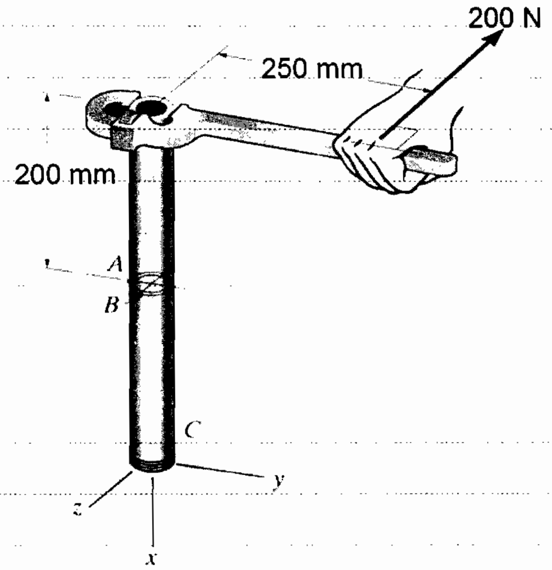
Problem #1:

Given:



The figure shown

$D_{out} = 60$ mm ; $D_{in} = 50$ mm

Required:

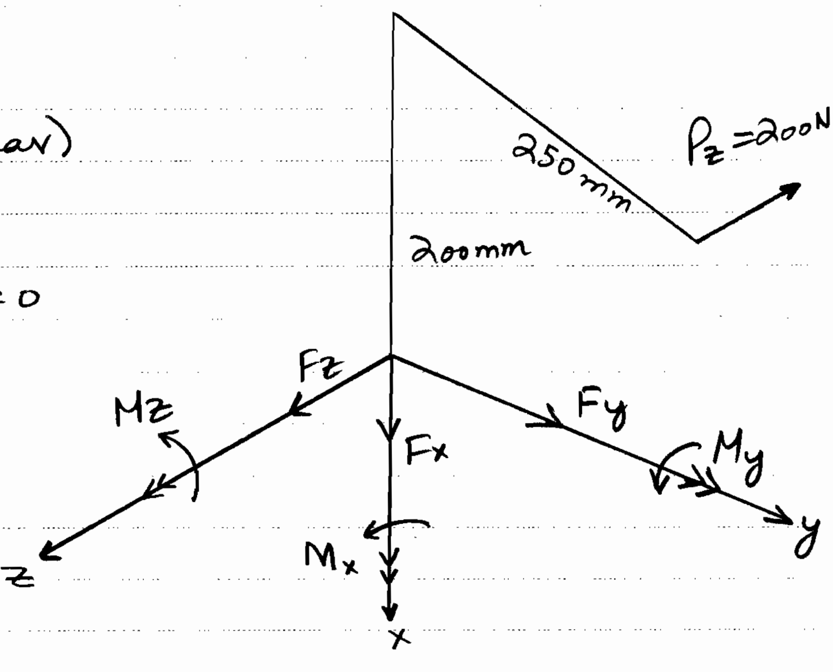
a) State of stress @ B
b) Principal normal stresses using the transformation Eqs & show the element.

Solution:

To determine the state of stress at B, we first need to calculate the internal forces at a section through Point B as shown in the FBD.

$\sum \vec{F} = 0 \Rightarrow$
$F_x = F_y = 0$
$F_z = 200$ N (shear)

$\sum M_x = 0 \Rightarrow$
$M_x - 200(0.25) = 0$

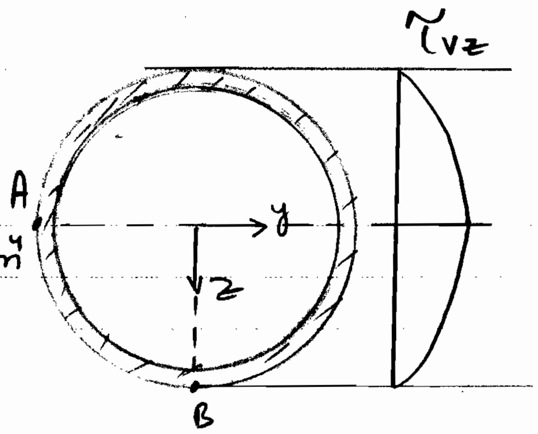


$M_x = 50 \ N.m = T$ (twisting)

$\Sigma M_y = 0 \Rightarrow \quad M_y - 200(0.2) = 0 \Rightarrow$
$M_y = 40 \ N.m$ (bending)

