CE 203 STRUCTURAL MECHANICS I

First Semester 2012 / 2013 (**121**)

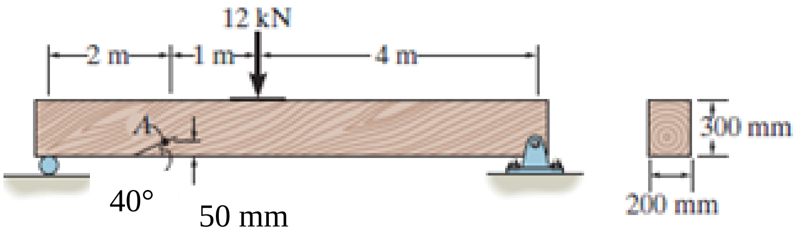
HOMEWORK NO. 14 (Key Solution)

- **Textbook Sections Covered:** Chapter 9 (Transformation of Stress)
- **DUE DATE:** **Monday, 25 December 2012**

Problem # 1 :- problem 9-23 **with the revised data**.

Given Data:

- The shown wood beam and its cross section.
- Point A is located at 50 mm from the bottom.
- The angle of the inclination of grain is 40 degrees.
- Using transformation equations for the solution.

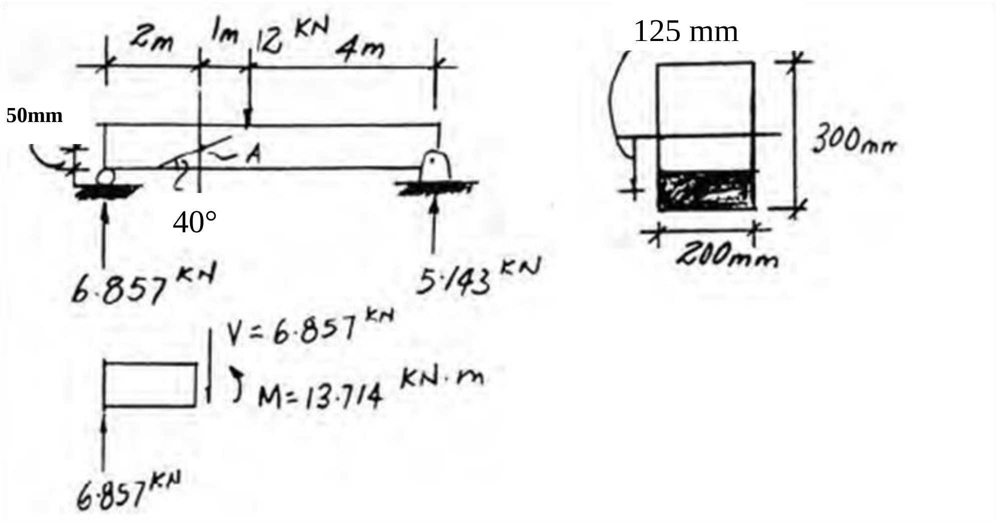


Required:

❖ Determining the normal and shear stress that acts perpendicular and parallel to the grain due to the loading.

Solution:

By solving the problem to find the reactions, shear and bending moment, the results are as shown below:-



$$I = \frac{0.2 * 0.3^3}{12} = 0.45 * 10^{-3}\ m^4$$

$$Q_A = \bar{y}A' = 0.125 * 0.2 * 0.05 = 1.25 * 10^{-3}\ m^3$$

$$\sigma_A = \frac{M * y_A}{I} = \frac{13.714 * 10^3 * 0.1}{0.45 * 10^{-3}} = 3.05\ MPa\ \ (Tension)$$

$$\tau_A = \frac{V * Q_A}{It} = \frac{6.875 * 10^3 * 1.25 * 10^{-3}}{0.45 * 10^{-3} * 0.2} = 0.0955\ MPa$$

$$\sigma_x = 3.05\ MPa,\ \ \sigma_y = 0\ \ \ , \tau_{xy} = -0.0955\ MPa\ \ ,\ \ \theta = 90° + 40° = 130°$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} Cos\ (2\theta) + \tau_{xy} Sin\ (2\theta)$$

