Solution of HW # 2

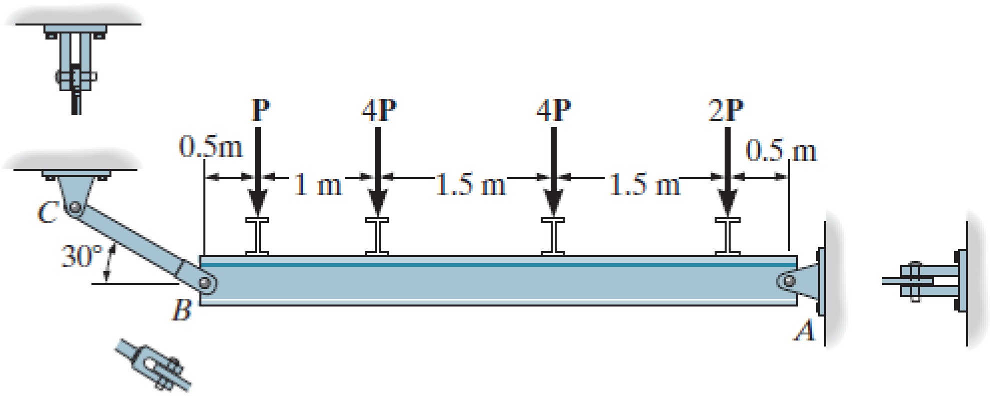
Problem #1:

Given:

The figure shown

Pin at A: $D = 20\ mm$; $\tau_{allow} = 30\ MPa$

Pins at B and C: $D = 25\ mm$; $\tau_{allow} = 20\ MPa$



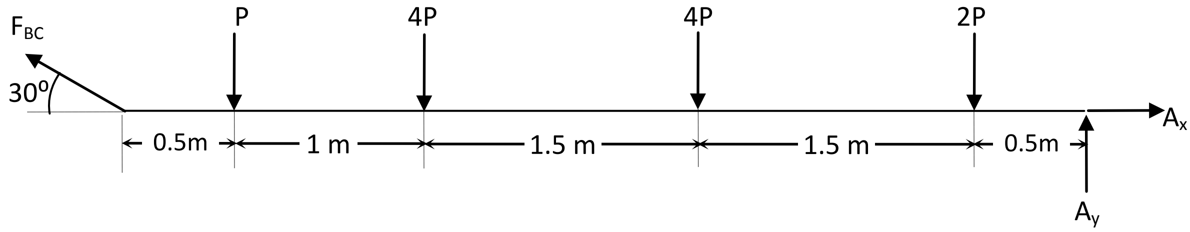
Required:

P_{max} *that can be safely applied*

Solution:

*Note that BC is a "**two-force member**". (How?! So what?!)*

The FBD is drawn below. We need to calculate the reactions/forces on the pins in terms of P



FBD

$+\circlearrowleft \Sigma M_A = 0$ (Why *start with this equation?!)*

$\Rightarrow 2P(0.5) + 4P(2) + 4P(3.5) + P(4.5) - F_{BC} sin30°(5) = 0 \Rightarrow$

$F_{BC} = 11P$

Solution of HW # 2

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow A_x - 11PCos30° = 0$

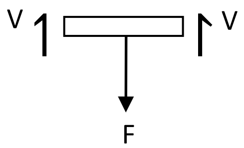
$\Rightarrow A_x = 9.52628P$

$+\uparrow \Sigma F_y = 0 \Rightarrow A_y - 2P - 4P - 4P - P + 11PSin30° = 0 \Rightarrow$

$\Rightarrow A_y = 5.5P$

Note that all pins at A, B, and C are in ***double*** *shear. See the original figure.*

$$\tau = \frac{V}{A}$$



For the pin at A, we need to take the resultant of A_x and A_y. (Why?!)

