King Fahd University of Petroleum & Minerals

CIVIL ENGINEERING DEPARTMENT

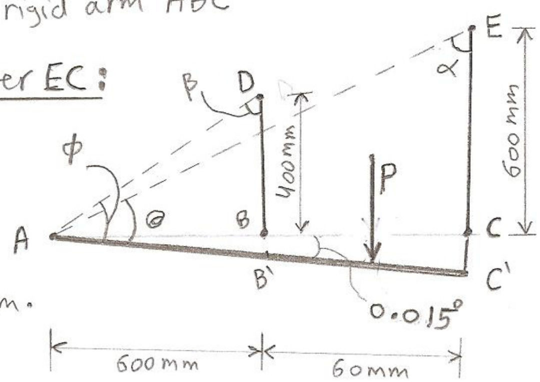
CE 203 - Term 121

Solution of HOMEWORK NO. 3

1- Solve problem F2-2 in the textbook (p. 74) using the given revised data: angle of rotation is 0.015 degrees (instead of .02).

Solution I:

The figure shows the orientation of the rigid arm ABC after rotation. From the figure for member EC:



$$\alpha = \tan^{-1}\left(\frac{1200\ mm}{600\ mm}\right) = 63.435^\circ$$

$$\theta = 90 - \alpha + 0.015 = 26.58^\circ$$

$$L_{AE} = \sqrt{(600mm)^2 + (1200mm)^2} = 1341.641\ mm.$$

Applying the law of Cosines:

$$L_{C'E} = \sqrt{(L_{AE})^2 + (L_{AC'})^2 - 2(L_{AE})\cdot(L_{AC'})\cos(26.58^\circ)}$$

$$= \sqrt{(1341.641mm)^2 + (1200mm)^2 - 2(1341.641mm)\cdot(1200\ mm)\cdot\cos(26.58^\circ)}$$

$$= 600.3132\ mm$$

Normal strain for member CE, $\epsilon_{CE} = \dfrac{L_{C'E} - L_{CE}}{L_{CE}} = \dfrac{600.3132mm - 600mm}{600\ mm}$

$$= \boxed{5.22 \times 10^{-4}\ mm/mm}$$

For member DB:

$$\beta = \tan^{-1}\left(\frac{600\ mm}{400\ mm}\right) = 56.31^\circ$$

$$\phi = 90 - \beta + 0.015 = 33.705^\circ$$

$$L_{AD} = \sqrt{(400mm)^2 + (600mm)^2} = 721.110mm$$

$$L_{B'D} = \sqrt{(721.11mm)^2 + (600mm)^2 - 2(721.11mm)(600mm)\cdot\cos(33.705^\circ)} = 400.156mm$$

$$= 400.156\ mm.$$

Normal strain for member BD, $\epsilon_{BD} = \dfrac{L_{B'D} - L_{BD}}{L_{BD}} = \dfrac{400.156\ mm - 400\ mm}{400\ mm}$

$$= 3.90 \times 10^{-4}\ mm/mm.$$

Problem 1 (con't).

Solution II:

For small strains, the same results can be obtained by approximating the elongation of both wires,

For member CE:

$$\Delta L_{CE} = \theta L_{AC} = \left[\left(\frac{0.015^\circ}{180^\circ}\right)\cdot(\pi \text{ rad})\right]\cdot(1200\text{mm}) = 0.31416 \text{ mm}$$

$$\Rightarrow \epsilon_{CE} = \frac{\Delta L_{CE}}{L_{CE}} = \frac{0.31416 \text{ mm}}{600 \text{ mm}} = \underline{5.236 \times 10^{-4} \text{ mm/mm}}.$$

For member BD:

$$\Delta_{BD} = \left[\left(\frac{0.015^\circ}{180^\circ}\right)\cdot(\pi \text{ rad})\right]\cdot(600\text{ mm}) = 0.157 \text{ mm}$$

$$\Rightarrow \epsilon_{BD} = \frac{\Delta L_{BD}}{L_{BD}} = \frac{0.157 \text{ mm}}{400 \text{ mm}} = \underline{3.927 \times 10^{-4} \text{ mm/mm}}.$$

2- Using the strain results of Problem (1) above, determine the tension in each cable given that for each cable: cross sectional area = 20 mm^2 & E = 20 GPa. (Hint : you do not need equilibrium equations in this case)

Solution :