$\Sigma M_z = 0 \Rightarrow \quad M_z = 0$

$$\sigma_x^B = \pm \cancel{\frac{F_x}{A_x}}^{\,0} \pm \cancel{\frac{M_z y}{I_z}}^{\,0} \pm \frac{M_y z}{I_y}$$



$$I_y = \frac{\pi}{4}\left[(30)^4 - (25)^4\right] = 3.29376(10)^{-7} m^4$$

$$\Rightarrow \sigma_x^B = \frac{40\,(0.03)}{3.29376(10)^{-7}} \Rightarrow$$

$$\boxed{\sigma_x^B = 3.6432 \text{ MPa} \quad \text{"T"}}$$

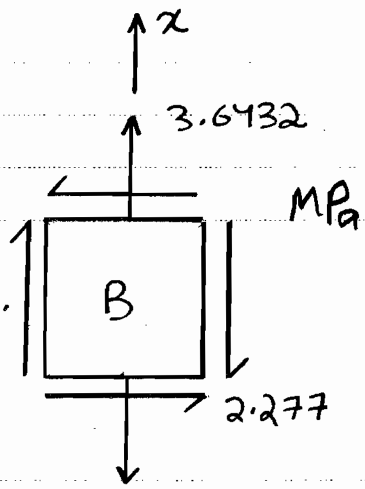
$$\tau^B = \cancel{\tau_{vz}}^{\,0} + \cancel{\tau_{vy}}^{\,0} + \tau_T \Rightarrow$$

as $Q_A = 0$ as $V_y = 0$

$$J = 2I \Rightarrow \tau^B = \frac{Tr}{J} = \frac{50(0.03)}{2(3.29376)(10)^{-7}} \Rightarrow$$

$$\boxed{\tau^B = 2.2770 \text{ MPa}}$$

The state of stress at B is as shown.



Note that all other stress components are zero. also note that τ is $\ominus$ according to your text book sign convention.

$$\sigma_{1,2\;max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{3.6432 + 0}{2} \pm \sqrt{\left(\frac{3.6432 - 0}{2}\right)^2 + (-2.277)^2}$$

$$= 1.8216 \pm 2.9160 \Rightarrow$$

$$\boxed{\sigma_1 = \sigma_{max} = 4.738 \text{ MPa "T"}}$$

$$\boxed{\sigma_2 = \sigma_{min} = -1.094 \text{ MPa "C"}}$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)} = \frac{-2.277}{\left(\frac{3.6432}{2}\right)} \Rightarrow$$

$$\boxed{\theta_{p_1} = -25.67^\circ \;;\; \theta_{p_2} = +64.33^\circ}$$

To Know θ_{p_1} Corresponds to σ_1 or σ_2, we need to Substitute its value into the "general" transformation equation to see

we get σ_1 or σ_2 $\Rightarrow$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta_{p_1} + \tau_{xy} \sin 2\theta_{p_1}$$

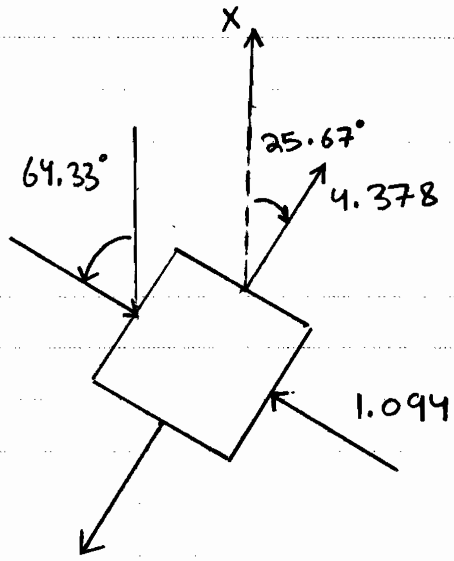
$\Rightarrow \sigma_{x_1} = 4.738$ MPa $\Rightarrow$

$\theta_{p_1} = -25.67^\circ$ is the direction of σ_1. $\Rightarrow$

The element of the principal stresses is drawn

Note that $\underline{\underline{\tau}}$ @ principal plane is always $\underline{\underline{\text{Zero}}}$.

$\underline{\underline{\text{MPa}}}$



Note that $\sigma_x + \sigma_y = 3.6432 + 0 = 3.64$

$\sigma_1 + \sigma_2 = 4.738 - 1.094 = 3.64$

Always $\sigma_{axis} + \sigma_{\perp\,axis} = \text{constant}$

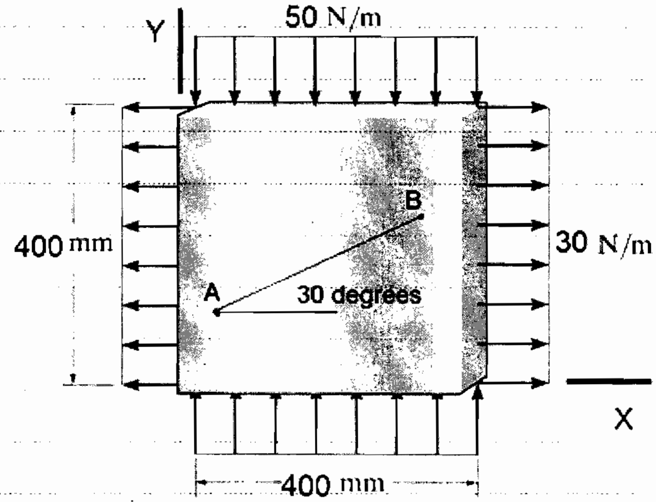
Use it to check your

answers.

Problem # 2:

Given:

The loading on the Plate shown.
$t = 20$ mm



Required:

σ & τ on plane AB using the eqs. Show the element.

