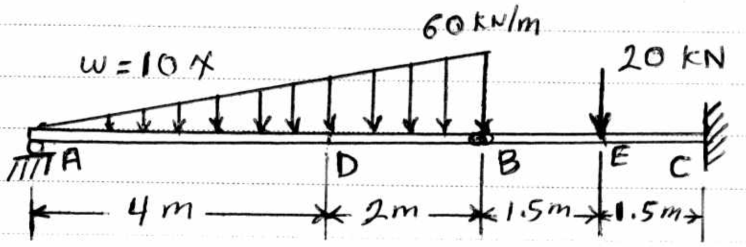
Problem # 1:

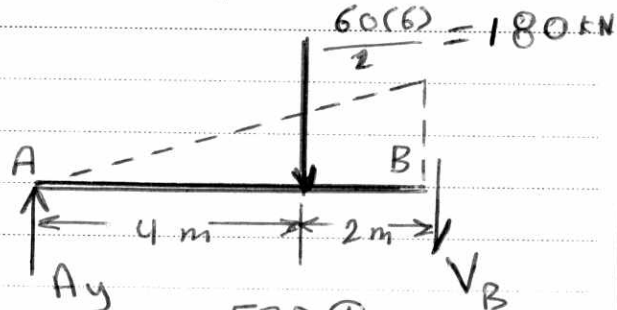
Given:
The figure shown



Required:
The internal forces at D and E. E is just to the right of the 20-kN load.

Solution:
First, we need to find the reactions; at least the one at A. (Why ?!)
Note that we need to separate the beam from the internal hinge/pin at B in order to find the reaction(s). (Why ?!)

In FBD①,
$\curvearrowleft + \sum M_B = 0 \Rightarrow$
$180(2) - 6A_y = 0$
$\Rightarrow A_y = 60\ kN$

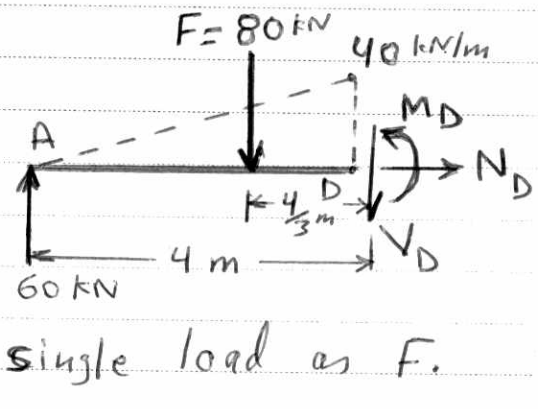


FBD①

Now, we can take a section through D and take the left part, as shown in FBD②.

FBD②

$F = 40\left(\frac{4}{2}\right) = 80\ kN$



Note that if the point is within a distributed load, then we "cut" first after that we replace the distributed load on that part only by an equivalent single load as F.

$\xrightarrow{+} \sum F_x = 0 \Rightarrow \boxed{N_D = 0}$

$+\uparrow \Sigma F_y = 0 \Rightarrow$

$60 - 80 - V_D = 0 \Rightarrow \boxed{V_D = -20 \text{ kN}}$

$\circlearrowleft + \Sigma M_D = 0 \Rightarrow M_D + 80(\tfrac{4}{3}) - 60(4) = 0 \Rightarrow$

$$\boxed{M_D = 133\tfrac{1}{3} \text{ kN.m}}$$

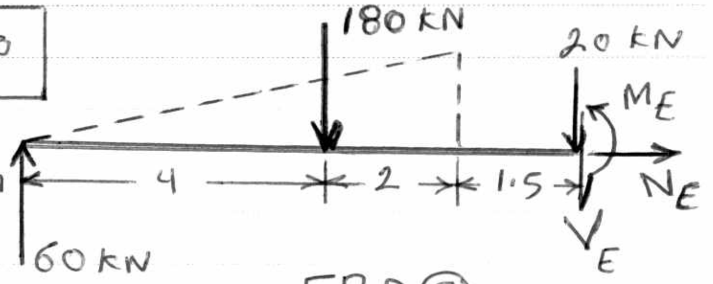
Now, we determine the internal forces @ E. Taking the left part, FBD ③ is drawn.