Using Hooke's Law, $\sigma = E \cdot \epsilon$

— For Cable (wire) CE :

$$\sigma_{CE} = E \cdot \epsilon_{CE} = (20 \times 10^{9}\ Pa) \cdot (5.22 \times 10^{-4}\ \tfrac{mm}{mm})$$

$$= 10.44 \times 10^{6}\ Pa = 10.44\ MPa.$$

$$\therefore F_{CE} = \sigma_{CE} \cdot A_{CE} = (10.44 \times 10^{6}\ Pa) \cdot (20 \times 10^{-6}\ m^2)$$

$$= \underline{208.8\ N.}\ \text{(Tension Force)}.$$

— For Cable (wire) BD :

$$\sigma_{BD} = E \cdot \epsilon_{BD} = (20 \times 10^{9}\ Pa) \cdot (3.90 \times 10^{-4}\ \tfrac{mm}{mm})$$

$$= 7.8 \times 10^{6}\ Pa = 7.8\ MPa.$$

$$F_{BD} = \sigma_{BD} \cdot A_{BD} = (7.8 \times 10^{6}\ Pa) \cdot (20 \times 10^{-6}\ m^2)$$

$$= \underline{156.0\ N.}\ \text{(Tension Force)}.$$

3- Using the table given in problem 3-1 in your textbook (p. 98), plot the stress-strain diagram for the material **accurately and to scale**. Use the following revised data : diameter = 180 mm (instead of 150) and the length = 350mm (instead of 300). Use the plot to determine the following properties : Modulus of Elasticity , the proportional limit, the ultimate stress , the failure stress, and the modulus of toughness.

Solution:

The stress strain values of a concrete cylinder were calculated and tabulated in the next page using the following equations:

$$Sress,\ \sigma = \frac{Load}{Area} = \text{load} / (\pi\ (0.09\text{m})^2)$$

$$Strain,\ \varepsilon = \frac{Contraction\ (mm)}{Gage\ Length\ (mm)} = \frac{Contraction\ (mm)}{350\ mm}$$

- **The plotted stress strain diagram is shown in the next page, and it represents the initial portion of a typical stress strain diagram of concrete.**

Note: the ultimate strength of typical concrete is 35 MPa with the corresponding strain of 0.002 mm/mm.

- **The modulus of elasticity, E (approximately) =** $\frac{6.25\ MPa-0}{0.0003\ mm/mm-0} = 20.833\ GPa$
- **The proportional limit: it is not easy to determine.**
- **The maximum stress from the diagram = 10.4138 MPa.**
- **Both the ultimate stress (σ_u) and the failure stress (σ_f) cannot be obtained since the complete diagram up to failure is not shown .**
- **For the given portion of the diagram,**

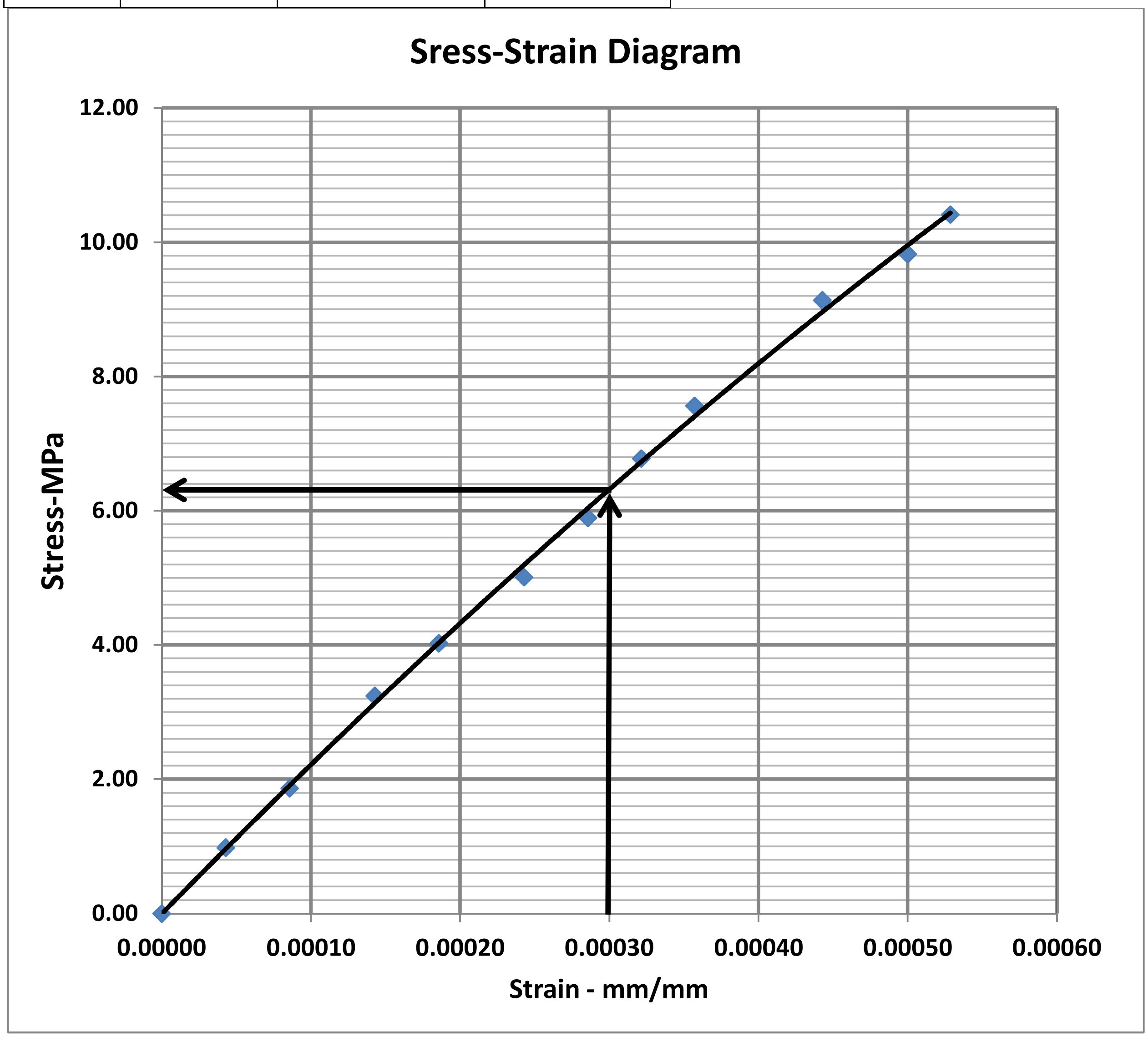
  the modulus of toughness, U_t (approximately) = the entire area under the curve