Solution:

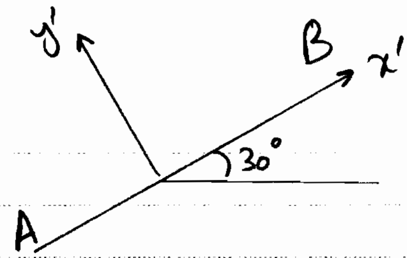
First we need to calculate the stresses.

$$\sigma_x = \frac{30\,N/m}{0.02\,m} = 1.5 \text{ KPa "T"}$$

$$\sigma_y = \frac{-50\,N/m}{0.02\,m} = -2.5 \text{ KPa "C"}$$

$$\tau_{xy} = 0$$

$$\theta = +30° \Rightarrow$$



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \cancel{\tau_{xy}}^{0} \sin 2\theta$$

$$= \frac{1.5 + (-2.5)}{2} + \frac{1.5 - (-2.5)}{2} \cos 2(30°) \Rightarrow$$

$$\boxed{\sigma_{x'} = 0.5 \text{ KPa}}$$

Note: you may use ⊖ 60° angle so that you get σ_I and $\tau_{\parallel}$ to AB

Solution of HW # 14

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{1.5 - 2.5}{2} - \frac{1.5 + 2.5}{2}\cos 60 \Rightarrow$$

$$\boxed{\sigma_{y'} = -1.5 \text{ KPa}} \longleftarrow \underline{\underline{\text{Normal to AB}}}$$

Check: $\sigma_x + \sigma_y = 1.5 - 2.5 = -1$

$\sigma_{x'} - \sigma_{y'} = 0.5 - 1.5 = -1$ $\underline{\underline{\text{OK}}}$!

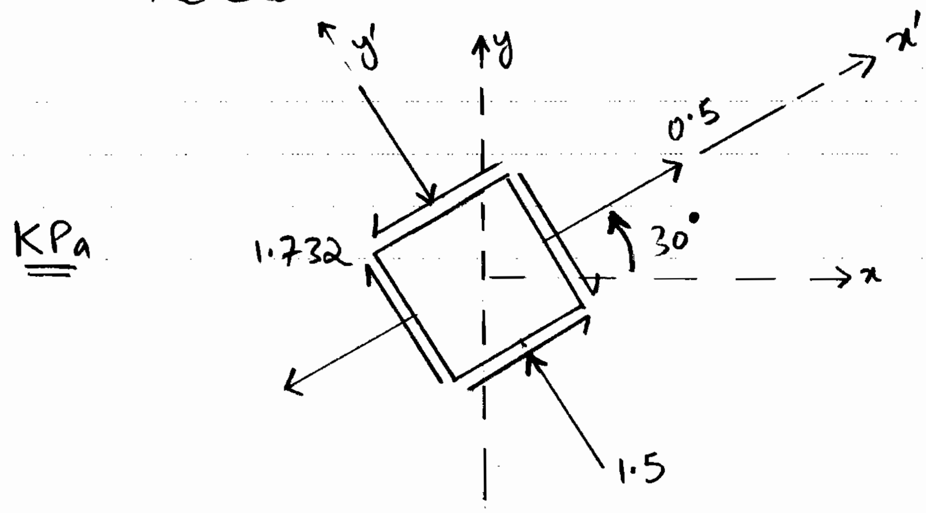
$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{1.5 + 2.5}{2}\right)\sin 60^\circ \Rightarrow$$

$$\boxed{\tau_{x'y'} = -1.732 \text{ KPa}}$$

The other $\tau_{y'x'}$ is $\boxed{\tau_{y'x'} = 1.732 \text{ KPa}}$ Parallel $\underline{\underline{\text{to AB}}}$

The element is shown.

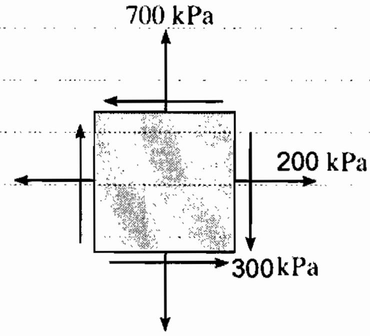


Note that $\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$ "always"

Problem # 3:

Given:

The state of stress on the element shown.



Required:

using the eqs:

i) stresses @ $\theta = 20°$ clockwise
ii) Principal stresses & direction
iii) Maximum shear & direction

Show all on properly-oriented elements.

Solution:

i) $\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$

$\sigma_x = +200$; $\sigma_y = +700$; $\tau_{xy} = -300$ "kPa"

$\theta = -20° \Rightarrow$

$\sigma_{x'} = \frac{200 + 700}{2} + \frac{200 - 700}{2} \cos(-40) - 300 \sin(-40)$

