

HW #4 - Solution

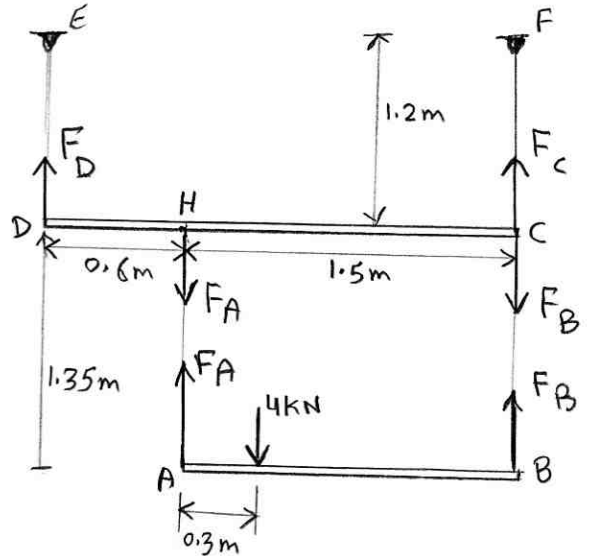
Pr #1 :

Given : $A_{AH} = A_{BC} = 40 \times 10^{-6} \text{ m}^2$.

$E_{AH} = E_{BC} = 100 \times 10^9 \text{ Pa}$.

and $A_{DE} = A_{CF} = 50 \times 10^{-6} \text{ m}^2$.

$E_{DE} = E_{CF} = 150 \times 10^9 \text{ Pa}$.



- From the F.B.D. :

For the rigid member AB;

$$+\uparrow \sum M_B = 0; + (4 \text{ kN}) (1.2 \text{ m}) - F_A (1.5 \text{ m}) = 0$$

$$\Rightarrow F_A = 3.2 \text{ kN}.$$

$$+\uparrow \sum F_y = 0; F_A + F_B - 4 \text{ kN} = 0 \Rightarrow F_B = 0.8 \text{ kN}.$$

F.B.D.

Similarly, for rigid member DC;

$$+\uparrow \sum M_C = 0; + (3.2 \text{ kN}) (1.5 \text{ m}) - F_D (2.1 \text{ m}) = 0$$

$$\Rightarrow F_D = 2.286 \text{ kN}.$$

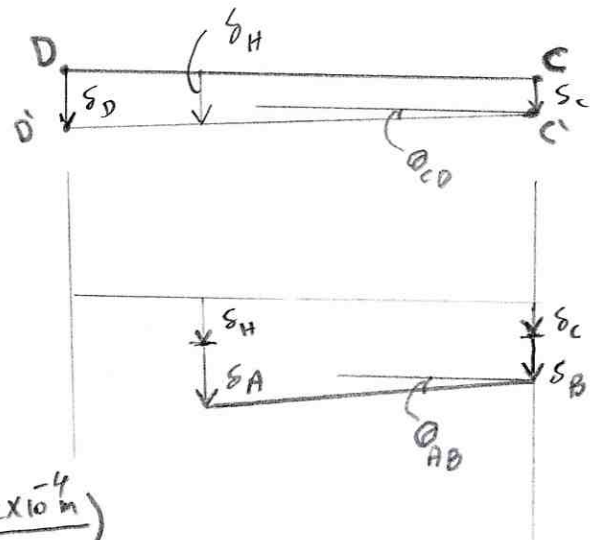
$$+\uparrow \sum F_y = 0; F_D + F_C - 3.2 \text{ kN} - 0.8 \text{ kN} = 0$$

$$\Rightarrow F_C = 1.714 \text{ kN}.$$

- The displacement of member DC;

$$\delta_C = \frac{F_C \cdot (1.2 \text{ m})}{A_{CF} \cdot E_{CF}} = \frac{(1.714 \times 10^3 \text{ N}) \cdot (1.2 \text{ m})}{(50 \times 10^{-6} \text{ m}^2) \cdot (150 \times 10^9 \text{ Pa})} = 2.742 \times 10^{-4} \text{ m}.$$

$$\delta_D = \frac{F_D \cdot (1.2 \text{ m})}{A_{DE} \cdot E_{DE}} = \frac{(2.286 \times 10^3 \text{ N}) \cdot (1.2 \text{ m})}{(50 \times 10^{-6} \text{ m}^2) \cdot (150 \times 10^9 \text{ Pa})} = 3.658 \times 10^{-4} \text{ m}.$$