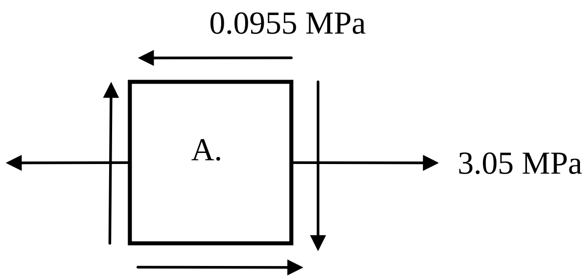
$$\sigma_{x'} = \frac{3.05 + 0}{2} + \frac{3.05 - 0}{2} Cos\ (2 * 130) + (-0.0955) Sin\ (2 * 130)$$

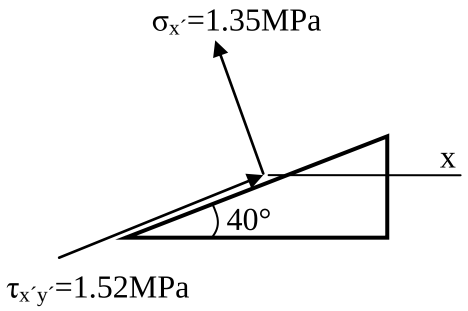
$$\boldsymbol{\sigma_{x'} = 1.35\ MPa.}\qquad \boldsymbol{Ans}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} Sin\ (2\theta) + \tau_{xy} Cos\ (2\theta)$$

$$\tau_{x'y'} = -\frac{3.05 - 0}{2} Sin\ (2 * 130) + (-0.0955) Cos\ (2 * 130)$$

$$\boldsymbol{\tau_{x'y'} = 1.52\ MPa}\qquad \boldsymbol{Ans}$$



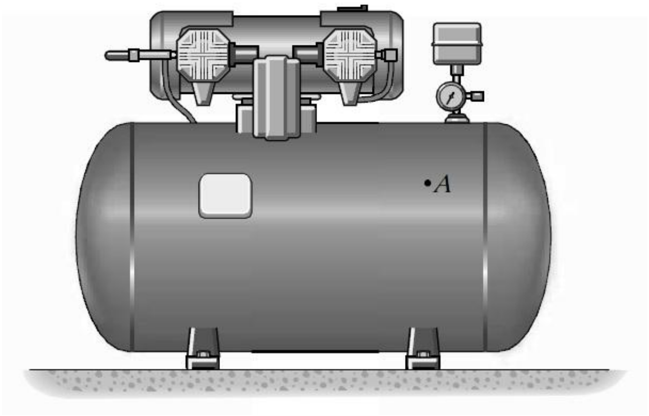


State of stress at point A

Problem # 2 :

Given Data:

- The shown figure (cylindrical tank of the air compressor).
- Internal radius is 60 mm and the wall thickness is 4 mm.
- Allowable normal stress is 50 MPa
- Allowable shear stress is 12 MPa.
- Using transformation equations for the solution.



Required:

- Determination the largest internal pressure that can be applied to the tank.

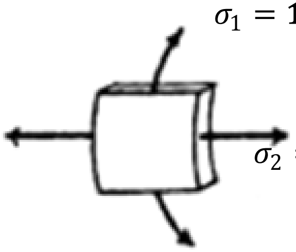
Solution:

For the cylindrical tank, there are two stresses (hoop stress and longitudinal stress).

$Since\ \frac{r}{t} = \frac{60}{4} = 15 > 10\ ,\ \ thin\ wall\ analysis \quad \Rightarrow$

$$hoop\ stress\ \ \sigma_1 = \frac{pr}{t} = \frac{p * 60}{4} = \mathbf{15p}\ (\boldsymbol{MPa})$$

$$longitudinal\ stress\ \ \sigma_2 = \frac{pr}{2t} = \frac{p * 60}{8} = \mathbf{7.5\ p}\ (\boldsymbol{MPa})$$