$\Rightarrow \boxed{\sigma_{x'} = 451.3 \text{ kPa "T"}}$

$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$

$= \frac{200 + 700}{2} - \frac{200 - 700}{2} \cos(-40) + 300 \sin(-40)$

$$\Rightarrow \boxed{\sigma_{y'} = 448.7 \text{ KPa "T"}}$$

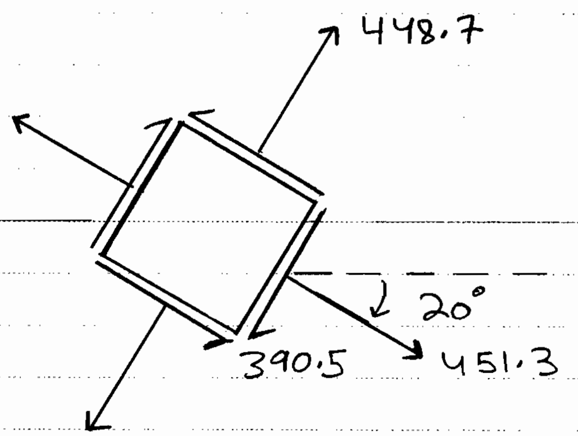
$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{200-700}{2}\right)\sin(-40) - 300\cos(40)$$

$$\Rightarrow \boxed{\tau_{x'y'} = -390.510 \text{ KPa}}$$

The element is shown.

KPa



ii)

$$\sigma_{1,2\ max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy}^2)}$$

Thus,

$$\sigma_{1,2} = \frac{200+700}{2} \pm \sqrt{\left(\frac{200-700}{2}\right)^2 + (-300)^2}$$

$$= 450 \pm 390.512 \Rightarrow$$

$$\boxed{\sigma_1 = \sigma_{max} = 840.5 \text{ KPa "T"}}$$

$$\boxed{\sigma_2 = \sigma_{min} = 59.49 \text{ KPa "T"}}$$

Check:

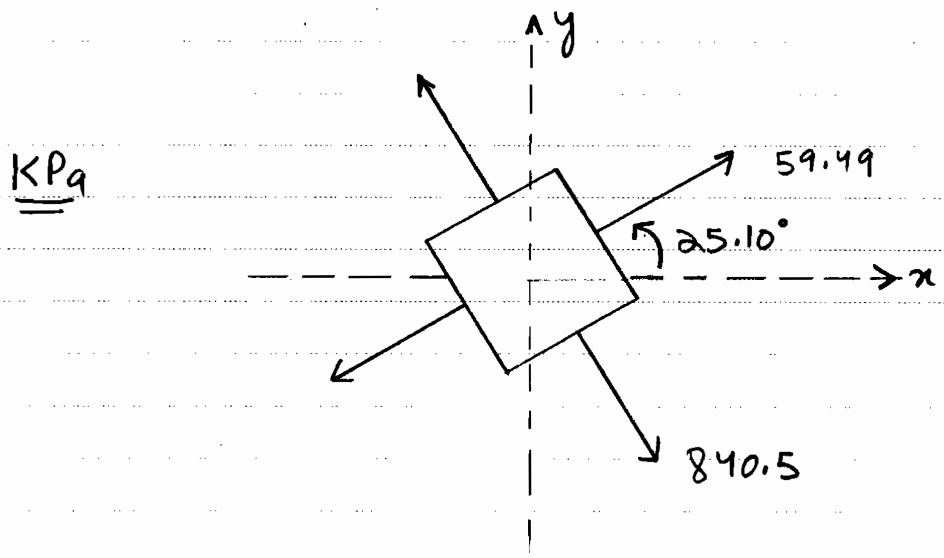
$\sigma_x + \sigma_y = 200 + 700 = 900$ KPa
$\sigma_{x'} + \sigma_{y'} = 451.3 + 448.7 = 900$ KPa ok
$\sigma_{max} + \sigma_{min} = 840.5 + 59.5 = 900$ KPa ok

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)} = \frac{-300}{\left(\frac{200-700}{2}\right)} = +1.2 \Rightarrow$$

$\boxed{\theta_{p_1} = 25.10° \text{ "CCW"}}$; $\boxed{\theta_{p_2} = 115.1°}$

As in P#1, $\sigma(25.10°) = 59.49$ KPa $= \sigma_{min}$

The element is shown.



Note that $\tau = 0$ (Why?!)

iii)

$$\tau_{\substack{max\\min}} = \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$= \pm\ 390.51$ KPa $\Rightarrow$

$$\boxed{\tau_{max} = 390.5 \text{ KPa} \ ; \ \tau_{min} = -390.5 \text{ KPa}}$$

$$\tan 2\theta_s = \frac{-(\sigma_x-\sigma_y)/2}{\tau_{xy}} \Rightarrow$$

$\theta_{s_1} = -19.90°$; $\theta_{s_2} = 70.10°$

(Also we can get it by knowing that
$\theta_s = \theta_p \pm 45$
$= 25.1 \pm 45 = 70.1°, -19.9°$)

