King Fahd University of Petroleum & Minerals

CIVIL ENGINEERING DEPARTMENT

CE 203 STRUCTURAL MECHANICS I

First Semester 2012 / 2013 (**121**)

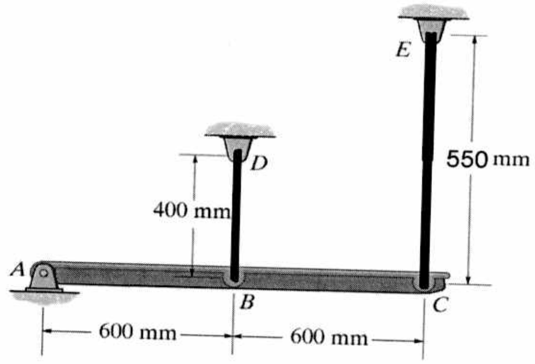
HOMEWORK NO. 6 SOLUTION

1

Req: Stresses if $\Delta T = 70^\circ C$

Solution:

When temp is increased both rods will expand but are not free to expand since the rigid beam is connected to both. As a result one will devlop tension while the other will develop compression. By inspection CE will be in tension (how!)

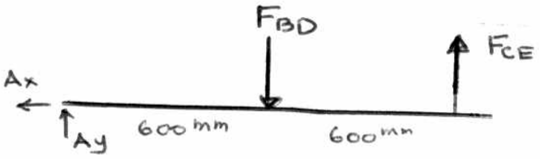


Consider FBD

$\circlearrowleft \sum M_A = 0$

$$600\, F_{BD} - 1200\, F_{CE} = 0$$

$$F_{BD} = 2 F_{CE} \quad \text{—(1)}$$



FBD beam ABC

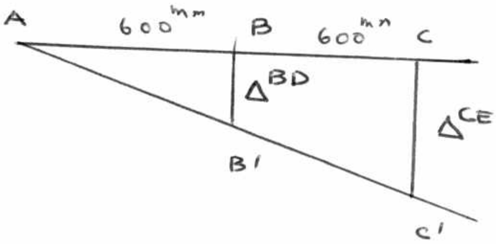
Statically indeterminate, use compatibility to come up with an extra Equation

from deformation geometry

$$\frac{\Delta^{CE}_{total}}{1200} = \frac{\Delta^{BD}_{total}}{600}$$

or

$$\Delta^{CE}_{total} = 2\,\Delta^{BD}_{total}$$



Deformation geometry

$$\alpha L_{CE} \Delta T - \frac{F_{CE} L_{CE}}{AE} = 2\left[\alpha L_{BD} \Delta T + \frac{F_{BD} L_{BD}}{AE}\right] \quad \text{—(2)}$$

From table:

$\alpha = 24\times10^{-6} /^\circ c$; $E = 68.9\, GPa$, and $A = \frac{\pi}{4}(12)^2$

substituting and solving (1) & (2) one gets:

$F_{CE} = 6.09\, kN$ ∴ $\sigma_{CE} = 13.5\, MPa$ tension

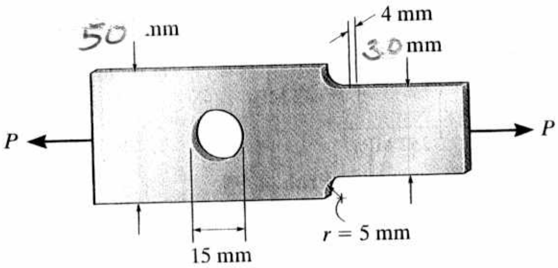
$F_{BD} = 12.18\, kN$ $\sigma_{BD} = 26.9\, MPa$ Compression

#2 (Problem 4-91 Modified)

Req: Max P that can be applied

Solution:

→ If hole controls then the maximum stress will be



$$\sigma_{max} = K_{hole} \frac{P}{t(W-2r)} \leq \sigma_{all} \quad -①$$

to find K we use Fig 4.25

Note that the x-axis reads $\frac{2r}{W}$

$\because$ for $\frac{2r}{W} = \frac{15}{50} = 0.3$ the figures gives $K \cong 2.3$ (hole)

use values in ①

$$2.3 \frac{P}{4(50-15)} \leq 147 \quad \Rightarrow \quad P \leq 8{,}948 \text{ N}$$