$\sigma_1 = \mathbf{15p}$

$\sigma_2 = \mathbf{7.5p}$

$$maximum\ shear\ stress\ \tau_{\max\ in\ plane} = \sqrt{\left(\frac{15p - 7.5p}{2}\right)^2 + 0} = 3.75\ p\ MPa.$$

Now using the allowable normal stress $\sigma_{av} = 15\ p\ MPa = 50\ MPa\ \Rightarrow p = 3.33\ MPa.(1)$

And the allowable shear stress $\tau_{\max\ in\ plane} = 3.75\ p\ MPa = 12\ MPa \quad \Rightarrow p = 3.2\ MPa.\ (2)$

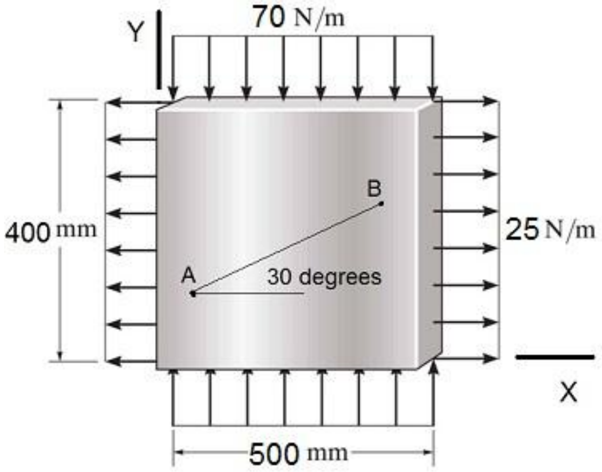
So, the largest internal pressure that can be applied is min(3.33, 3.2 MPa)

is 3.2 MPa Ans.

Problem # 3 :

Given Data:

- The shown figure (square plate subjected to distributed loads).
- Plate thickness =15 mm.
- Using the transformation equations for the solution.



Required:

a. Determining the normal and shear stresses acting on plane AB and Showing the results on an element.
b. Determining the maximum shear stress obtained by rotating the element and Showing the results on an element.

Solution:

$$\sigma_x = \frac{25}{15 * 10^{-3}} = 1.667 kPa,$$

$$\sigma_y = \frac{-70}{15 * 10^{-3}} = -4.667\ kPa, \tau_{xy} = 0\ , \theta = 30° + 90° = 120°$$

The stresses in the plane parallel to AB can be found as follows:-

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} Cos\ (2\theta) + \tau_{xy} Sin\ (2\theta)$$

$$\sigma_{x'} = \frac{1.667 - 4.667}{2} + \frac{1.667 + 4.667}{2} Cos\ (2 * 120°) + (0) Sin\ (2 * 120°)$$

$$\boldsymbol{\sigma_{x'} = -3.0835\ kPa.} \qquad \boldsymbol{Ans}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} Cos\ (2\theta) - \tau_{xy} Sin\ (2\theta)$$

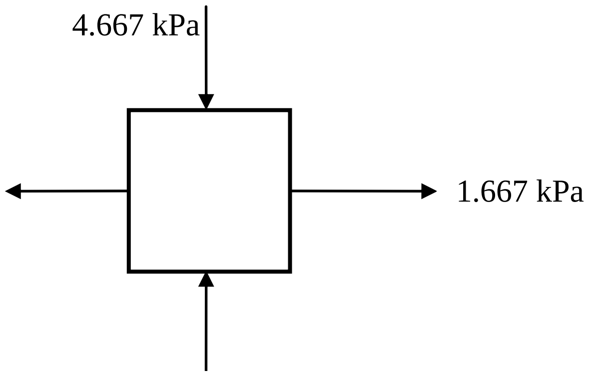
$$\sigma_{y'} = \frac{1.667 - 4.667}{2} - \frac{1.667 + 4.667}{2} Cos\ (2 * 120°) - (0) Sin\ (2 * 120°)$$