$$\tau(70.1) = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

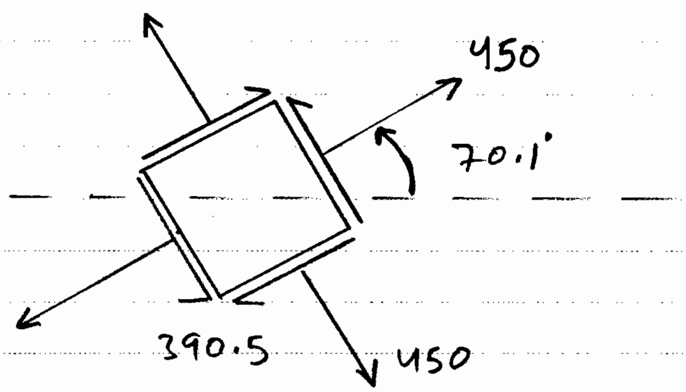
$= +390.5 \Rightarrow$

$\theta_{s2} = 70.1$ is the direction of τ_{max}

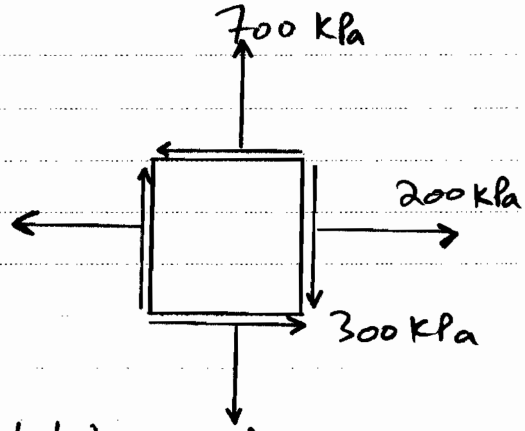
$$\sigma(\text{on plane of } \tau_{max}) = \frac{\sigma_x+\sigma_2}{2} = 450 \text{ KPa}$$

The element is drawn

KPa



Problem # 4:



Given:

As in Prob # 3

Required:

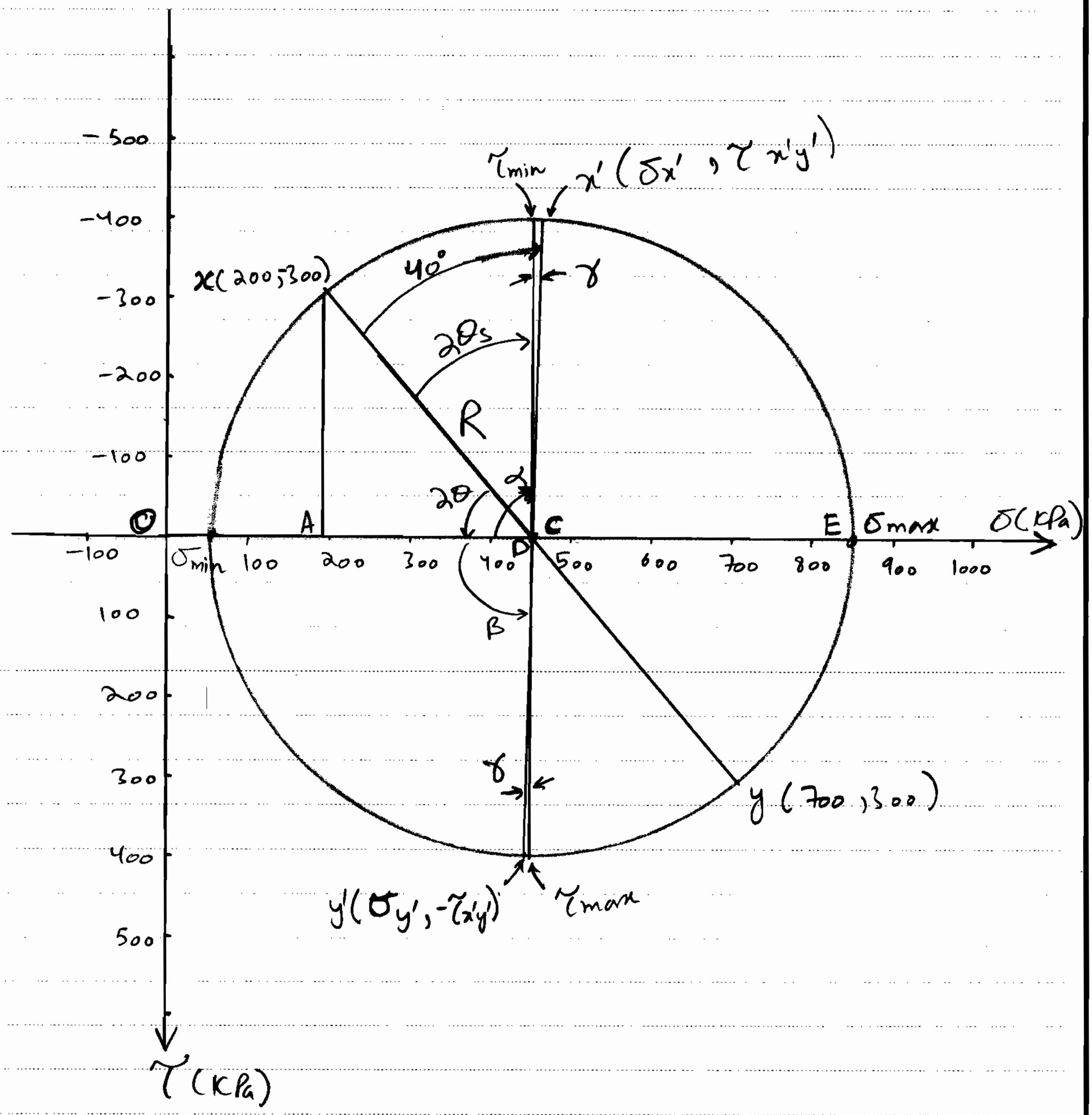
As in Prob # 3 but using Mohr's circle

Solution:

The points on the $\sigma - \tau$ plane are:

$$X(\sigma_x, \tau_{xy}) = X(200, -300)$$
$$Y(\sigma_y, -\tau_{xy}) = Y(700, 300)$$

with "appropriate" scale (take advantage of the whole page; do not let the circle to be "too small" or it goes beyond the page. How to know the best scale; look at the values of the stresses => "guess" σ_{max} and σ_{min} => put the scale; details if you ask your instructor, but you need to practice this!), σ-axis drawn horizontal and τ vertical with "down" as ⊕.



O = origin
C = center of circle
XC = yC = radius
} ⇒ Draw the circle.

OC = averge of x and y $= \frac{200+700}{2} = 450$ KPa
$AC = OC - OA = 450 - 200 = 250$ KPa
The triangle XAC can be used to calculate R.

Thus,

$$R = \sqrt{\overline{XA}^2 + \overline{AC}^2} \Rightarrow$$

$$R = \sqrt{(-300)^2 + (250)^2} = 390.51 \text{ KPa}$$

i) σ & τ @ 20° CW from $\underline{\underline{X}}$ =>

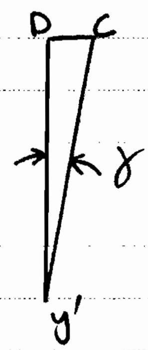
on the circle we go $\underline{\underline{2}}$ (20) = 40° from $\underline{\underline{X}}$ (not the horizontal axis, σ), as shown on the circle. [CW]

$$\tan 2\theta = \left|\frac{XA}{AC}\right| = \frac{300}{250} \Rightarrow 2\theta = 50.194°$$

$$\alpha = 2\theta + 40 = 90.194$$

$$(\text{or } \beta = 180 - 90.194 = 89.806°)$$

(we may use the triangle Dy'C to calculate $\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$ =>
$\gamma = 90 - 89.806 = 0.19443°$)



$$\sigma_{y'} = OC - R\cos\beta$$
$$= 450 - 390.51 \cos 89.806 \Rightarrow$$

$$\boxed{\sigma_{y'} = 448.7 \text{ KPa "T"}}$$

$$\sigma_{x'} = OC + R\cos\beta \Rightarrow$$

$$\boxed{\sigma_{x'} = 451.3 \text{ KPa "T"}}$$

$\tau_{x'y'} = R \sin\beta \Rightarrow$ $\boxed{\tau_{x'y'} = 390.58 \text{ MPa}}$

ii) $\sigma_{max} = OC + R = 450 + 390.51 \Rightarrow$

$\boxed{\sigma_{max} = 840.5 \text{ KPa "T"}}$

$\sigma_{min} = OC - R = 450 - 390.51 \Rightarrow$

$\boxed{\sigma_{min} = 59.49 \text{ KPa "T"}}$

Direction of $\sigma_{min} = \theta_{P_1} = \frac{1}{2}(2\theta) \Rightarrow$

$\boxed{\theta_{P_1} = 25.10^\circ \text{ ccw from } x \text{ to } \sigma_{min}}$

Direction of $\sigma_{max} = \frac{1}{2}(180 - 2\theta) \Rightarrow$

$\boxed{\theta_{P_2} = 64.90^\circ \text{ Cw from } x \text{ to } \sigma_{max}}$

iii) $\tau_{max} = +R \Rightarrow$ $\boxed{\tau_{max} = 390.51 \text{ KPa}}$

$\tau_{min} = -R \Rightarrow$ $\boxed{\tau_{min} = -390.51 \text{ KPa}}$

$2\theta_s = 90 - 2\theta \Rightarrow$ $\boxed{\theta_{s_1} = 19.9^\circ \text{ cw from } x \text{ to } \tau_{min}}$

$\theta_{s_1} = 90 - 19.9 \Rightarrow$ $\boxed{\theta_{s_2} = 70.1^\circ \text{ ccw from } x \text{ to } \tau_{max}}$

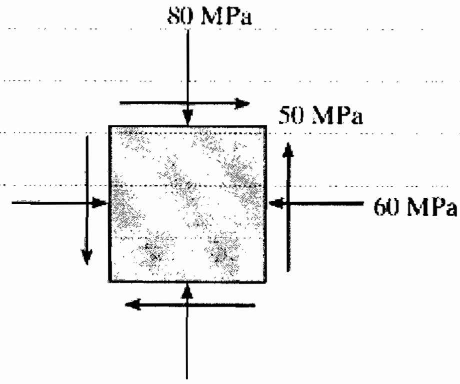
All elements are as shown in Problem#3

Solution of HW # 14

Problem # 5:

Given:

The state of stress on the element shown



Required:

i) σ & τ on element oriented 55° ccw
ii) Principal stresses and directions
iii) Max shear and direction
show all on elements.

Solution:

The steps in problem #4 will be followed.
X(−60, 50) ; y(−80, −50) => The axes are drawn with a good scale.