The angle of tilt of member CD:

$$\theta_{CD} = \tan^{-1} \left(\frac{\delta_D - \delta_C}{2.1 \text{ m}} \right) = \tan^{-1} \left(\frac{3.658 \times 10^{-4} \text{ m} - 2.742 \times 10^{-4} \text{ m}}{2.1 \text{ m}} \right)$$

$$= \tan^{-1} (4.3619 \times 10^{-5}) = \underline{\underline{2.5 \times 10^{-3} \text{ deg.}}}$$

- The displacement of member AB:

$$\delta_H = \delta_C + \frac{1.5\text{m}}{2.1\text{m}} (\delta_D - \delta_C) = 3.396 \times 10^{-4} \text{ (in member DC)}.$$

$$\delta_A \text{ (due to } F_A) = \frac{F_A \cdot (1.35\text{m})}{A_{AH} \cdot E_{AH}} = \frac{(3.2 \times 10^3 \text{ N}) \cdot (1.35\text{m})}{(40 \times 10^{-6} \text{ m}^2) \cdot (100 \times 10^9 \text{ Pa})} = 1.08 \times 10^{-3} \text{ m}.$$

$$\delta_B \text{ (due to } F_B) = \frac{F_B \cdot (1.35\text{m})}{A_{BC} \cdot E_{BC}} = \frac{(0.8 \times 10^3 \text{ N}) \cdot (1.35\text{m})}{(40 \times 10^{-6} \text{ m}^2) \cdot (100 \times 10^9 \text{ Pa})} = 2.7 \times 10^{-4} \text{ m}.$$

$$\text{The displacement of A (down)} = \delta_H + \delta_A = 3.396 \times 10^{-4} \text{ m} + 1.08 \times 10^{-3} \text{ m} \\ = 1.4196 \times 10^{-3} \text{ m}.$$

$$\text{The displacement of B (down)} = \delta_C + \delta_B \\ = 2.742 \times 10^{-4} \text{ m} + 2.7 \times 10^{-4} \text{ m} \\ = 5.442 \times 10^{-4} \text{ m}.$$

The angle of tilt of member AB:

$$\theta_{AB} = \tan^{-1} \left(\frac{\delta_A - \delta_B}{1.5\text{m}} \right) = \tan^{-1} \left(\frac{1.4196 \times 10^{-3} \text{ m} - 5.442 \times 10^{-4} \text{ m}}{1.5\text{m}} \right) \\ = \tan^{-1} (8.754 \times 10^{-4}) \\ = \underline{\underline{0.0502^\circ}}.$$

Pr # 2 :

$$\text{Given: } (\sigma_{\text{all}})_{BC} = 80 \text{ MPa}$$

$$r_{BC} = 3 \text{ mm}$$

$$E = 100 \text{ GPa}$$

Since the maximum movement (down) of point D is 4 mm

\Rightarrow the corresponding displacement (down)

$$\text{of point B} = \frac{4 \text{ mm} \times 1250 \text{ mm}}{2000 \text{ mm}} = 2.5 \text{ mm}$$

$\therefore \sigma_{\text{all}}$ for cable BC = 80 MPa

$$\Rightarrow (\epsilon_{\text{max}})_{BC} = \frac{80 \times 10^6 \text{ Pa}}{100 \times 10^9 \text{ Pa}} = 0.8 \times 10^{-3} \frac{\text{mm}}{\text{mm}}$$