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow \boxed{N_E = 0}$

$+\uparrow \Sigma F_y = 0 \Rightarrow$

$60 - 180 - 20 - V_E = 0$

$\Rightarrow \boxed{V_E = -140 \text{ kN}}$



FBD③

$\circlearrowleft + \Sigma M_D = 0 \Rightarrow M_E + 180(3.5) - 60(7.5) = 0 \Rightarrow$

$$\boxed{M_D = -180 \text{ kN.m}}$$

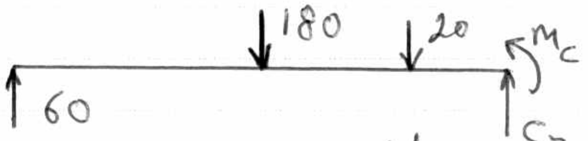
Extra

If the right part is chosen, then the FBD can be drawn after determining the reactions at C. This can be done by taking the FBD of the whole beam after calculating A_y. $\Rightarrow$

$\Sigma F_y = 0 \Rightarrow C_y = 140 \text{ kN} \uparrow$

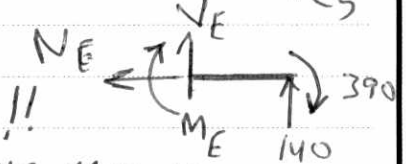
$\Sigma M_c = 0 \Rightarrow M_c = -390 \text{ kN.m}$

$= 390 \circlearrowright$



Then, the right part is drawn. $\Rightarrow$

* Note the directions of the internal forces!!

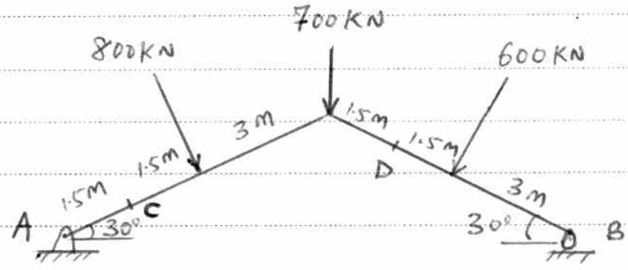


$\Sigma F_x = 0$; $\Sigma F_y = 0$; $\Sigma M_c = 0 \Rightarrow N_E = 0$, $V_E = -140$, $M_E = -180$

Problem #2

Given:

The figure shown



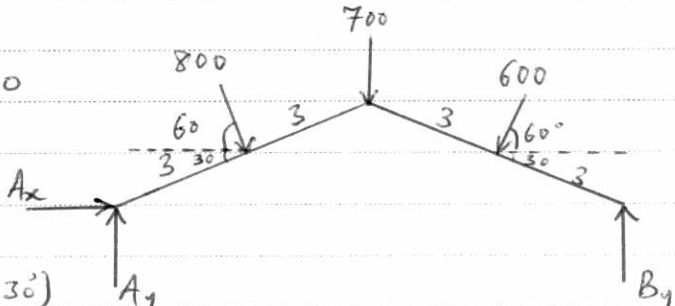
Required:

The internal forces at C and D

Solution:

We need to find the reactions first. (Why?!)

FBD for the entire structure is drawn



$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$A_x + 800 \cos 60^\circ - 600 \cos 60^\circ = 0$

$A_x = -100 \text{ KN}$

$= 100 \text{ KN} \leftarrow$

$+\circlearrowleft \Sigma M_A = 0 \Rightarrow$

$B_y(6\cos 30^\circ + 6\cos 30^\circ) - 700\,(6\cos 30^\circ)$

$+ 600 \cos 60^\circ (3 \sin 30^\circ) - 600 \sin 60^\circ (3\cos 30^\circ + 6\cos 30^\circ) - 800(3) = 0$

$B_y = 927.35 \text{ KN}$

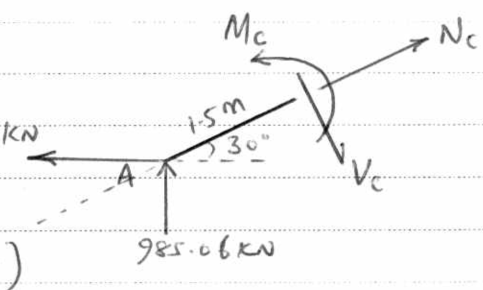
$+\uparrow \Sigma F_y = 0 \Rightarrow$

$A_y + 927.35 - 700 - 800 \sin 60^\circ - 600 \sin 60^\circ = 0 \Rightarrow$

$A_y = 985.06 \text{KN}$

Now, we make a section through point C and take the left part. (Why?!)