  = (0.5) X ($10.4138\text{X}10^6$) X (0.0005286mm/mm)

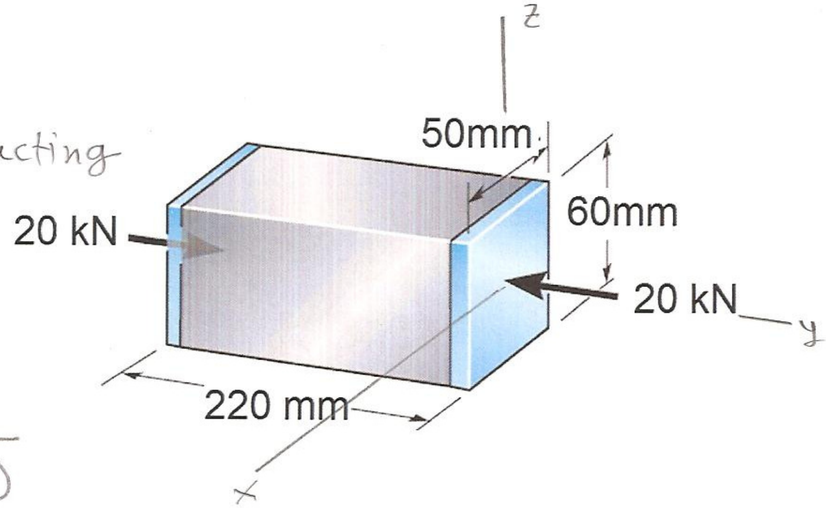
  = 0.00275 MJ/m^3

Stress Strain Data

Load (kN)	Stress (MPa)	Contraction (mm)	Strain (mm/mm)
0.0	0.0000	0.0000	0.0000000
25.0	0.9824	0.0150	0.0000429
47.5	1.8666	0.0300	0.0000857
82.5	3.2420	0.0500	0.0001429
102.5	4.0280	0.0650	0.0001857
127.5	5.0104	0.0850	0.0002429
150.0	5.8946	0.1000	0.0002857
172.5	6.7788	0.1125	0.0003214
192.5	7.5648	0.1250	0.0003571
232.5	9.1367	0.1550	0.0004429
250.0	9.8244	0.1750	0.0005000
265.0	10.4138	0.1850	0.0005286



4- The given solid block has the initial dimensions shown in the figure. Determine the final dimensions of the block, and the change in volume of the block. Given : E = 80 GPa, $\nu = 0.35$. (*Note : the loads are applied on rigid thin plates only to distribute the stress over the area)*



Solution :

The compression stress acting on the block along y-axis

$$\therefore \sigma_y = \frac{-20\,KN}{A_{thinplate}}$$

$$= \frac{-20 \times 10^3\ N}{(0.05m).(0.06m)}$$

$$- 6.667 \times 10^6\ Pa\ (C)$$

Using Hooke's law $\Rightarrow \epsilon_y = \frac{\sigma_y}{E} = \frac{-6.667 \times 10^6\ Pa}{80 \times 10^9\ Pa} = -83.338 \times 10^{-6}\ \frac{m}{m}$

$$\Rightarrow \delta_y = \epsilon_y \cdot L_y = (-83.338 \times 10^{-6}\ \tfrac{mm}{mm}).(220\,mm) = -0.018334\,mm$$