→ If fillet controls then the maximum stress will be

$$\sigma_{max} = K_{fillet} \frac{P}{ht} \leq \sigma_{all} \quad -②$$

to find K we use Fig 4.26

$\frac{W}{h} = \frac{50}{30} \simeq 1.67$; $\frac{r}{h} = \frac{5}{30} = 0.17$

reading K from the figure, one gets

$K_{fillet} \simeq 1.9$

use values in ②

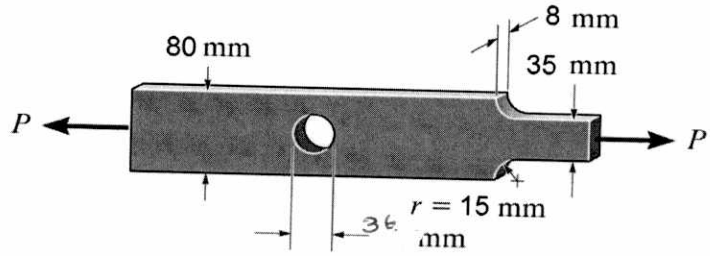
$$1.9 \frac{P}{30 \times 4} \leq 147 \qquad \underline{P \leq 9{,}284 \text{ N}}$$

$\because$ hole controls and $P_{max} = 8{,}948$ N

#3

Required stresses at various locations.

Solution



(a) Average stress near left End $= \dfrac{P}{wt}$

$= \dfrac{P}{80 \times 8}$

(b) Average stress near right End $= \dfrac{P}{ht}$

$= \dfrac{P}{35 \times 8}$

(c) max stress near hole $\approx K \dfrac{P}{(w-2r)t}$

to find K use Fig. 4-25

with $\dfrac{2r}{w} = \dfrac{36}{80} = 0.45 \xrightarrow{\text{read}} K \simeq 2.15$

$\sigma_{\substack{max \\ hole}} = 2.15 \dfrac{P}{(80-36)\,8} = 6.1\,P$

(d) max stress near fillet $= K \dfrac{P}{ht}$

to find K use fig 4-26

with

$\dfrac{w}{h} = \dfrac{80}{35} = 2.3$

$\Rightarrow K = 1.45$

$\dfrac{r}{h} = \dfrac{15}{35} = 0.43$

$\because \sigma_{\substack{max \\ fillet}} = 1.45 \dfrac{P}{35 \times 8} = 5.2\,P$

(e) Maximum stress will be near the hole

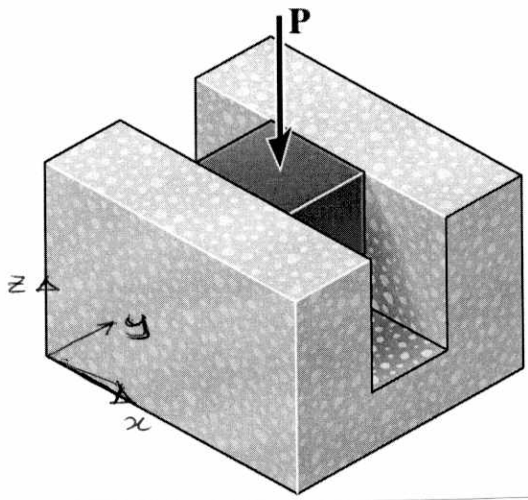
$= 6.1\,P$

#4) Req. Effective Modulus of Elasticity under shown confinement

Solution

$\sigma_z = -\frac{P}{A}$; $\sigma_x = 0$; σ_y unknown

ε_z unknown ; ε_x unknown ; $\varepsilon_y = 0$



$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$

substituting we get

$$\boxed{\sigma_y = \nu\,\sigma_z} \quad —①$$

Let calculate ε_z,

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu^2\sigma_z)$$

$$\boxed{\varepsilon_z = \frac{1-\nu^2}{E}\,\sigma_z} \quad —②$$

The above relations between stress and strain in z-direction is compared to their relation with no confinement (i.e. $\varepsilon_z = \frac{\sigma_z}{E}$) to conclude that the Effective modulus of Elasticity is $\frac{E}{1-\nu^2}$