From the FBD,



$\nearrow^{+} \Sigma F_x = 0 \Rightarrow$

$N_c - 100 \cos 30^\circ + 985.06 \cos 60^\circ = 0$

$\Rightarrow \boxed{N_c = -405.9 \text{ KN}}$ (what does the $\ominus$ sign mean?!)

$\nwarrow^{+} \Sigma F_y = 0 \Rightarrow$

$-V_c + 100 \sin 30^\circ + 985.06 \sin 60^\circ = 0 \Rightarrow$

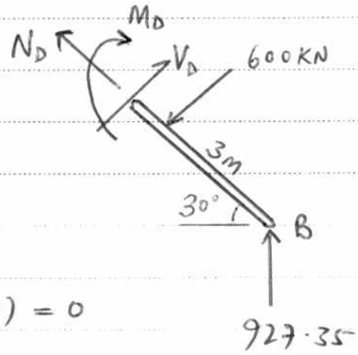
$\boxed{V_c = 903.1 \text{ KN}}$

$+\circlearrowleft \Sigma M_A = 0 \Rightarrow M_c - V_c(1.5) = 0 \Rightarrow$

$\boxed{M_c = 1355 \text{ KN.m}}$

Solution of HW # 1

For Point D, we take the right part. (Why?!)



$+\nwarrow \quad \Sigma F_x = 0 \Rightarrow N_D + 927.35 \cos 60° = 0$

$\Rightarrow \quad \boxed{N_D = -463.7 \text{ KN}}$

$+\swarrow \quad \Sigma F_y = 0 \Rightarrow -V_D + 600 - 927.35 \sin 60° = 0$

$\Rightarrow \quad \boxed{V_D = -203.1 \text{ KN}}$

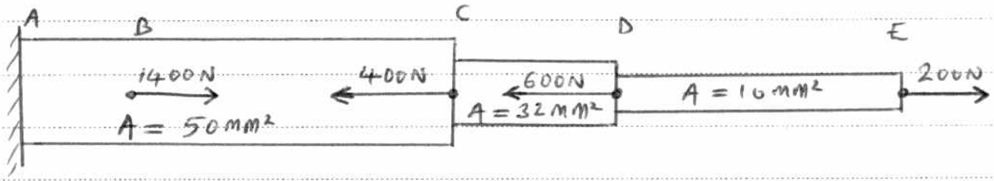
$+\circlearrowleft \quad \Sigma M_B = 0 \Rightarrow -M_D - (-203.11)(4) + 600(3) = 0$

$\Rightarrow \quad \boxed{M_D = 2612 \text{ KN.m}}$

Problem #3

Given:

The figure shown.



Required:

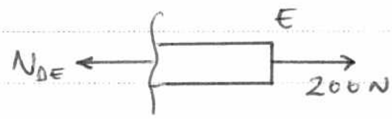
The values and locations of the maximum tensile and compressive stresses

Solution:

We need to check at 4 different locations. (why?!). Thus, we need to make 4 sections and 4 FBD's, as shown. Note that we took the right parts in all. We may take the left parts in some of them as it may be easier; however, in this case, we have to find the reaction at A.

DE

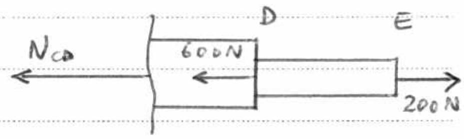
$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow \quad N_{DE} = 200\,N \ (T)$



$\sigma_{DE} = N/A = \dfrac{200}{10(10)^{-6}} = 20\,MPa\ (T)$

CD

$\xrightarrow{+} \Sigma f_x = 0 \Rightarrow \quad 200 - 600 - N_{CD} = 0$



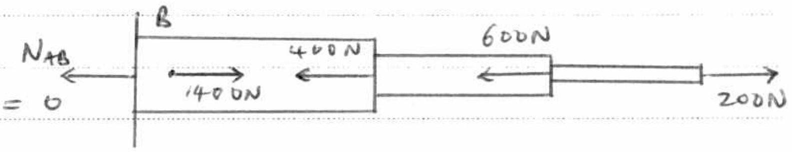
$\Rightarrow \quad N_{CD} = -400\,N = 400\,N \ (C)$