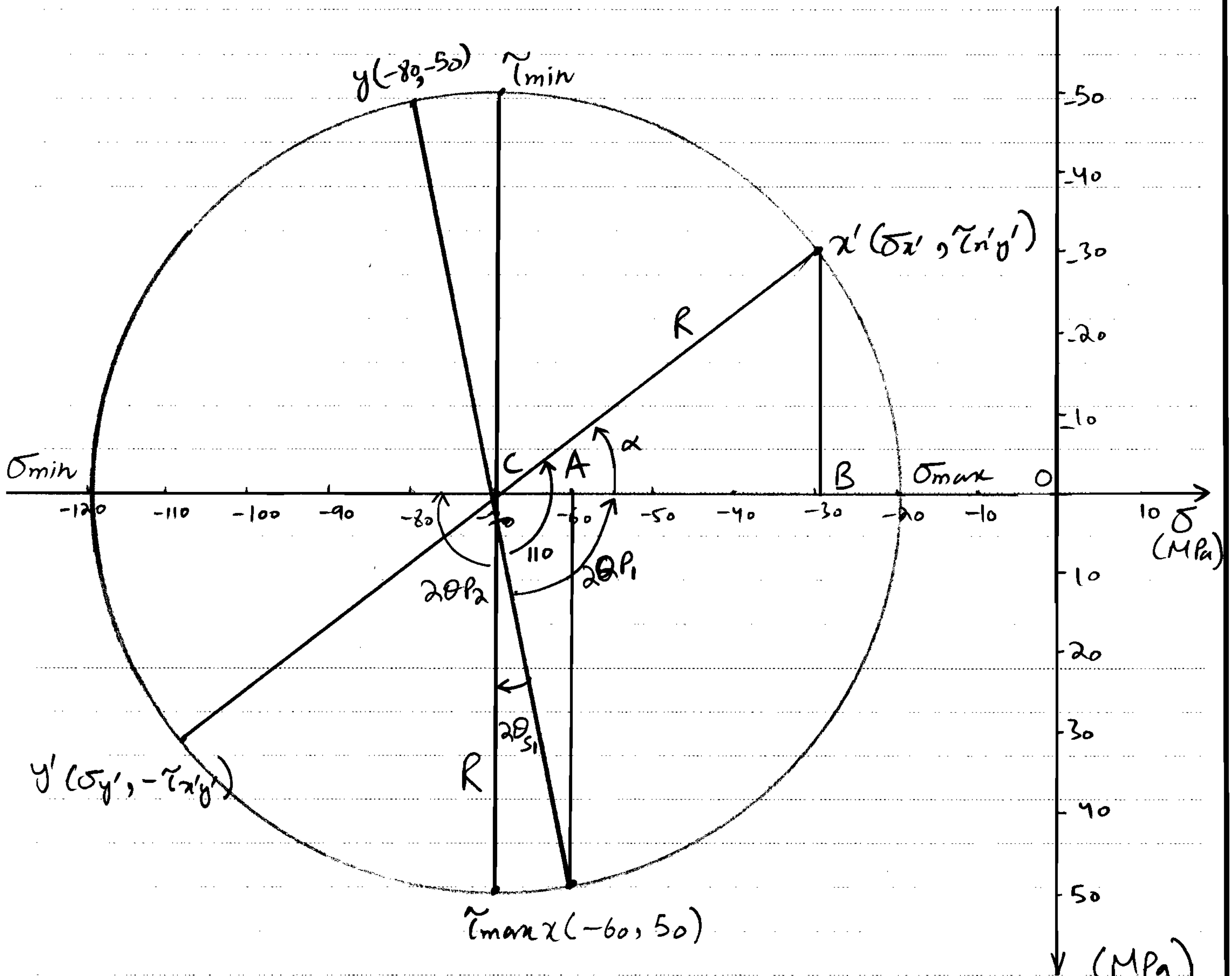
$$OC = -\frac{60-80}{2} = -70 \text{ MPa}$$

$$|AC| = OC - OA = 70 - 60 = 10 \text{ MPa}$$

$$R = \sqrt{\overline{CA}^2 + \overline{Ax}^2} = \sqrt{(10)^2 + (50)^2} = 50.990 \text{ MPa}$$

i) we go 2(55) 110° ccw from X to X′ as shown on the circle

we use the triangle CX′B =>



$$\alpha = 110 - 2\theta_{P_1}$$

$$2\theta_{P_1} = \tan^{-1}\frac{50}{10} \Rightarrow 2\theta_{P_1} = 78.69^\circ \Rightarrow$$

$$\alpha = 31.310^\circ$$

$$|CB| = R\cos\alpha = 43.564 \text{ MPa} \Rightarrow$$

$$\sigma_{x'} = OC - CB = -70 - (-43.564) \Rightarrow$$

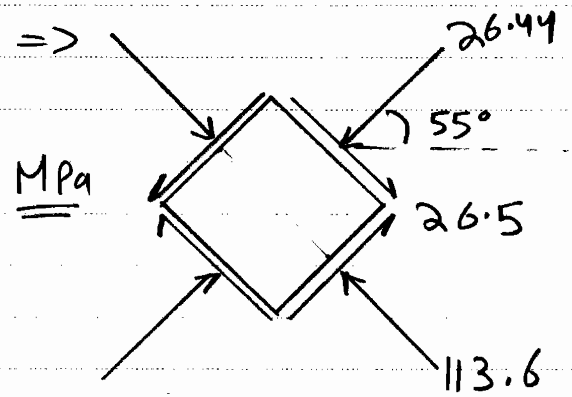
$$\boxed{\sigma_{x'_1} = -26.44\,MPa \text{ "C"}}$$

