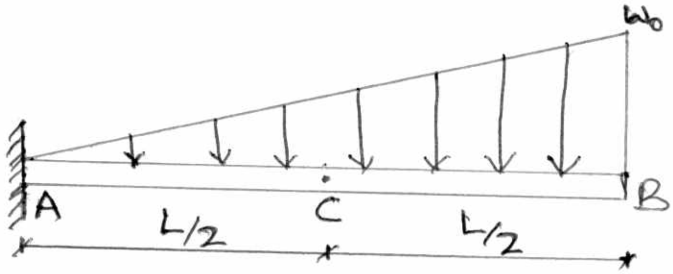
CE 203-121
Solution of HW #9

Problem #1

Given:

The beam shown

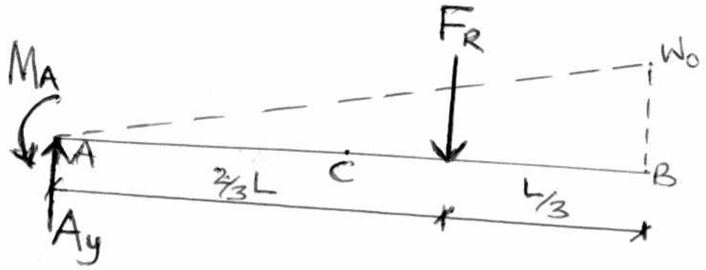


Required:

Internal shear force and moment at point C.

Solution

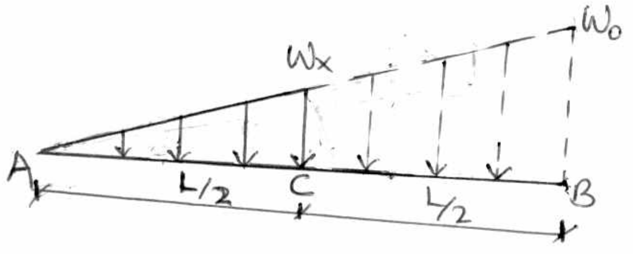
FBD:



Reactions: $F_R = \frac{1}{2} \times W_0 \times L = \frac{W_0 L}{2}$

$+\uparrow \Sigma F_y = 0;\quad A_y - \frac{W_0 L}{2} = 0 \quad \therefore A_y = \frac{W_0 L}{2}$

$+\circlearrowleft \Sigma M_A = 0;\quad M_A - \frac{W_0 L}{2}\left(\frac{2}{3}L\right) = 0 \quad \therefore M_A = \frac{W_0 L^2}{3}$

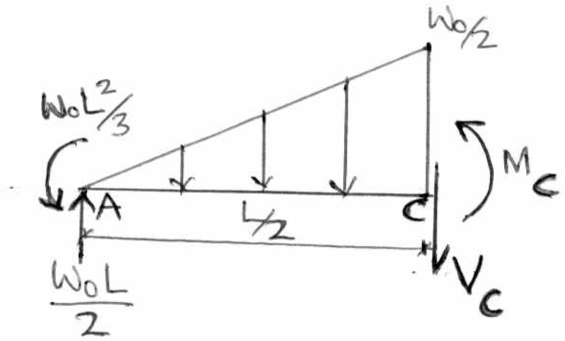


Using similar triangle,

$$\frac{W_x}{W_0} = \frac{L/2}{L}$$

$$\therefore W_x = \frac{W_0}{2}$$

Section at C:



$$+\uparrow \Sigma F_y = 0; \quad \frac{w_oL}{2} - \frac{1}{2}\left(\frac{w_o}{2}\right)\left(\frac{L}{2}\right) - V_c = 0$$

$$\boxed{\therefore V_c = \frac{3}{8} w_o L}$$

$$+\circlearrowright \Sigma M_c = 0; \quad \frac{w_oL^2}{3} - \frac{w_oL}{2}\left(\frac{L}{2}\right) + \frac{1}{2}\left(\frac{w_o}{2}\right)\frac{L}{2}\left(\frac{1}{3} \times \frac{L}{2}\right) + M_c = 0$$

$$\boxed{\therefore M_c = -\frac{5}{48} w_o L^2}$$

Problem #2

Given:

The beam shown in Problem #1

Required:

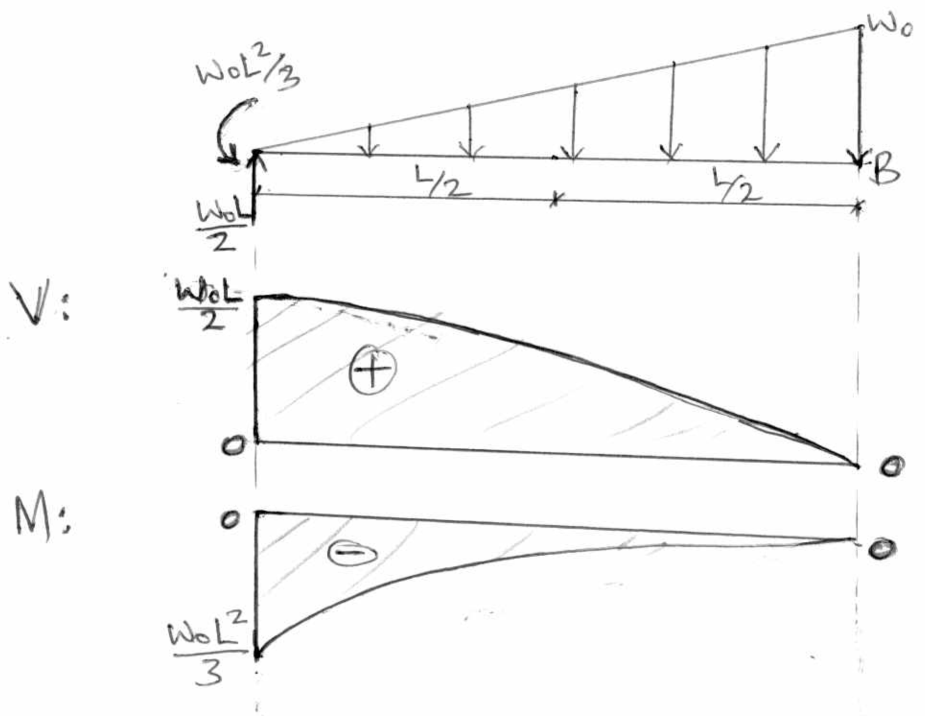
SFD and BMD by graphical method

Solution

From Problem #1, the reactions are:

$$A_y = \frac{W_oL}{2}, \quad M_A = \frac{W_oL^2}{3}$$

The SFD and BMD are shown as follows:



Areas

$$\Delta V_{A\to B} = A_{load_{A\to B}} = -\tfrac{1}{2} \times L \times W_o = -\frac{W_oL}{2}$$

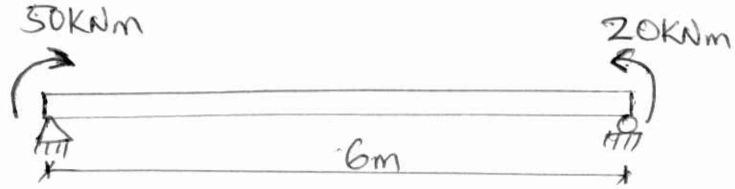
$$V_B = V_A + \Delta V_{A\to B} = \frac{W_oL}{2} - \frac{W_oL}{2} = 0$$

$$\Delta M_{A\to B} = A_{V_{A\to B}} = \tfrac{2}{3}\left(\frac{W_oL}{2}\right)(L) = \frac{W_oL^2}{3}$$

$$M_B = M_A + \Delta M_{A\to B} = -\frac{W_oL^2}{3} + \frac{W_oL^2}{3} = 0$$

Problem #3

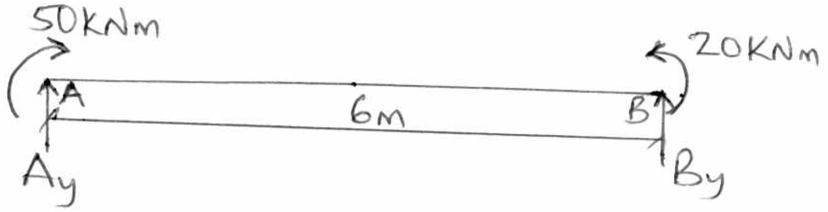
Given:
The beam shown



Required:
SFD and BMD using graphical method

Solution

FBD:



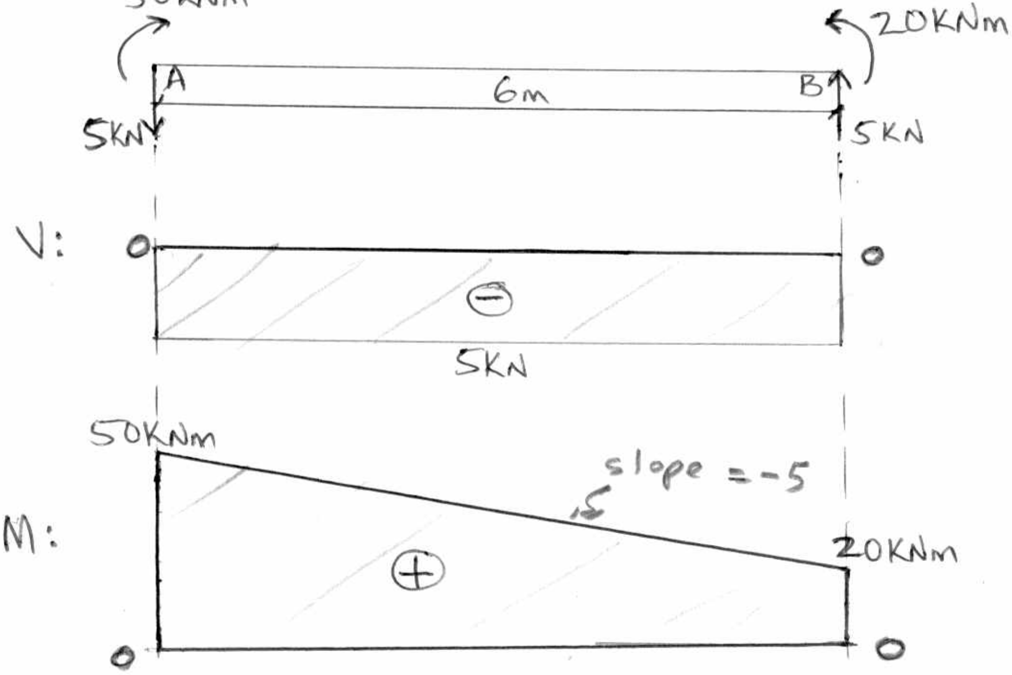
Reactions:

$+\uparrow \sum F_y = 0; \quad A_y + B_y = 0 \quad \therefore A_y = -B_y$

$+\circlearrowleft \sum M_B = 0; \quad 20 - 50 - A_y(6) = 0 \quad \therefore A_y = -5KN$

$\therefore B_y = 5KN$

The SFD and BMD are shown as follows:



Areas:

$$V_A = A_y = -5KN$$

$$\Delta V_{A\to B} = A_{load\,A\to B} = 0$$

$$V_B = V_A + \Delta V_{A\to B} = -5 + 0 = -5KN$$

$$M_A = 50KNm$$

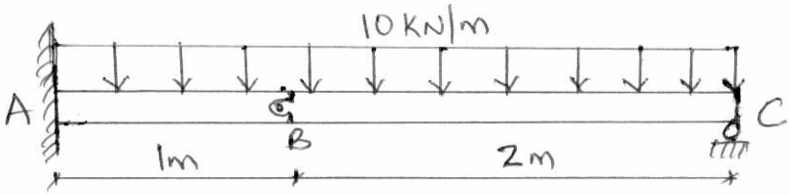
$$\Delta M_{A\to B} = A_{V_{A\to B}} = -5\times 6 = -30KNm$$

$$M_B = M_A + \Delta M_{A\to B} = 50 - 30 = 20KNm$$

Problem #4

Given:

The beam shown:



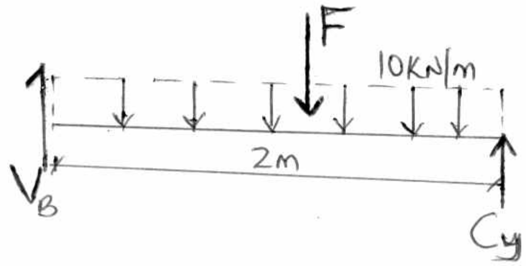
Required:

The SFD and BMD using graphical method

Solution

To calculate the reactions, we need to separate the beam at point B,

By considering the right part of point B,

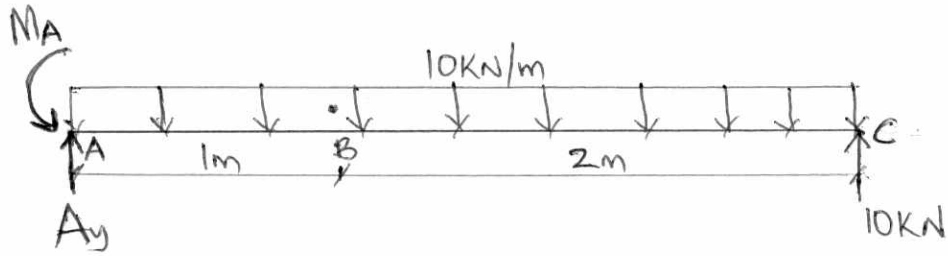


$\Rightarrow F = 10 \times 2$

$= 20 KN$

$+\circlearrowleft \Sigma M_B = 0; \quad 2C_y - (10 \times 2) \times 1 = 0 \quad \therefore C_y = 10 KN$

Now, considering the entire beam,

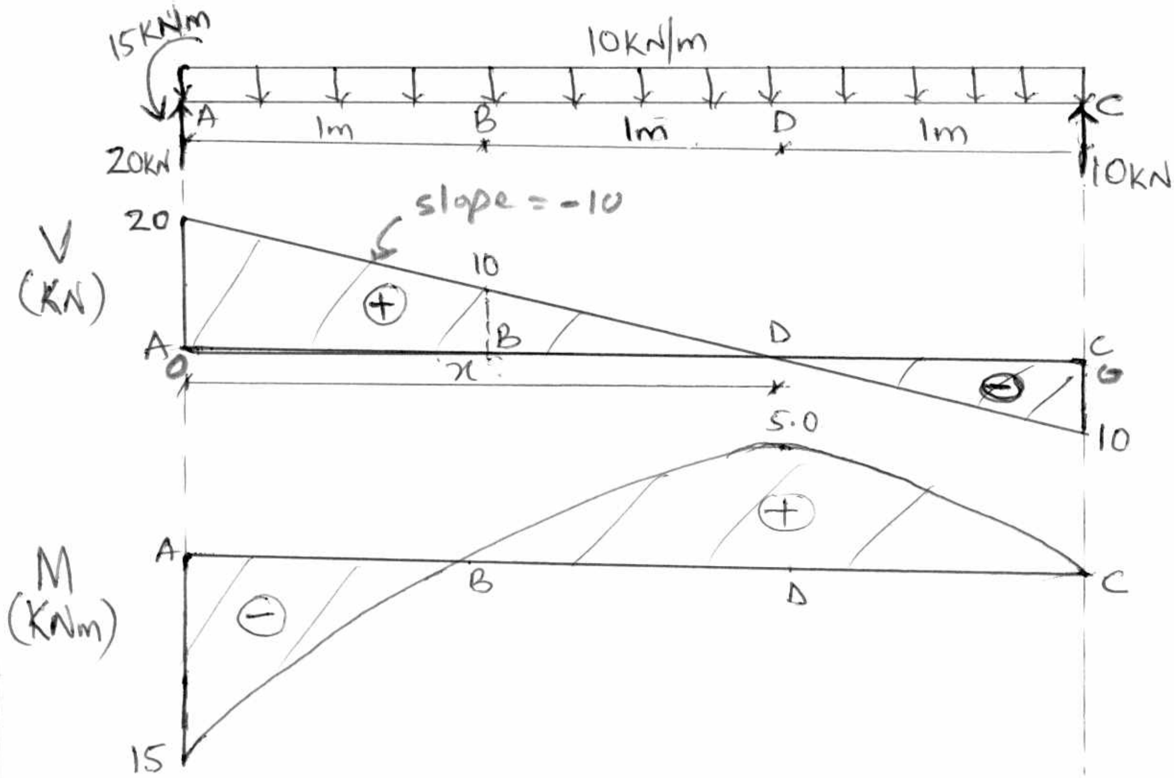


$+\uparrow \Sigma F_y = 0; \quad A_y + 10 - 10 \times 3 = 0 \quad \therefore A_y = 20 KN$

