# CE 203 STRUCTURAL MECHANICS I

First Semester 2012 / 2013 (**121**)

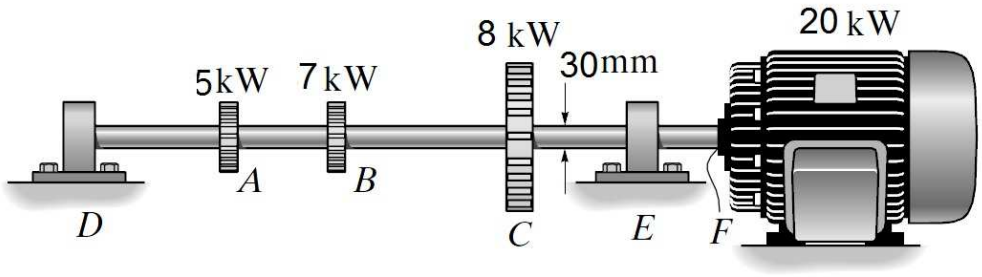
## HOMEWORK NO. 7 (Key Solution)

- **Textbook Sections Covered: 5.1 & 5.3 ,** Torsion of circular shafts

**Problem # 1:-**

***Given Data:***

- The shown figure.
- Smooth bearings at *D* and *E (induce zero resistance torque).*
- Motor delivers 20 KW of Power to the shaft.
- Turning at 60 rev/sec.
- Gears *A*, *B*, and *C* remove 5 kw, 7 kw, and 8 kw respectively

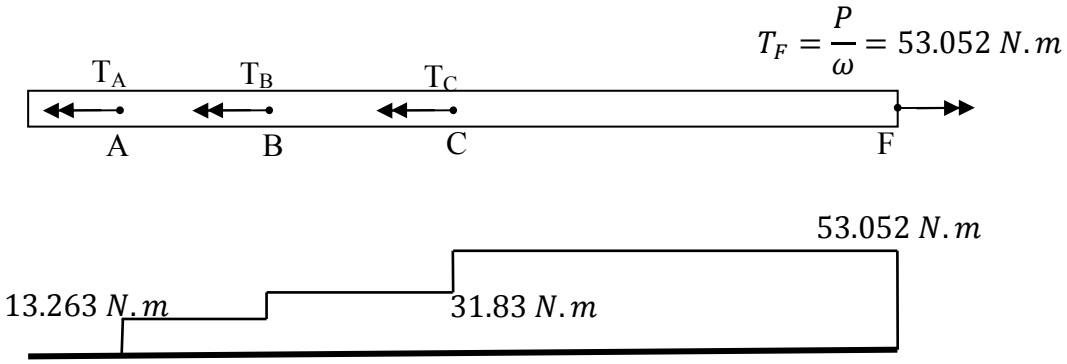



***Required:***

- **The maximum shear stress developed in the shaft within regions *CF* and *BC*.**

***Solution:***

$\omega = 60\frac{rev}{se}\left[2\pi\frac{rad}{rev}\right] = 120\pi\frac{rad}{s}$ As it is shown in the following free body diagram and torque diagram,




*Free body diagram and torque diagram*

$$T_F = \frac{P}{\omega} = 20 * \frac{10^3}{120\pi} = 53.052\ N.m$$
$$T_A = \frac{P}{\omega} = 5 * \frac{10^3}{120\pi} = 13.263\ N.m$$
$$T_B = \frac{P}{\omega} = 7 * \frac{10^3}{120\pi} = 18.568\ N.m$$
$$T_C = \frac{P}{\omega} = 8 * \frac{10^3}{120\pi} = 21.22\ N.m$$