$$\sigma_y = OC + CB = -70 - 43.564 \Rightarrow$$

$$\boxed{\sigma_y = -113.6 \text{ MPa "C"}}$$

$$\tau_{x'y'} = x'B = R \sin 2$$

$$\Rightarrow \boxed{\tau_{x'y'} = -26.50 \text{ MPa}}$$



The element is shown.

ii) $\sigma_{max} = OC + R = -70 + 50.99 \Rightarrow$

$$\boxed{\sigma_{max} = -19.01 \text{ MPa "C"}}$$

$$\sigma_{min} = OC - R \Rightarrow \boxed{\sigma_{min} = -120.99 \text{ MPa "C"}}$$

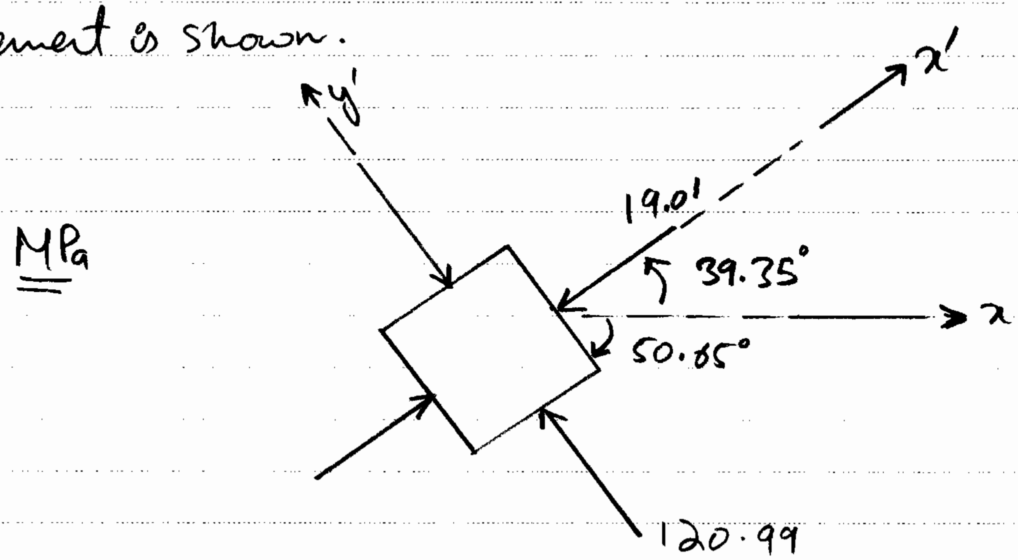
$$2\theta_{P_1} = 78.69^\circ \Rightarrow$$

$$\boxed{\theta_{P_1} = 39.35^\circ \text{ ccw from } x \text{ to } \sigma_{max}}$$

$$2\theta_{P_2} = 180 - 2\theta_{P_1} \Rightarrow$$

$$\boxed{\theta_{P_2} = 50.65^\circ \text{ cw from } x \text{ to } \sigma_{min}}$$

The element is shown.



iii) $\tau_{max} = +R \Rightarrow \boxed{\tau_{max} = 50.99 \text{ MPa}}$

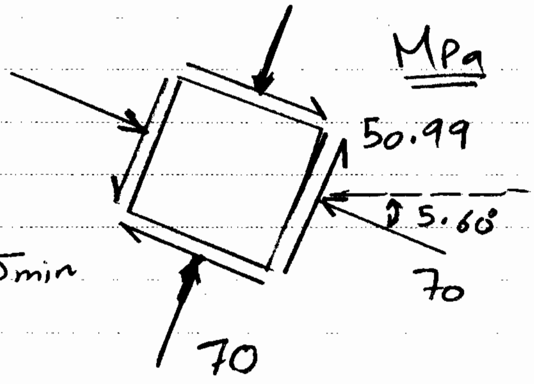
$\tau_{min} = -R \Rightarrow \boxed{\tau_{min} = -50.99 \text{ MPa}}$

$2\theta_{s_1} = 90 - 2\theta_{P_1} \Rightarrow$

$\boxed{\theta_{s_1} = 5.66° \text{ CW from X to } \tau_{max}}$

$\theta_{s_2} = 90 - \theta_{s_1} \Rightarrow \boxed{\theta_{s_2} = 84.34° \text{ CCW from X to } \tau_{min}}$

$\boxed{\sigma = OC = -70 \text{ MPa} \quad \text{"C"}}$



The element is shown.

Note that:

$\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'} = \sigma_{max} + \sigma_{min}$
$= \sigma_{shear} + \sigma_{shear} = -140$

Use it as a check!