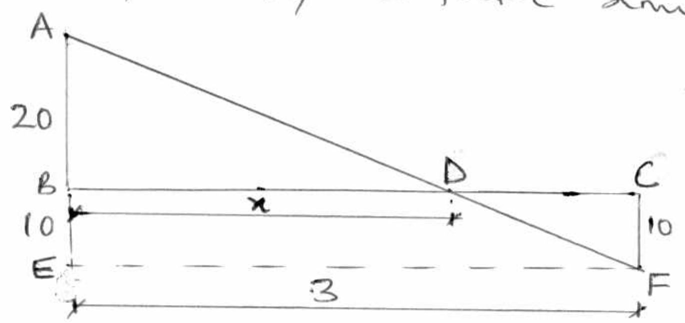
$+\circlearrowleft \Sigma M_A = 0; \quad M_A - (10 \times 3) \times 1.5 + 10 \times 3 = 0$

$\therefore M_A = 15 KN.m$

The SFD and BMD are shown below:



To find x, we take similar triangles ABD and AEF,



$$\Rightarrow \frac{x}{20} = \frac{3}{20+10}$$

$$\therefore x = 2m$$

<u>Areas</u>

$$\Delta V_{A\to C} = A_{load\,A\to C} = -10(3) = -30KN$$

$$V_C = V_B + \Delta V_{B\to C} = 20 - 30 = -10KN$$

$$\Delta M_{A\to B} = A_{V_{A\to B}} = \tfrac{1}{2}\times(20+10)\times 1 = 15KNm$$

$$\Rightarrow M_B = M_A + \Delta M_{A\to B} = -15 + 15 = 0$$

$$\Delta M_{B\to D} = A_{V_{B\to D}} = \tfrac{1}{2}\times 10\times 1 = 5KNm$$

$$\Rightarrow M_D = M_B + \Delta M_{B\to D} = 0 + 5 = 5KNm$$

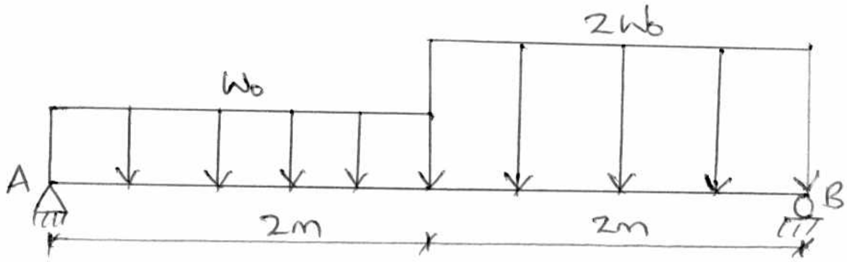
$$\Delta M_{D\to C} = A_{V_{D\to C}} = -\tfrac{1}{2}\times 10\times 1 = -5KNm$$

$$\Rightarrow M_C = M_D + \Delta M_{D\to C} = 5 - 5 = 0$$

Problem #5

Given:

The beam shown

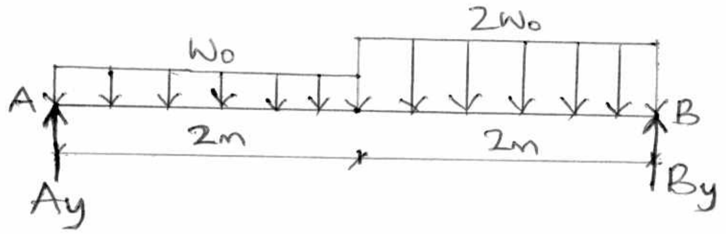


Required:

SFD and BMD using graphical method.