$V_A = R_A = \sqrt{(9.52628P)^2 + (5.5P)^2} = 11P$ *(As expected = F_{BC}. How?!)*

$$Set\ \tau_A \equiv 30 = \frac{11P}{2\left(\frac{\pi}{4}\right)(20)^2} \Rightarrow$$

$P_{max}^{(1)} = 1713.6\ N = 1.714\ kN.$

For the pins at B and C, both are under the force F_{BC}. ⇒

$$\tau_B = \tau_C = \frac{F_{BC}}{A} \Rightarrow$$

$$\frac{11P}{2\left(\frac{\pi}{4}\right)(25)^2} \equiv 20 \Rightarrow$$

$P_{max}^{(2)} = 1785.0\ N = 1.785\ kN.$

From $P_{max}^{(1)}$ and $P_{max}^{(2)}$ we choose the ***smaller*** *value for the* ***maximum allowable*** *P.(Why?!).*

$\Rightarrow \boxed{\boldsymbol{P_{max} = 1.714\ kN}}$

In our case in this problem, we can not tell which one "controls", the pin at A, or the pins at B and C. (Why?!). Thus we had to check both.

In some cases, it may be possible. (When, and how?!)

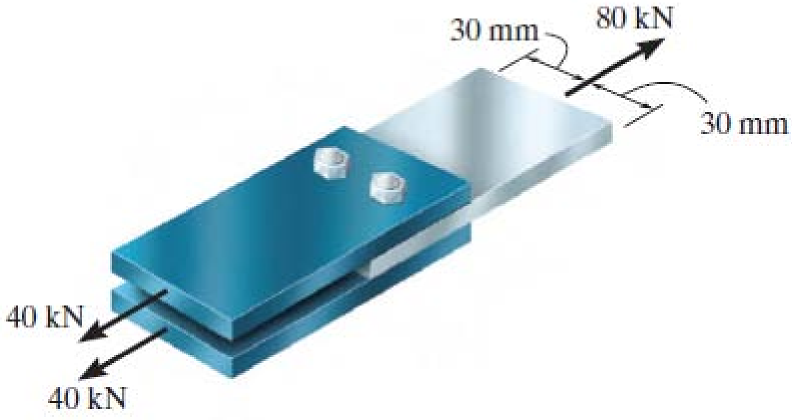
Solution of HW # 2

<u>Problem #2:</u>

Given:

The figure shown

$\tau^{bolt}_{Ultimate} = 400\ MPa$**,** *factor of safety =2.0*



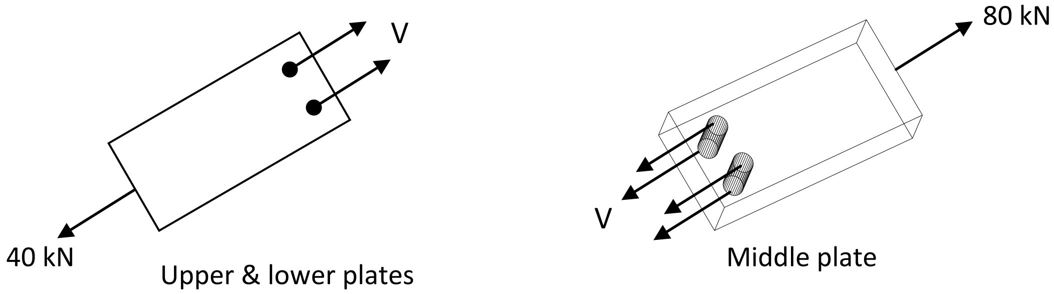
Required:

$D_{required}$ *for the bolts*

Solution:

$$\tau = \frac{V}{A}$$

If we consider the middle plate, the bolts are in ***<u>double shear</u>****; if we consider the upper or lower plate, the bolts are in* ***<u>single shear</u>****. See the FBDs below.*