$\sigma_{CD} = \dfrac{400}{32(10)^{-6}} = 12.5\ MPa\ (C)$

BC

$\xrightarrow{+} \Sigma f_x = 0 \Rightarrow$

$200 - 600 - 400 + 1400 - N_{AB} = 0$

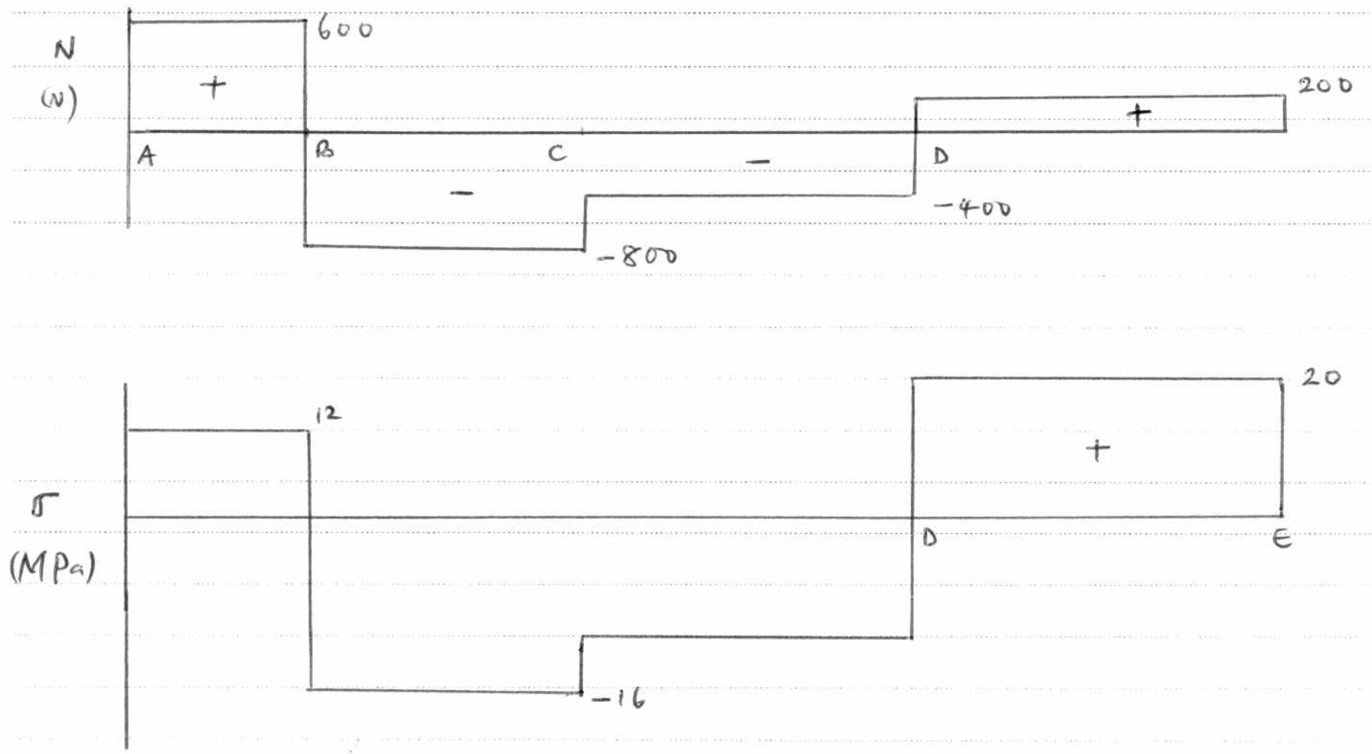


$\Rightarrow \quad N_{AB} = 600\,N \ (T)$

$\sigma_{CD} = \dfrac{600}{50} = 12\ MPa\ (T)$

Thus, $\boxed{\sigma^{T}_{Max} = 20\,MPa \text{ in } DE}$ $\boxed{\sigma^{C}_{Min} = 16\ MPa \text{ in } BC}$

As an extra work, we may draw the variation of N and σ along the members as show below


N
(N)
600
+
200
+
A
B
C
–
D
–
–400
–800
20
12
+
σ
(MPa)
D
E
–16

Problem # 4

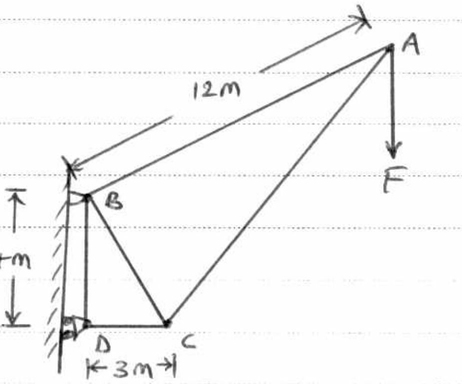
Given:

The truss shown

$A_{each} = 500\ mm^2$

$\sigma^{T}_{max} = 150\ MPa$

$\sigma^{T}_{max} = 180\ MPa$



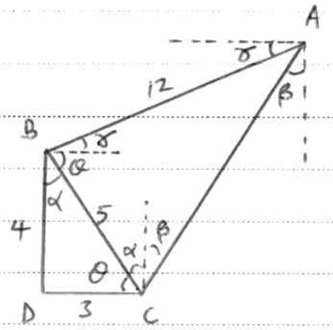
Required:

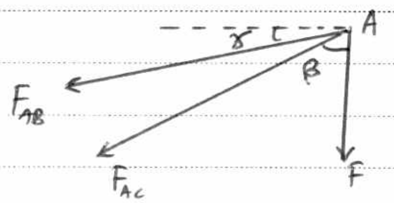
Maximum force F that can be applied

Solution:

In order to find the stresses in the members, we need to calculate the forces in these members. As you took in statics, it is better to use the method of joints and start with joint A. (why?!). Note that, no need to calculate the reactions. (why?!).

From the geometry and the FBD of joint A





$$\theta = \tan^{-1} 4/3 = 53.130^\circ$$

$$\gamma = 90 - \theta = 36.870^\circ$$

$$\alpha = 90 - \theta = 36.870^\circ$$

$$(\alpha + \beta) = \tan^{-1} \frac{12}{5} = 67.380^\circ$$

$$\Rightarrow \beta = 67.380^\circ - 36.870^\circ = 30.510^\circ$$

$$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$$

$$-F_{AB} \cos\gamma - F_{AC} \sin\beta = 0$$

$$\xrightarrow{+} \Sigma F_y = 0 \Rightarrow$$

$$-F_{AB} \sin\gamma - F_{AC} \cos\beta - F = 0$$

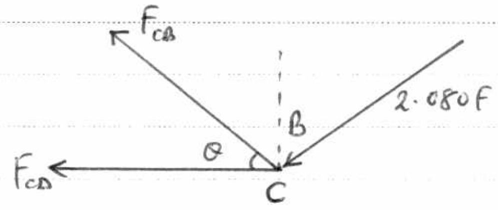
$$\Rightarrow F_{AC} = -\frac{\cos\gamma}{\sin\beta} F_{AB} \Rightarrow$$

$$-F_{AB} \sin\gamma - \frac{\cos\gamma}{\sin\beta} \cos\beta\, F_{AB} - F = 0 \Rightarrow$$

$F_{AB} = 1.320\,F$ (T)

$\Rightarrow F_{AC} = -2.080\,F$ (c)

[Note that we have to write the forces in the Members as a function (in terms) of F. Why?!]



Now, joint C:

Start with

$+\uparrow \Sigma F_y = 0$ (why?!)

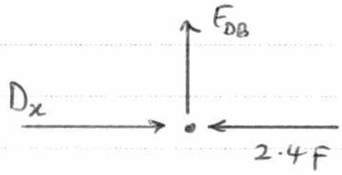
$\Rightarrow F_{CB} \sin\theta - 2.080F \cos\beta = 0$

$\Rightarrow F_{CB} = 2.240\,F$ (T)

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$

$-F_{CD} - 2.08F \sin\beta - 2.24\,F \cos\theta = 0 \Rightarrow$

$F_{CD} = -2.40F$ (C)



Now, joint D:

Note that F_{DB} is a zero-force member $\Rightarrow F_{DB} = 0$

At this stage, we calculated the forces in all Members. Since the area of all Members is the same,