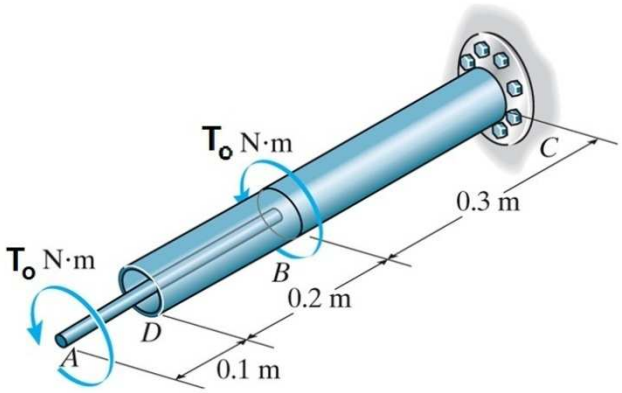
$$(\boldsymbol{\tau_{max}})_{\boldsymbol{CF}} = \frac{T_{CF}C}{J} = \frac{53.052 * 0.015}{\frac{\pi}{2} * 0.015^4} = 10.007 * 10^6\,\frac{N}{m^2} \equiv \mathbf{10.007\ MPa} \qquad \boldsymbol{Ans}$$

$$(\boldsymbol{\tau_{max}})_{\boldsymbol{BC}} = \frac{T_{BC}C}{J} = \frac{31.83 * 0.015}{\frac{\pi}{2} * 0.015^4} = 6.004 * 10^6\,\frac{N}{m^2} \equiv \mathbf{6.004\ MPa} \qquad \boldsymbol{Ans}$$

**Problem # 2:-**

***Given Data:***

- The shown figure.
- Solid rod AB (d=12 mm).
- Rigid disk at B.
- Tube DC outer diameter is 40 mm and a thickness of 6 mm.
- The max allowable stress is 50 MPa.

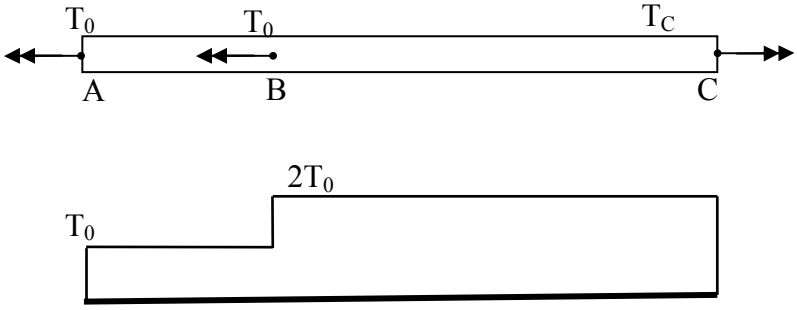



***Required:***

- **The largest value of torque $T_0$ that can be safely applied.**

***Solution:***

As it is shown in the following free body diagram and torque diagram,




*Free body diagram and torque diagram*

$$J_{BC} = \frac{\pi}{2} * (0.02^4 - 0.014^4) = 60.792 * 10^{-9}\ \pi\ m^4$$

$$J_{DA} = \frac{\pi}{2} * (0.006^4) = 0.648 * 10^{-9}\ \pi\ m^4$$

$$From\ Equilibrium,\ T_{BC} = 2T_0\ ; \qquad and \qquad T_{AB} = T_0$$

$$\tau_{BC} = \frac{2T_0 * 0.02}{60.792 * 10^{-9}\ \pi} = 50 * 10^6\ \frac{N}{m^2} \quad \Rightarrow \quad \boldsymbol{(T_0)_{BC} = 238.730\ N.m}$$

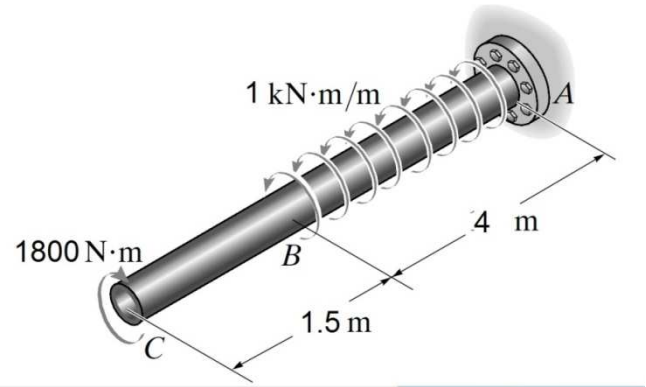
$$\tau_{AB} = \frac{T_0 * 0.006}{0.648 * 10^{-9}\ \pi} = 50 * 10^6\ \frac{N}{m^2} \quad \Rightarrow \quad \boldsymbol{(T_0)_{AB} = 16.965\ N.m}$$

$$\boldsymbol{\therefore\ The\ max\ Torque\ T_0\ that\ can\ be\ safely\ applied\ is\ 16.965\ N.m \quad Ans}$$