\Rightarrow The max. allow displacement of B = $(\epsilon_{\text{max}})_{BC} \cdot L_{BC}$

$$= (0.8 \times 10^{-3} \frac{\text{mm}}{\text{mm}}) \cdot (1000 \text{ mm})$$

$$= 0.8 \text{ mm} < 2.5 \text{ mm}$$

$\Rightarrow \therefore$ The max force in cable BC = $(\sigma_{\text{all}})_{BC} \cdot A_{BC}$

$$= (80 \times 10^6 \text{ Pa}) \cdot [\pi \cdot (3.0 \times 10^{-3} \text{ m})^2]$$

$$= \underline{\underline{2,261.95 \text{ N}}}$$

By applying equilibrium equ:

$$\curvearrowleft \sum M_A = 0 ; +P \cdot (2000 \text{ m}) - (2,261.95 \text{ N}) \cdot (1250 \text{ mm}) = 0$$

$$\Rightarrow \boxed{P = 1,413.72 \text{ N}}$$

Pr #3

$$\delta = \frac{P \cdot L}{AE}$$

From the table in the inside cover of the textbook:

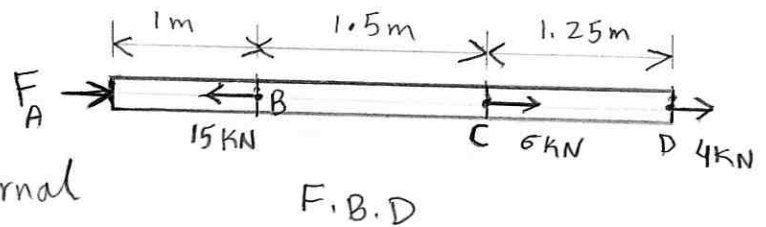
For A36 steel $\Rightarrow E_{st} = 200 \text{ GPa}$

For C8340 Red Brass $\Rightarrow E_{br} = 101 \text{ GPa}$

The cross-section area of the rod, $A = \frac{\pi}{4} (0.01 \text{ m})^2$

Internal Forces:

By applying the section method and equilibrium equations, the internal forces and in the three segments can be obtained as shown below:



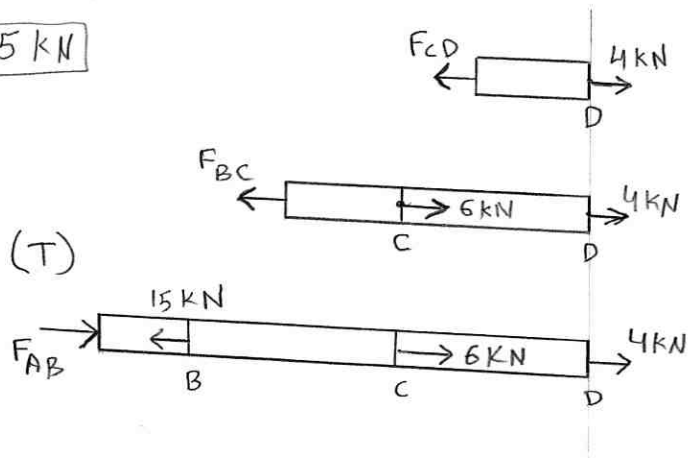
For the entire rod; $\sum F_x = 0$; $+F_A - 15 \text{ kN} + 6 \text{ kN} + 4 \text{ kN} = 0$

$\Rightarrow F_A = 5 \text{ kN}$

In segment CD: $F_{CD} = +4 \text{ kN (T)}$

In segment BC: $F_{BC} = 6 \text{ kN} + 4 \text{ kN} = +10 \text{ kN (T)}$

In segment AB: $F_{AB} = 15 \text{ kN} - 6 \text{ kN} - 4 \text{ kN} = 5 \text{ kN (C)}$



a) Total change in length $= \delta_{B/A} + \delta_{C/B} + \delta_{D/C} = \frac{F_{AB} \cdot L_{AB}}{A \cdot E_{st}} + \frac{F_{BC} \cdot L_{BC}}{A \cdot E_{br}} + \frac{F_{CD} \cdot L_{CD}}{A \cdot E_{st}}$

$$= \frac{(-5 \times 10^3 \text{ N}) \cdot (1 \text{ m})}{\frac{\pi}{4} (0.01 \text{ m})^2 \cdot 200 \times 10^9 \text{ Pa}} + \frac{(10 \times 10^3 \text{ N}) \cdot (1.5 \text{ m})}{\frac{\pi}{4} (0.01 \text{ m})^2 \cdot 101 \times 10^9 \text{ Pa}} + \frac{(4 \times 10^3 \text{ N}) \cdot (1.25 \text{ m})}{\frac{\pi}{4} (0.01 \text{ m})^2 \cdot (200 \times 10^9 \text{ Pa})}$$