σ^T_{max} will be at F^T_{Max} and

σ^c_{Max} will be at F^c_{max}. Thus,

$F^T_{Max} = F_{CB} = 2.240\,F \Rightarrow$

We set $\sigma^T_{max} \equiv \dfrac{2.24\,F_{Max}}{A} \Rightarrow$

$$150\,(10)^6 = \frac{2.24\,F_{Max}}{500\,(10)^{-6}} \Rightarrow$$

$F^{①}_{Max} = 33.48$ KN from T

$F^c_{max} = F_{CD} = 2.40F \Rightarrow$

Set $\sigma^c_{Max} = \dfrac{2.40F}{A} \Rightarrow$

$$180\,(10)^6 = \frac{2.4\,F_{max}}{500\,(10)^{-6}} \Rightarrow$$

$F^{②}_{Max} = 37.5$ KN from C

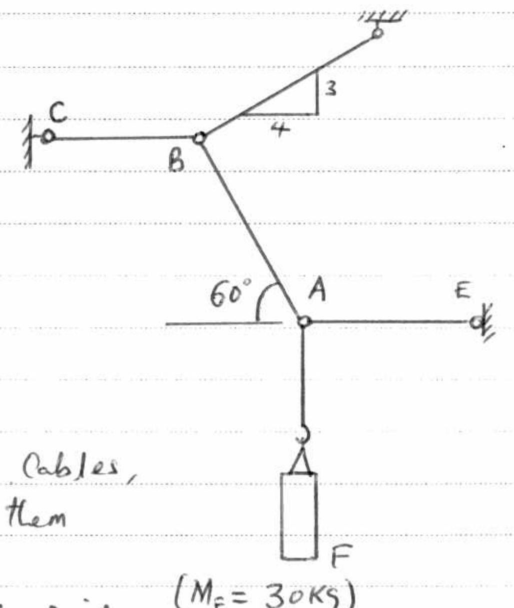
For Maximum F from $F^{①}_{Max}$ and $F^{②}_{Max}$, we choose the smaller one (why?!) $\Rightarrow$ $\boxed{F_{max} = 33.48\text{ KN}}$

Problem #5

Given:

The figure shown

$A_{cable} = 100\ mm^2$



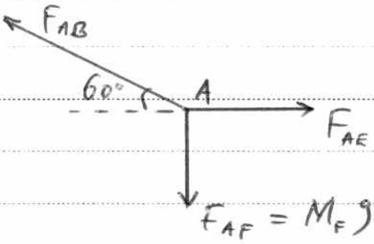
Required:

The normal stresses in the Cables

Solution:

In order to find the stresses in the Cables, we need to calculate the forces in them using "statics" Methods.

Let's start by drawing FBD at point (Particle) A. (Why?!)



$\sigma_{AF} = \frac{F_{AF}}{A} = \frac{30(9.81)}{100(10)^{-6}}$

$\Rightarrow \sigma_{AF} = 2.943\ MPa\ (T)$

$+\uparrow \Sigma F_y = 0$ (Why?!)

$\Rightarrow F_{AB} \sin 60 - 30(9.81) = 0$

$\Rightarrow F_{AB} = 339.83\ N$

$\sigma = \frac{F}{A} \Rightarrow$

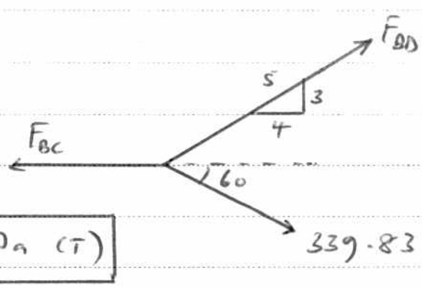
$\sigma_{AB} = \frac{339.83}{100(10)^{-6}} \Rightarrow \boxed{\sigma_{AB} = 3.398\ MPa\ (T)}$

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow F_{AE} - F_{AB} \cos 60 = 0 \Rightarrow$

$F_{AE} = 169.91\ N \Rightarrow$

$\sigma_{AE} = \frac{169.91}{100(10)^{-6}} \Rightarrow \boxed{\sigma_{AE} = 1.699\ MPa\ (T)}$

Now, we draw the FBD at B. $\Rightarrow$



$+\uparrow \Sigma F_y = 0 \Rightarrow$

$F_{BD}(3/5) - 339.83 \sin 60° = 0$

$\Rightarrow F_{BD} = 490.50\ N$

$\Rightarrow \sigma_{BD} = \frac{490.50}{100(10)^{-6}} \Rightarrow \boxed{\sigma_{BD} = 4.905\ MPa\ (T)}$

$\xrightarrow{+} \sum F_x = 0 \Rightarrow$

$339.83 \cos 60 + 490.50 (4/5) - F_{BC} = 0 \Rightarrow$

$F_{BC} = 562.32 \, N \Rightarrow$

$\sigma_{BC} = \dfrac{562.32}{100(10)^{-6}} \Rightarrow \boxed{\sigma_{BC} = 5.623 \, MPa \quad (T)}$