→ dilatation, e, $= \varepsilon_x + \varepsilon_y + \varepsilon_z$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_z + \sigma_y) = 0 - \frac{\nu}{E}(\sigma_z + \nu\sigma_z)$$

$$\therefore \varepsilon_x = \frac{-\nu(1+\nu)}{E}\sigma_z$$

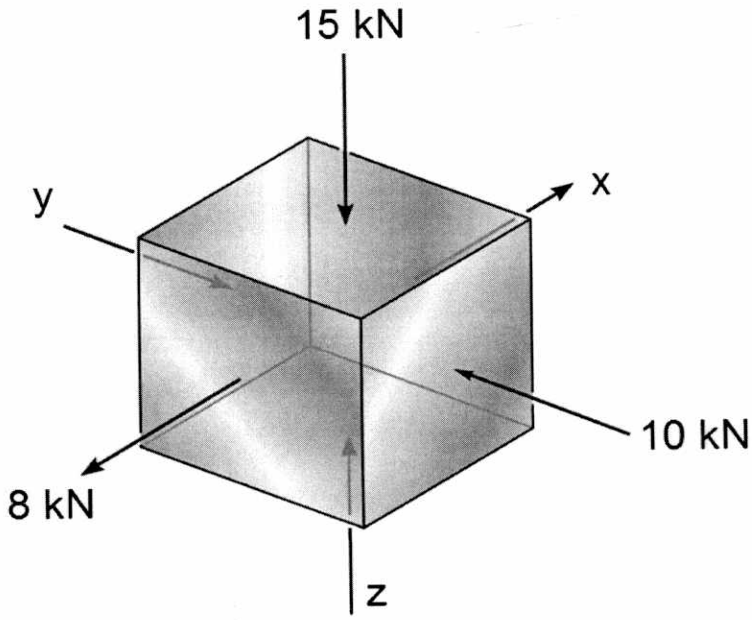
$$\varepsilon_y = 0$$

$$\varepsilon_z = \frac{1-\nu^2}{E}\sigma_z \qquad \text{from ②}$$

$$\therefore e = \left[-\frac{\nu(1+\nu)}{E} + \frac{1-\nu^2}{E}\right]\sigma_z$$

$$e = \frac{(1-2\nu)(1+\nu)}{E}\sigma_z$$

5) Required Final dimensions & ΔV



$$\sigma_x = \frac{8 \times 10^3}{100^2} = 0.8 \text{ MPa}$$

$$\sigma_y = -\frac{10 \times 10^3}{100^2} = -1 \text{ MPa}$$

$$\sigma_z = \frac{-15 \times 10^3}{100^2} = -1.5 \text{ MPa}$$

Calculate strains

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$$

$$\varepsilon_x = \frac{1}{10^4}\left[0.8 - 0.3(-1 - 1.5)\right]$$

$$\underline{\varepsilon_x = 0.155 \times 10^{-3}} \Rightarrow \Delta_x = \varepsilon_x L_x = \underline{.0155 \text{ mm}}$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu(\sigma_x + \sigma_z)\right)$$

$$= \frac{1}{10^4}\left(-1 - 0.3(0.8 - 1.5)\right)$$

$$\underline{\varepsilon_y = -0.079 \times 10^{-3}} \Rightarrow \Delta_y = \varepsilon_y L_y = \underline{-.0079 \text{ mm}}$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu(\sigma_x + \sigma_y)\right)$$

$$= \frac{1}{10^4}\left(-1.5 - \nu(.8 - 1)\right)$$

$$\underline{\varepsilon_z = -0.144 \times 10^{-3}} \Rightarrow \Delta_z = \varepsilon_z L_z = \underline{-.0144 \text{ mm}}$$

The new dimensions will be:

$L_x = 100.0155$ mm ; $L_y = 99.9921$ mm ; $L_z = 99.9856$ mm.

$$e = \text{dilatation} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$e = -0.068 \times 10^{-3}$$

but

$$e \approx \frac{\Delta V}{V}$$

$$\therefore \quad \Delta V = (100)^3 * (-.068 \times 10^{-3}) = 68 \text{ mm}^3$$