Both will give us the ***<u>same answer</u>***. (Why, and how*?!)*

$$Allowable\ stress = \frac{Ultimate\ Stress}{Factor\ of\ Safety}$$

$$\Rightarrow \tau_{allow} = \frac{400}{2} = 200\ MPa$$

Solution of HW # 2

*Taking the **lower/upper** plate,*

$$\tau = \frac{V}{A} \Rightarrow 200(10)^6 \equiv \frac{40(10)^3}{\underbrace{2}_{\text{Two bolts}}\left(\frac{\pi}{4}\right)D^2}$$

$$\Rightarrow \boxed{\boldsymbol{D_{required} = 0.01128\ m = 11.3\ mm}}$$

OR

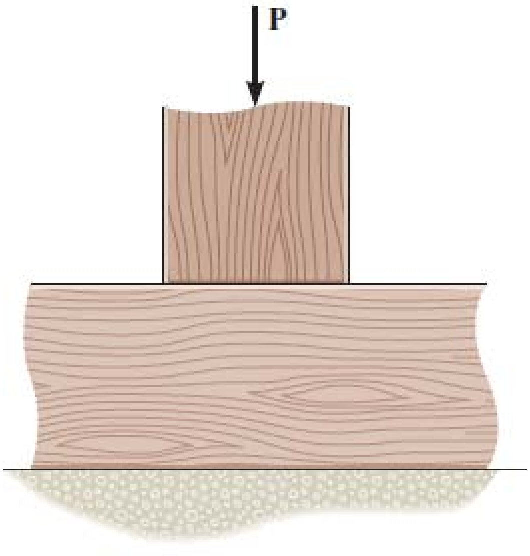
*Taking the **middle** plate,*

$$200(10)^6 \equiv \frac{80(10)^3}{\underbrace{2}_{\text{Double shear}}\underbrace{(2)}_{\text{Two bolts}}\left(\frac{\pi}{4}\right)D^2}$$

$$\Rightarrow D = 11.3\ mm, as\ above$$

Solution of HW # 2

<u>Problem #3:</u>



Given:

The figure shown

60 mm × 60mm oak post; pine block

$\sigma_{bearing_{allow}} = 35\ MPa$ *for the oak*

$\sigma_{bearing_{allow}} = 20\ MPa$ *for the pine*

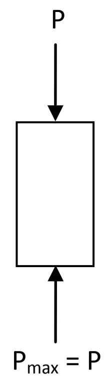
Required:

- *P*
- *If a rigid bearing plate is used between the oak and pine,* $\boldsymbol{A_{plate}}$ *required, so the maximum load can be supported.*
- P_{max}.

Solution:

For the first requirement, clearly the pine block "controls". (Why, and how?!)

Thus,



$$\sigma_b = \frac{P}{A} \equiv \left(\sigma_{b_{allow}}\right)_{pine} \Rightarrow$$

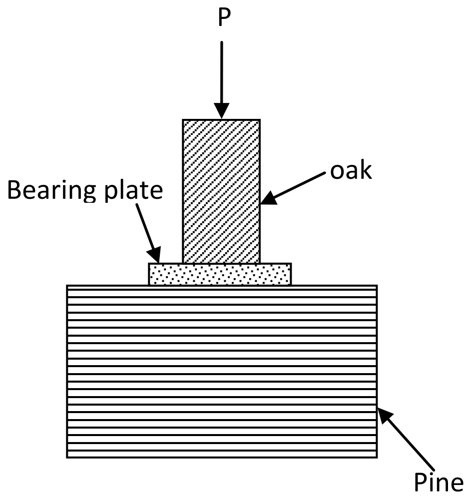
$$\frac{P_{max}}{60 \times 60} \equiv 20 \Rightarrow \boxed{\boldsymbol{P_{max_{allow}} = 72\ kN}}$$

We need to find $\boldsymbol{P_{max}}$ *(which is for the oak) for the second requirement. (Why?!)*

$$= \frac{P_{max}}{A} \equiv \left(\sigma_{max_{allow}}\right)_{oak} \Rightarrow$$

$$\frac{P_{max}}{60 \times 60} \equiv 35 \Rightarrow \boxed{\boldsymbol{P_{max} = 126\ kN}}$$

Now, we set $\left(\sigma_b\right)_{pine} \equiv 20\ MPa$ *(Why?!)* $\Rightarrow$



$$\frac{P_{max}}{A_{bearing\ plate}} \equiv \sigma_{allow_{pine}}$$

$$\frac{126 \times 10^3}{A_{bearing\ plate}} \equiv 20 \Rightarrow \boxed{\boldsymbol{A_{bearing\ plate} = 6300\ mm^2}}$$