Solution

FBD:



Reactions:

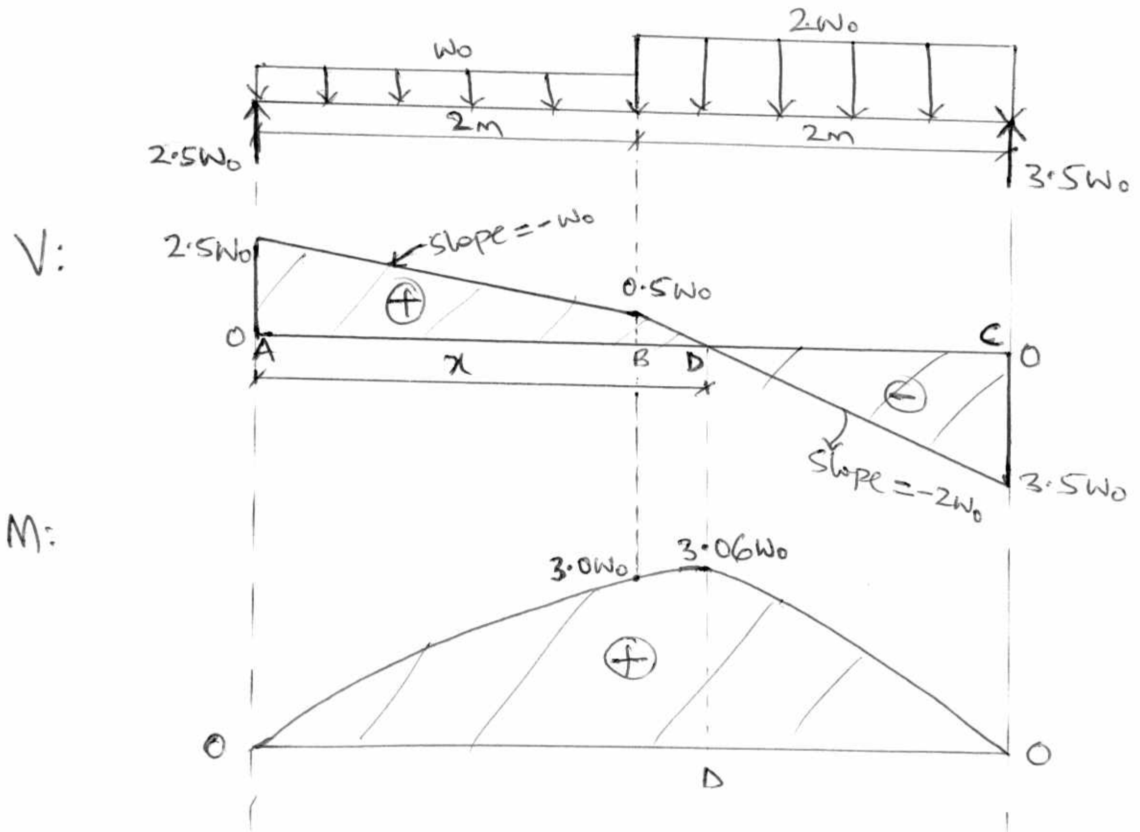
$+\circlearrowright \Sigma M_A = 0;\ B_y(4) - (W_o \times 2)\times 1 - (2W_o \times 2)\times 3 = 0$

$\therefore B_y = 3.5 W_o$

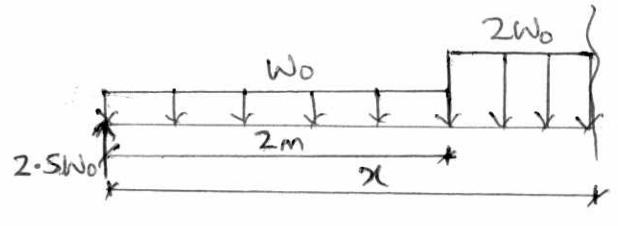
$+\uparrow \Sigma F_y = 0;\ A_y + 3.5W_o - W_o \times 2 - 2W_o \times 2 = 0$

$\therefore A_y = 2.5 W_o$

The SFD and BMD are shown below:



To find x, we need to find the equation where $SF=0$,



$$\Rightarrow 2.5W_0 - 2(W_0) - (2W_0)(x-2) = 0$$

$$2.5W_0 - 2W_0 - 2W_0 x + 4W_0 = 0$$

$$\therefore x = 2.25\text{m}$$

<u>Areas</u>

$$\Delta V_{A\to B} = A_{load\,A\to B} = -W_0 \times 2 = -2W_0$$

$$V_B = V_A + \Delta V_{A\to B} = 2.5W_0 - 2W_0 = 0.5W_0$$

$$\Delta V_{A\to C} = A_{load\,A\to C} = -W_0 \times 2 - 2W_0 \times 2 = -6W_0$$

$$V_C = V_A + \Delta V_{A\to C} = 2.5W_0 - 6W_0 = -3.5W_0$$

$$\Delta M_{A\to D} = A_{V\,A\to D} = \tfrac{1}{2}(2.5W_0 + 0.5W_0)\times 2 + \tfrac{1}{2}\times 0.5W_0 \times 0.25$$

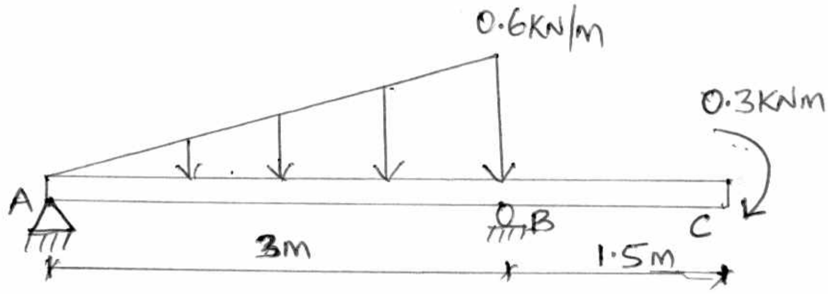
$$= 3.06W_0$$

$$M_D = M_A + \Delta M_{A\to D} = 0 + 3.06 = 3.06W_0$$

Problem #6

Given:

The beam shown

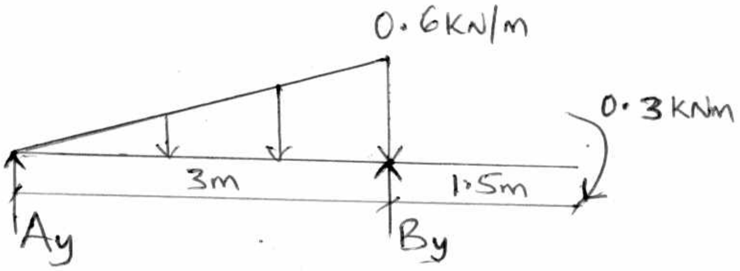


Required:

SFD and BMD using graphical method

Solution

FBD:



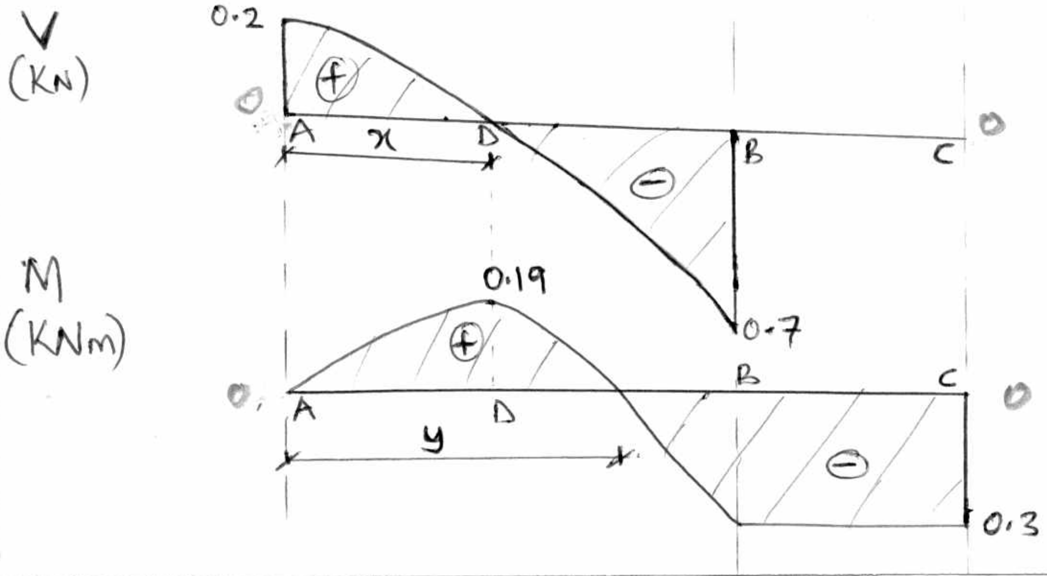
Reactions:

$+\circlearrowright \Sigma M_A = 0;\quad B_y(3) - 0.3 - (\tfrac{1}{2} \times 0.6 \times 3) \times 2 = 0$

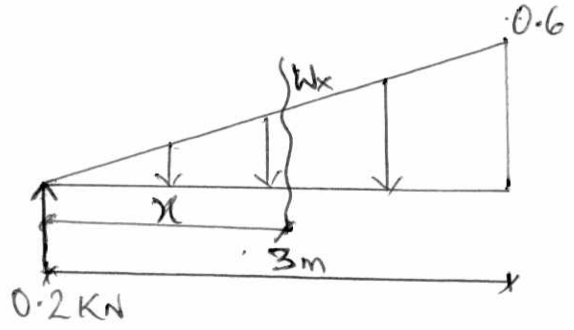
$\therefore B_y = 0.7 KN$

$+\uparrow \Sigma F_y = 0;\quad A_y + 0.7 - \tfrac{1}{2} \times 0.6 \times 3 = 0 \qquad \therefore A_y = 0.2 KN$

The SFD and BMD are shown below,



To find the distance x, at a point where $SF = 0$,



$\Rightarrow V_x = 0 = 0.2 - \frac{1}{2} \times W \times x$

$\Rightarrow W = 0.4/x$

Using similar triangles,

$$\frac{W}{0.6} = \frac{x}{3}$$

$$\Rightarrow x = \frac{3W}{0.6} = \frac{3}{0.6}\left(\frac{0.4}{x}\right)$$

$$\therefore x^2 = 2$$

$$\Rightarrow x = 1.41m$$

Areas

$$\Delta V_{A\to B} = A_{load\,A\to B} = \frac{-0.6\times 3}{2} = -0.9KN$$

$$V_B = V_A + \Delta V_{A\to B} = 0.2 - 0.9 = -0.9KN$$

$$\Delta M_{A\to D} = A_{V_{A\to D}} = \frac{2}{3}(0.2)(1.41) = 0.188 \simeq 0.19KNm$$

$$M_D = M_A + \Delta M_{A\to D} = 0 + 0.19 = 0.19KNm$$

$$\Delta M_{D\to B} = \frac{2}{3}(3.0)(0.2+0.7) - 3(0.7) - 0.19$$

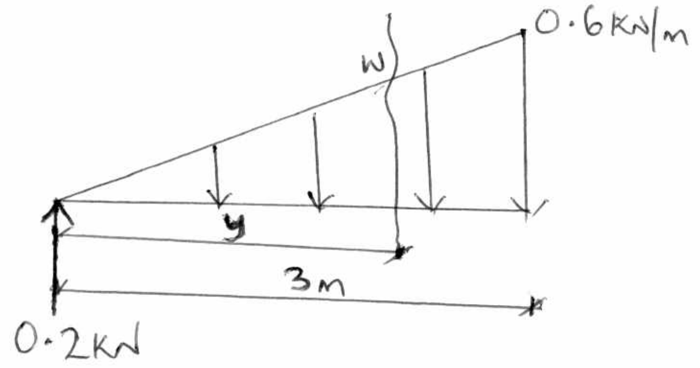
$$= -0.49KNm$$

$$M_B = M_D + \Delta M_{D\to B}$$

$$= 0.19 - 0.49 = -0.3KNm$$

Note:

To determine the distance 'y' at which $M = 0$,



$$\Rightarrow M_x = 0 = 0.2y - \left(\tfrac{1}{2} \times w \times y\right) \times \left(\tfrac{1}{3} \times y\right) = 0$$

$$\Rightarrow w = \frac{1.2}{y}$$

Using similar triangles,

$$\frac{w}{0.6} = \frac{y}{3}$$

$$\Rightarrow y = \frac{3w}{0.6} = \frac{3}{0.6}\left(\frac{1.2}{y}\right)$$

$$\Rightarrow y^2 = 6$$

$$\therefore y = 2.45\text{m}$$