$$= -318.3 \times 10^{-6} \text{ m} + 1,891 \times 10^{-6} \text{ m} + 318.3 \times 10^{-6} \text{ m} = +1,891 \times 10^{-6} \text{ m}$$

b) $\delta_{D/B} = \delta_{D/C} + \delta_{C/B} = 1,891 \times 10^{-6} \text{ m} + 318.3 \times 10^{-6} \text{ m} = +2,209.3 \times 10^{-6} \text{ m}$

c) $\delta_{D/C} = +318.3 \times 10^{-6} \text{ m}$

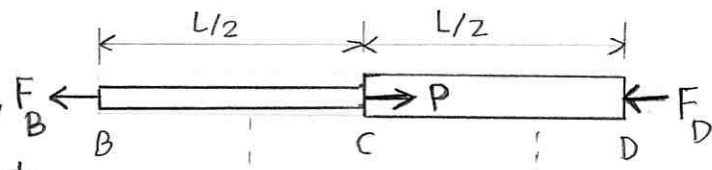
$\delta_{D/A} = \text{total change in length} = +1,891 \times 10^{-6} \text{ m}$

Pr #4 :

Given : $A_{BC} = 100 \text{ mm}^2$, $A_{CD} = 250 \text{ mm}^2$.

$P = 5 \text{ kN}$, $E = 100 \text{ GPa}$.

This problem is statically indeterminate, the equilibrium equation is not sufficient to determine the two reactions at B and D.



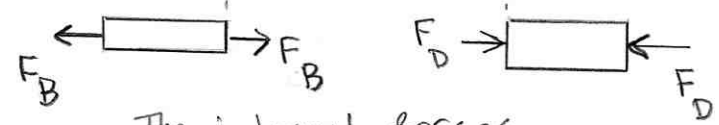
F.B.D.

From equilibrium equation;

$$\rightarrow \sum F_x = 0; P - F_D - F_B = 0$$

$$\Rightarrow \boxed{5 \text{ kN} = F_D + F_B} \quad \text{--- (1)}$$

Additional equation is needed to solve this problem.



The internal forces in both rods

Since both rods are fixed at both ends, then the compatibility condition becomes:

$$\delta_{B/D} = 0 \quad \text{--- (2)}$$

$$\delta_{B/D} = \delta_{B/C} + \delta_{C/D} = \frac{(F_B) \cdot (L/2)}{(A_{BC}) \cdot (E)} - \frac{(F_D) \cdot (L/2)}{(A_{CD}) \cdot (E)} = 0$$

$$\Rightarrow F_B = \frac{A_{BC}}{A_{CD}} \cdot F_D = \left(\frac{100 \text{ mm}^2}{250 \text{ mm}^2} \right) \cdot F_D$$

$$\Rightarrow \boxed{F_B = 0.4 F_D}$$

substitute into equ (1) $\Rightarrow 5 \text{ kN} = F_D + 0.4 F_D \Rightarrow F_D = 3.57 \text{ kN}$

$\therefore \sigma_{BC} = \frac{1.43 \times 10^3 \text{ N}}{100 \times 10^{-6} \text{ m}^2} = 14.3 \times 10^6 \text{ Pa} = \underline{14.3 \text{ MPa (T)}} \Rightarrow F_B = 1.43 \text{ kN}$

$\sigma_{CD} = \frac{-3.57 \times 10^3 \text{ N}}{250 \times 10^{-6} \text{ m}^2} = -14.28 \times 10^6 \text{ Pa} = -14.28 \text{ MPa (C)}$.

The displacement of point C = $\frac{F_B \cdot (L/2)}{A_{CB} \cdot E} = \frac{1.43 \times 10^3 \text{ N} \cdot (L/2)}{(100 \times 10^{-6} \text{ m}^2) \cdot (100 \times 10^9 \text{ Pa})} = \underline{7.15 \times 10^{-5} \text{ L}} \rightarrow$