**Problem # 3:-**

***Given Data:***

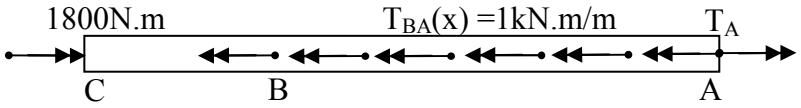
- The shown figure (hollow circular shaft).
- Outer radius = 50 mm ,
- Inner radius = 30 mm




***Required:***

- **The absolute maximum shear stresses in BC**
- **The absolute maximum shear stresses in BA.**
- **A diagram sketch for the value of the internal torque $T_R$ along axis CA**

***Solution:***



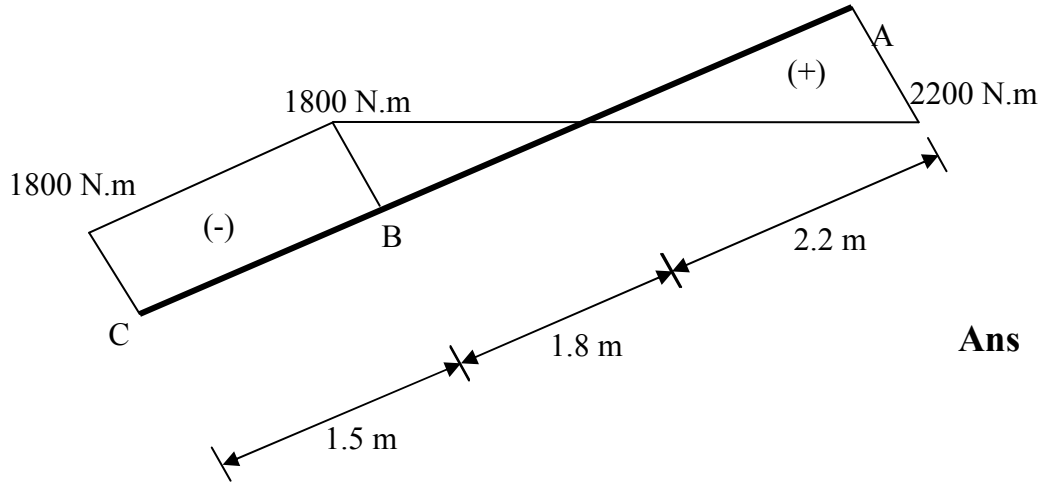

*Free body diagram*

$$\mathbf{T_A} + 1800 - 1 * 10^3 * 4 = 0; \quad \Rightarrow \quad \mathbf{T_A} = 2200 \text{ N.m}$$

$$\therefore \max T_{BA} = 2200 \text{ N.m} \quad ; \quad \text{and} \quad \max T_{BC} = 1800 \text{ N.m}$$

$$J_{BC} = J_{BA} = \frac{\pi}{2} * (0.05^4 - 0.03^4) = 2.72 * 10^{-6} \ \pi \ m^4$$

- $(\tau_{max})_{BC} = \frac{1800*0.05}{2.72*10^{-6}\ \pi} = \mathbf{10.532\ MPa}$. **Ans**
- $(\tau_{max})_{BA} = \frac{2200*0.05}{2.72*10^{-6}\ \pi} = \mathbf{12.873\ MPa}$. **Ans**
- **The diagram sketch for the value of the internal torque $T_R$ along axis CA is as follows:-**




**Ans**