1.2)}{30(10)^{-6}(193)(10)^9} = 1.7765\,(10)^{-3}\ \text{m}$$

From the geometry drawing shown,



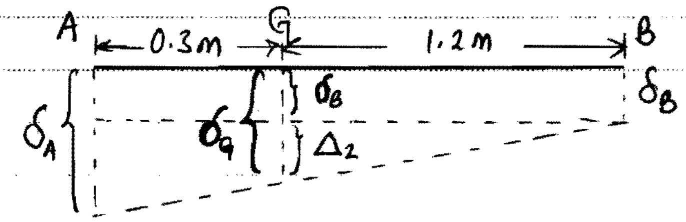
$\delta_H = \delta_c + \Delta_1$ (why needed?!)

$\delta_c = 1.7765\,(10)^{-3}$ m

$$\frac{\Delta}{\delta_D - \delta_C} = \frac{1.5}{2.1} \Rightarrow \Delta_1 = 4.230\,(10)^{-4}\ \text{m}$$

$\Rightarrow \delta_H = 2.1995\,(10)^{-3}$ m

In the figure shown,



$\delta_A = \delta_H + \ell_{AH}$

$\delta_B = \delta_c + \ell_{BC}$

$\delta_{load} = \delta_G = \delta_B + \Delta_2$

$$\frac{\Delta_2}{\delta_A - \delta_B} = \frac{1.2}{1.5}$$

$$\Rightarrow \delta_A = 2.1995\,(10)^{-4} + \left(\frac{16000\,(1.35)}{30(10)^{-6}(193)(10)^9}\right)$$

$\Rightarrow \delta_A = 5.9361 (10)^{-3}\, m$

$\delta_B = 1.7765 (10)^{-3} + \left(\dfrac{4000\,(1.35)}{30(10)^{-6}(193)(10)^{9}} \right)$

$= 2.7091 (10)^{-3}\, m$

$\Rightarrow \Delta_2 = 2.5768 (10)^{-3}\, m$

$\Rightarrow \boxed{\delta_{load} = 5.286 (10)^{-3}\, m = 5.286\ mm}$

CE 203 – 112

Solution of HW #4

Problem #2:

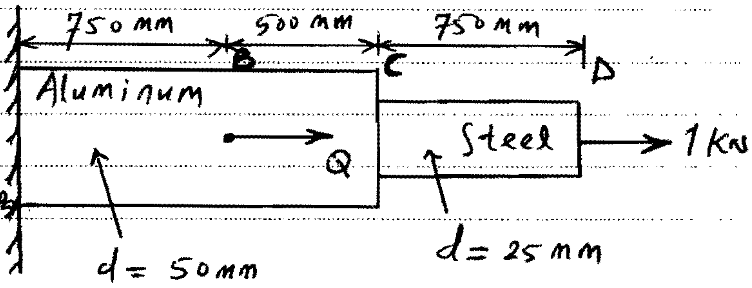
Given:

The figure shown

δ_{max} at right end $= 0.025\,mm$

σ_{max} in steel and aluminum $= 25\,MPa$

$E_{st} = 210\,GPa$; $A_{al} = 70\,GPa$



Required:

Q_{max}

Solution:

Two criteria must be satisfied (δ_{max} and σ_{max}).

Start with δ_{max} criterion $\Rightarrow$ find Q^{δ}_{max}

Then σ_{max} criterion $\Rightarrow$ find Q^{σ}_{max}

$\Rightarrow$ Choose $\min(Q^{\delta}_{max},\ Q^{\sigma}_{max})$ for Q_{max}. (Why?!)

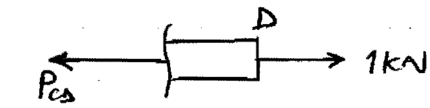
$\delta_D = \sum e_i$ (as left end is fixed)

$$\delta_D = e_{st} + e_{al}$$

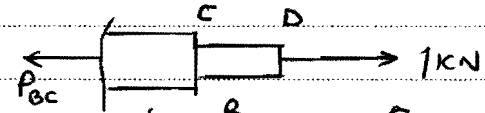
$$= \left(\frac{PL}{AE}\right)_{CD} + \left(\frac{PL}{AE}\right)_{BC} + \left(\frac{PL}{AE}\right)_{AB}$$

We need to get the internal forces from the FBD's. $\Rightarrow$

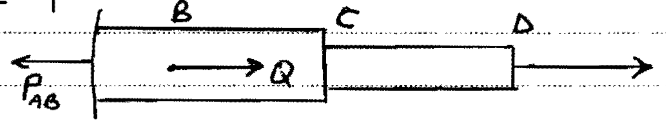
$P_{CD} = 1\,KN$ "T"