$\Rightarrow$ Final dimension along $y = L_y + \delta_y = \underline{219.98167\ mm}$

Using the Poisson's ratio, $\nu = 0.35 = -\frac{\epsilon_{lat}}{\epsilon_{long}}$

$$\Rightarrow \epsilon_x = \epsilon_z = -\nu \cdot \epsilon_y = -(0.35).(-83.338 \times 10^{-6}\ \tfrac{mm}{mm})$$

$$= 29.168 \times 10^{-6}\ \frac{mm}{mm}.$$

$$\Rightarrow \delta_x = \epsilon_x \cdot L_x = (29.168 \times 10^{-6}\ \tfrac{mm}{mm}).(50\,mm) = +0.001458\ mm.$$

$$\Rightarrow \delta_z = \epsilon_z \cdot L_z = (29.168 \times 10^{-6}\ \tfrac{mm}{mm}).(60\,mm) = +0.00175\ mm.$$

Final dimension along x-axis $= L_x + \delta_x = \underline{50.001458\ mm.}$

" " z-axis $= L_z + \delta_z = \underline{60.00175\ mm.}$

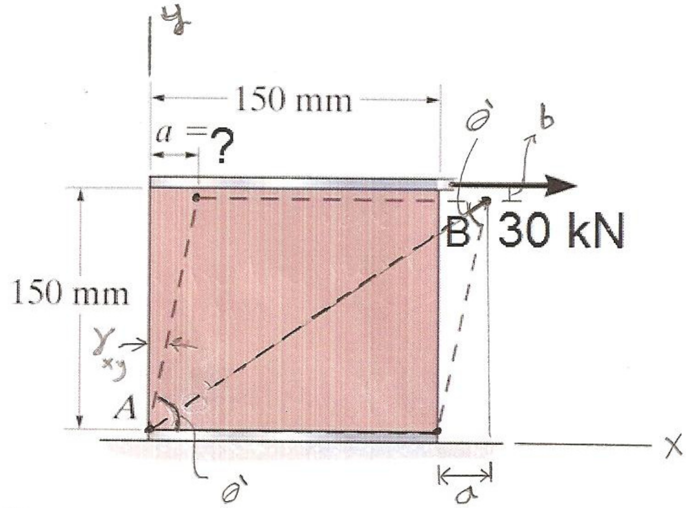
$\therefore$ The change in volume, ΔV = The final volume − initial volume

$= (219.98167mm).(50.00148mm).(60.00175mm) - (220mm)(50mm).(60mm)$

$= 659983.503\ mm^3 - 660000\ mm^3 = \underline{-16.497\ mm^3.}$

= The reduction in volume.

5- The given solid block has a thickness of 50 mm (in the third dimension , not visible in the figure). Determine the deformation distance a shown in the figure due to the application of the load. Also determine the average **normal strain** along the diagonal AB , and the average **shear strain** at corner B. Given G= 30 GPa.



Solution :

The shear stress applied on the block :

$$\tau = \frac{30 \times 10^3 \text{ N}}{(0.05 \text{ m})\cdot(0.15 \text{ m})} = 4.0 \times 10^6 \text{ Pa.}$$

Using Hooke's law for shear,

$$\tau = G.\gamma \Rightarrow \text{Shear strain, } \gamma_{xy} = \frac{\tau}{G} = \frac{4.0 \times 10^6 \text{ Pa}}{30 \times 10^9 \text{ Pa}} = \frac{4.0}{30} \times 10^{-3} \text{ rad.}$$

Since the shear strain, γ_{xy} is the change in angle $(\frac{\pi}{2} - \theta')$,

then, $\gamma_{xy} = \tan^{-1}\left(\frac{a}{150\text{mm}}\right) \cong \frac{a}{150\text{mm}}$ (small strain analysis)

$$\Rightarrow a = \left(\frac{4.0 \times 10^{-3}}{30} \text{rad}\right)\cdot(150 \text{ mm}) = \underline{0.02 \text{ mm.}}$$

The shear strain @ corner A = shear strain @ corner B

$$= + \frac{4}{30} \times 10^{-3} \text{ rad} = 0.00013333 \text{ rad.}$$

The normal strain along the diagonal AB :

$$L_{AB} = \sqrt{(150\text{mm})^2 + (150\text{mm})^2} = 212.132 \text{ mm (before deformation).}$$

$$L_{A'B'} = \sqrt{(150\text{mm} + 0.02\text{mm}) + (150 \text{ mm})^2} = 212.146 \text{ mm (after deformation).}$$

$$\Rightarrow \epsilon_{AB} = \frac{L_{A'B'} - L_{AB}}{L_{AB}} = \frac{212.146 - 212.132}{212.132} = \underline{+6.6 \times 10^{-5} \frac{\text{mm}}{\text{mm}}}$$