Solution of HW # 2

Problem #4:

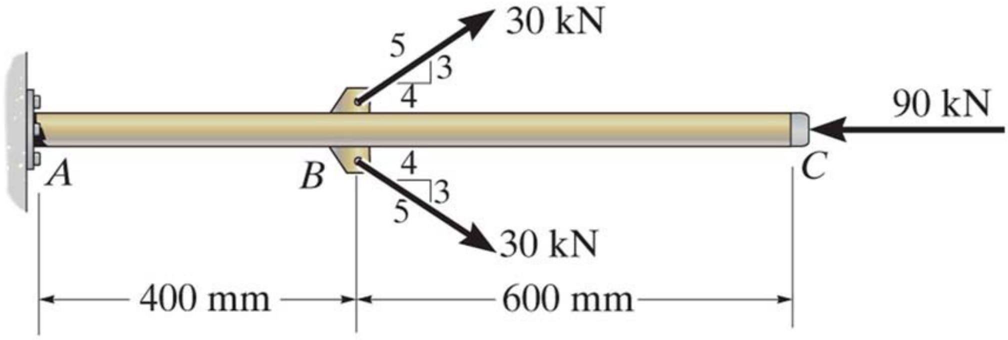
Given:

The figure shown

Circular cross section

$\sigma_{allow_{AB}} = 80\ MPa$

$\sigma_{allow_{BC}} = 130\ MPa$



Required:

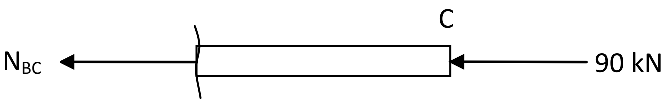
D_{min}

Solution:

$$\sigma = \frac{P}{A} = \frac{N}{A} \text{ where } N \text{ is the } \underline{\underline{\textbf{internal}}} \text{ force.}$$

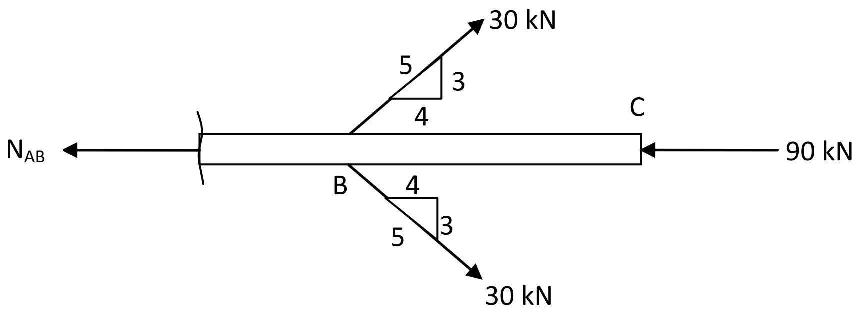
*We have two criteria, one for AB, and the other one for BC. By looking at the numbers (loads & allowable stresses), we **cannot** tell which one controls. (Why, and how?!)*

*First, we need to determine the **internal** forces from the FBD's below.*



$N_{BC} = -90\ kN = 90\ kN$ "C"

Solution of HW # 2



$$N_{AB} = -90 + 2(30)\left(\frac{4}{5}\right) = -42\ kN = 42\ kN\ "C"$$

$$\sigma_{AB} = \frac{N_{AB}}{A} \Rightarrow set\ 80 \equiv \frac{42(10)^3}{\frac{\pi}{4} \times D_{min}^2} \Rightarrow$$

$$D_{min}^{(1)} = 25.85\ mm$$

Note that the material behavior in tension is assumed to be the same as in compression.