$P_{BC} = 1\,KN$ "T"



$P_{AB} = 1 + Q$ "T"



Note that the al has two segments/parts/elements.

$$\Rightarrow \delta_D = \left(\frac{1000\,(0.75)}{\pi/4\,(0.025)^2(210)(10)^9}\right) + \left(\frac{1000\,(0.5)}{\pi/4\,(0.05)^2(70)(10)^9}\right) + \left(\frac{(1000+Q)\,(0.75)}{\pi/4\,(0.05)^2(70)(10)^9}\right)$$

$$= 7.27565(10)^{-6} + 3.6378\,(10)^{-6} + 5.45674(10)^{-6} + 5.45674(10)^{-9}Q$$

$$= 1.63702\,(10)^{-5} + 5.45674(10)^{-9}\,Q$$

Set $\delta_D \equiv \delta_{max} = 0.025(10)^{-3} \Rightarrow$ get Q^{δ}_{max}

$\Rightarrow \underline{\underline{Q^{\delta}_{max} = 1581\ N}}$

Now the stress criterion

$\sigma_{st} = \sigma_{CD} = \dfrac{1000}{\pi/4\,(0.025)^2} = 2.037\ MPa < 2.5$ $\underline{\underline{ok}}$

Note that $\sigma_{st} \neq F(Q)$

$\sigma_{Al}^{1} = \sigma_{BC} = \dfrac{1000}{\pi/4\,(0.05)^2} = 0.5093\ MPa < 2.5$ $\underline{\underline{ok}}$

$\sigma_{Al}^{2} = \sigma_{AB} = \dfrac{1000 + Q}{\pi/4\,(0.05)^2} \Rightarrow$

set $\sigma_{al}^{2} = \sigma_{max} = 2.5\ MPa \Rightarrow$ get Q^{σ}_{max}

$1000 + Q^{\sigma}_{max} = 2.5(10)^6(\pi/4)(0.05)^2 \Rightarrow$

$Q^{\sigma}_{max} = 3909\ N$

$Q_{max} = \min(Q^{\delta}_{max}, Q^{\sigma}_{max})$

$\Rightarrow \boxed{Q_{max} = 1581\ N}$

Note that this problem is statically determinate so geometric compatibility was not used.

Problem # 3

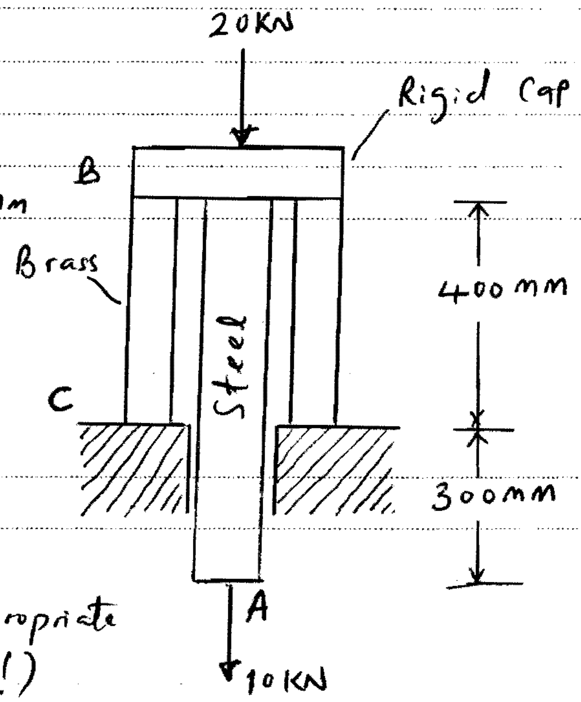
Given:

The figure shown

$D_{steel} = 12\ mm$

$D_{brass\ out} = 30\ mm$; $D_{brass\ in} = 20mm$

$E_{st} = 210\ GPa$; $E_{br} = 105\ GPa$



Required:

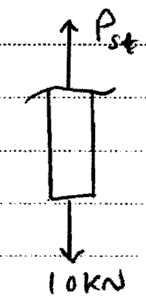
a) δ_A

b) σ_{st} and σ_{br}

Solution:

a) $\delta_A = e_{st} + e_{Br}$ (with appropriate signs !!)

$$e_{st} = \left(\frac{PL}{AE}\right)_{st}$$



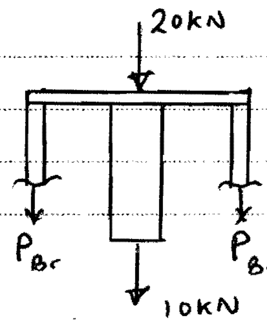
$P_{st} = 10\ KN$ "T"

$$e_{st} = \left[\frac{10(10)^3(0.7)}{\pi/4\,(0.012)^2(210)(10)^9}\right]$$

$= 2.9473\,(10)^{-4} m$ "extension"

P_{Br} (Total) $= -30\ KN$ "C"

$$e_{Br} = \left[\frac{-30\,(10)^3(0.4)}{\pi/4\,\{(0.03)^2 - (0.02)^2\}(105)(10)^9}\right]$$



$= -2.9103\ m$ "Contraction"

Note that the steel extension will move point A down, and the brass Contraction will also move point A down !! $\Rightarrow$

$\delta_A = (2.9473 + 2.9103)(10)^{-4}\ m \Rightarrow$

$$\boxed{\delta_A = 0.5858\ mm\ \text{down}}$$

b) $\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{10(10)^3}{\pi/4(0.012)^2} \Rightarrow \boxed{\sigma_{st} = 88.42 \text{ MPa } \text{"T"}}$

$$\sigma_{Br} = \frac{P_{Br}}{A_{Br}} = \frac{-30(10)^3}{\pi/4\left[(0.03)^2 - (0.02)^2\right]}$$

$$\Rightarrow \boxed{\sigma_{Br} = -76.39 = 76.39 \text{ MPa } \text{"C"}}$$

Note that this problem is statically determinate so geometric compatibility was not used.

Problem # 4:

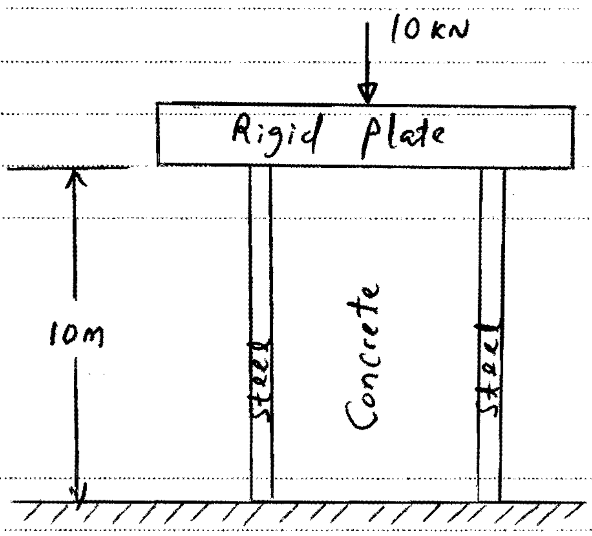
Given:

The figure shown

$D_{st\,out} = 4.5\,m$; $D_{st\,in} = 4.026\,m$

Concrete fill

$E_{st} = 210\,GPa$; $E_{con} = 21\,GPa$



Required:

σ in steel and concrete

Solution:

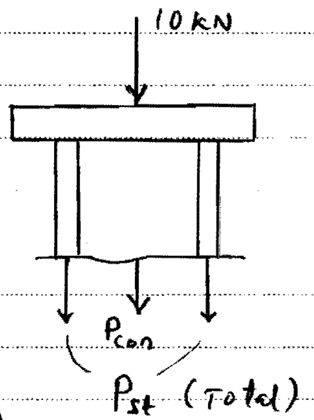
Note that the problem is statically indeterminate (SI) as there are two unknowns (P_{st} and P_{con}) and only one equilibrium equation. (why ?!). Thus, we must use the geometric compatibility concept to be able to solve SI problems.

First, start with

① Equilibrium

$+\downarrow \Sigma F_y = 0 \Rightarrow$

$$10 + P_{st} + P_{con} = 0 \qquad ①$$



[Note that internal forces are assumed "T" why ?!]