$$\sigma_{BC} = \frac{N_{BC}}{A} \Rightarrow \qquad set\ 130 \equiv \frac{90(10)^3}{\frac{\pi}{4} \times D_{min}^2} \Rightarrow$$

$$D_{min}^{(2)} = 29.69\ mm$$

*For the **minimum** required D, we choose the **maximum** of $D_{min}^{(1)}$ and $D_{min}^{(2)}$. (Why?!)* $\Rightarrow$

$$\boxed{\boldsymbol{D_{min} = 29.7\ mm}}$$

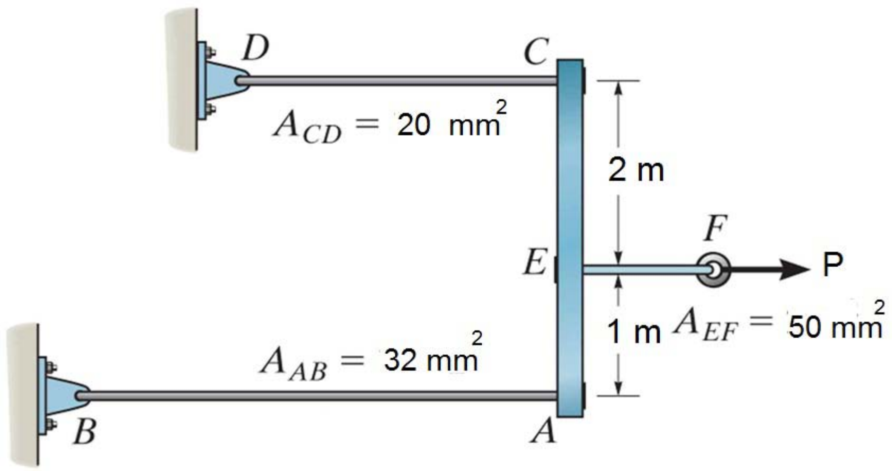
Solution of HW # 2

Problem #5:

Given:

The figure shown

$\sigma_{allow} = 20\ MPa\ in\ each\ cable.$



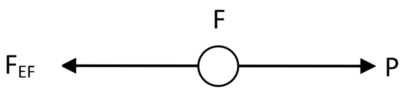
Required:

$P_{\max allow}$

Solution:

To find the stresses in the cables, we need to find the ***internal*** *forces in them. (Why?!)*

For cable EF, FBD (1) is drawn

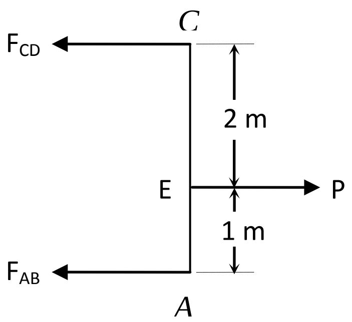


FBD (1)

$$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$$

$$P - F_{EF} = 0 \Rightarrow \quad F_{EF} = P$$

For the cables AB and CD, FBD (2) is drawn



FBD (2)

$$\circlearrowleft + \ \Sigma M_C = 0 \Rightarrow$$

$$P(2) - F_{AB}(3) = 0$$

$$\Rightarrow F_{AB} = \frac{2}{3}P$$

$$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$$

$$P - \frac{2}{3}P - F_{CD} = 0 \Rightarrow \quad F_{CD} = \frac{1}{3}P$$

Solution of HW # 2

By looking at the numbers (forces & areas), we cannot tell which cable "controls". (Why, and how?!). Thus we need to check all three cables.

Cable AB:

$$\sigma_{AB} = \frac{F_{AB}}{A} \Rightarrow$$

$$Set\ \sigma_{AB} \equiv 20\ MPa = \frac{\frac{2}{3}P}{32}$$

$$\Rightarrow P_{max}^{(1)} = 960\ N$$

Cable CD:

$$\sigma_{CD} \equiv 20\ MPa = \frac{\frac{1}{3}P}{20}$$

$$\Rightarrow P_{max}^{(2)} = 1200\ N$$

Cable EF:

$$\sigma_{EF} \equiv 20\ MPa = \frac{P}{50}$$

$$\Rightarrow P_{max}^{(3)} = 1000\ N$$

*For the **maximum allowable** load **P**, we choose the **minimum** of $P_{max}^{(1)}$, $P_{max}^{(2)}$, and $P_{max}^{(3)}$. (Why?!)*

$$\Rightarrow \boxed{\boldsymbol{P_{\max\ allow} = 960\ N}}$$

(which is controlled by cable AB)