Then

② Geometric Compatibility

$$e_{st} = e_{con} \qquad ② \text{ (why ?!)}$$

Now,

③ Material properties,

$$e = \frac{PL}{AE} \qquad ③$$

From eq. ③ into eq. ②,

$$\left(\frac{PL}{AE}\right)_{st} = \left(\frac{PL}{AE}\right)_{con}$$

$$\frac{P_{st}\,(10)}{\pi/4\left[(4.5)^2-(4.026)^2\right](210)(10)^9} = \frac{P_{con}\,(10)}{\pi/4\,(4.026)^2(21)(10)^9} \Rightarrow$$

$$P_{st} = 2.4933\,P_{con} \quad (4)$$

From eq. (4) into eq. (1), $\Rightarrow$

$$10 + 2.4933\,P_{con} + P_{con} = 0 \Rightarrow$$

$$P_{con} = -2.8626 \text{ KN} \quad \text{"C"}$$

$$\Rightarrow P_{st} = -7.1374 \text{ KN} \quad \text{"C"}$$

$$\sigma = P/A \Rightarrow$$

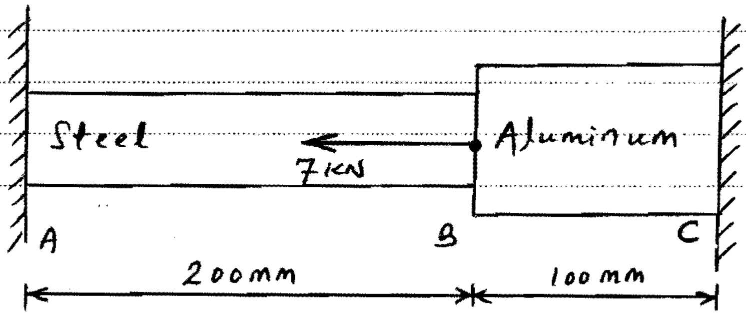
$$\sigma_{st} = \frac{-7.1374\,(10)^3}{\pi/4\left[(4.5)^2-(4.026)^2\right]} \Rightarrow \boxed{\sigma_{st} = 2.249 \text{ KPa} \quad \text{"C"}}$$

$$\sigma_{con} = \frac{-2.8626\,(10)^3}{\pi/4\,(4.026)^2} \Rightarrow \boxed{\sigma_{con} = 224.9 \text{ Pa} \quad \text{"C"}}$$

"Small" Stresses !

Problem #5:

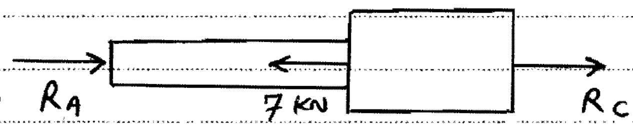
Given:
The figure shown
$A_{st} = 200\ mm^2$
$E_{st} = 210\ GPa$
$A_{al} = 400\ mm^2$
$E_{al} = 70\ GPa$



Required:
σ_{st} and σ_{al}

Solution:
This problem is like #4 as it is SI and geom. Compat. is needed. (show !!)

① Equilibrium
From the FBD,
$\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow$
$R_A + R_C - 7(10)^3 = 0$ ①

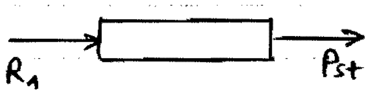


② Geom. Compat.
$$\Sigma e = e_{st} + e_{al} = 0 \quad ②$$ (why?!)

③ Material Behaviour
$$e = \frac{PL}{AE} \quad ③$$

From ③ into ②
$$\left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{al} = 0 \quad ④$$

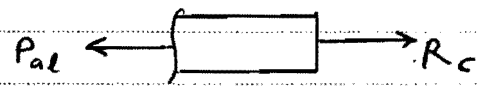
$$P_{st} = -R_A \quad ⑤$$



(assumed "T", why?!)

$P_{al} = R_c$ (6)

Thus, from (5) and (6) into (4)



$$\left[\frac{-R_A(0.2)}{200(10)^{-6}(210)(10)^9} + \frac{R_c(0.1)}{400(10)^{-6}(70)(10)^9}\right] = 0$$

$\Rightarrow \quad R_A = 0.75 R_c$ (7)

From eq. (7) into eq. (1),

$(1.75) R_c = 7(10)^3 \quad \Rightarrow \quad R_c = 4(10)^3 \text{ N}$

$\Rightarrow \quad R_A = 3(10)^3 \text{ N}$

Thus, $\sigma_{st} = \dfrac{P_{st}}{A_{st}} = \dfrac{-R_A}{A_{st}}$

$= \dfrac{-3(10)^3}{200(10)^{-6}} \Rightarrow$

$$\boxed{\sigma_{st} = -15 \text{ MPa} = 15 \text{ MPa} \quad \text{"C"}}$$

$\sigma_{al} = \dfrac{P_{al}}{A_{al}} = \dfrac{R_c}{A_{al}} = \dfrac{4(10)^3}{400(10)^{-6}} \Rightarrow$

$$\boxed{\sigma_{al} = 10 \text{ MPa} \quad \text{"T"}}$$

Are the answers reasonable? Why?! Think !!!