$$\boldsymbol{\sigma_{y'} = 0.0835\ kPa.} \qquad \boldsymbol{Ans}$$

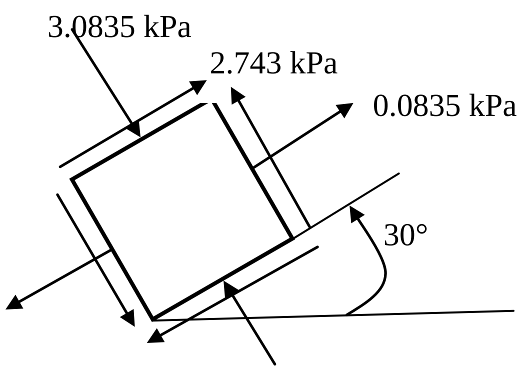
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} Sin\ (2\theta) + \tau_{xy} Cos\ (2\theta)$$

$$\tau_{x'y'} = -\frac{1.667 + 4.667}{2} Sin\ (2 * 120°) + (0) Cos\ (2 * 120°)$$

$$\boldsymbol{\tau_{x'y'} = 2.743\ \ kPa} \qquad \boldsymbol{Ans}$$



Given Plane state of stress



The results on the element on plane AB

The maximum shear stress is as follows:-

$$\tau_{\max\ in\ xy} = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + (\tau_{xy})^2}$$

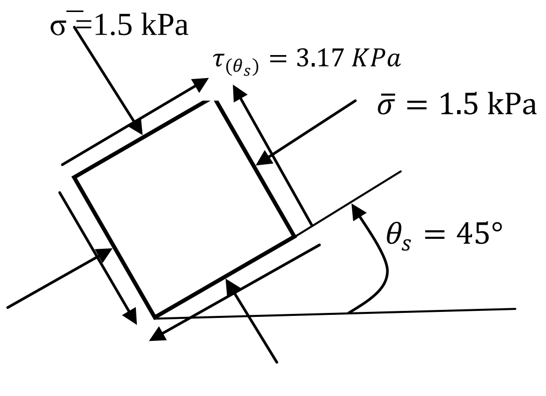
$$\tau_{\max\ in\ xy} = \sqrt{(\frac{1.667 + 4.667}{2})^2 + (0)^2}$$

$\boldsymbol{\tau_{\max\ in\ xy} = 3.167\ KPa.}$ **Ans.**

$$\bar{\sigma} = \left(\frac{1.667 - 4.667}{2}\right) = -1.5\ KPa$$

$$\tan(2\theta_s) = -\frac{\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\tau_{xy}} = -\infty \quad ; \quad \Rightarrow 2\theta_s = \frac{\pi}{2} \Rightarrow \ \theta_s = \frac{\pi}{4} = \mathbf{45}°$$

$$\tau_{(\theta_s)} = -\left(\frac{1.667 + 4.667}{2}\right) sin90 = -3.17$$

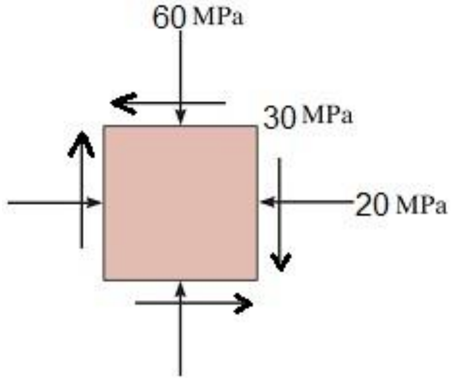


The results on the element

Problem # 4 :

Given Data:

- The shown element (plane state of stress)
- Using Mohr's circle for the solution.



Required:

i. Determining the equivalent state of stress if the element is rotated by 50 degrees clockwise.
ii. Determining the principal normal stresses and orientation.
iii. Determining the maximum shear stress and orientation.
iv. Showing the result of each part on a properly-oriented element

Solution:

$\sigma_x = -20MPa, \quad \sigma_y = -60MPa\,, \qquad and\ \tau_{xy} = -30MPa$

$$\sigma_{av} = \frac{\sigma_x + \sigma_y}{2} = \frac{-20-60}{2} = -40MPa$$

Now we can draw Mohr's Circle as follows:-

$R = \sqrt{20^2 + 30^2} = 36.1\ MPa.$

$\sigma_1 = 36.1 - 40 = -3.9\ MPa\ \textbf{\textit{Ans.}}$

$\sigma_2 = -36.1 - 40 = -76.1\ MPa\ \textbf{\textit{Ans.}}$

$2\theta_{p1} = \tan^{-1}\left(\frac{-30}{20}\right) = -56.3°$

$\theta_{p1} = 28.15°\ clockwise\ \ \textbf{\textit{Ans}}$

For the element rotated by 50° clockwise,

$Sin(100-56.3) = \frac{\tau_{x'y'}}{R} \Rightarrow$

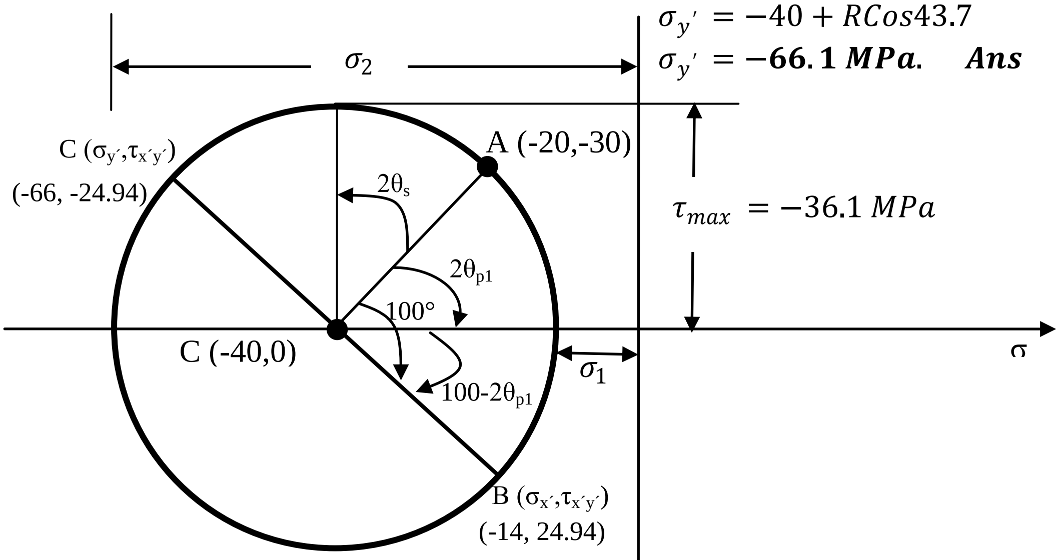
$\tau_{x'y'} = \mathbf{24.94\ MPa.}\quad \textbf{\textit{Ans}}$

$$Cos(100-56.3) = \frac{40-\sigma_{x'}}{R} \Rightarrow$$

$\sigma_{x'} = \mathbf{-13.9\ MPa.}\quad \textbf{\textit{Ans}}$

$\sigma_{y'} = -40 + RCos43.7$

$\sigma_{y'} = \mathbf{-66.1\ MPa.}\quad \textbf{\textit{Ans}}$



$\tau_{max} = R = -36.1\ MPa.\quad \textbf{\textit{Ans}}$

$2\theta_s = \tan^{-1}(-\left(\frac{40-20}{-30}\right)) = 33.7°$

$\theta_s = 16.85°\quad counterclockwise\ \textbf{\textit{Ans.}}$

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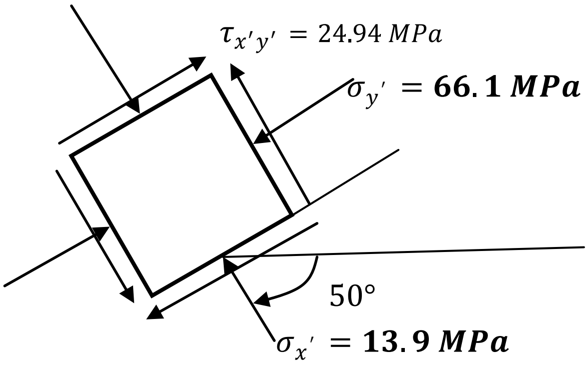
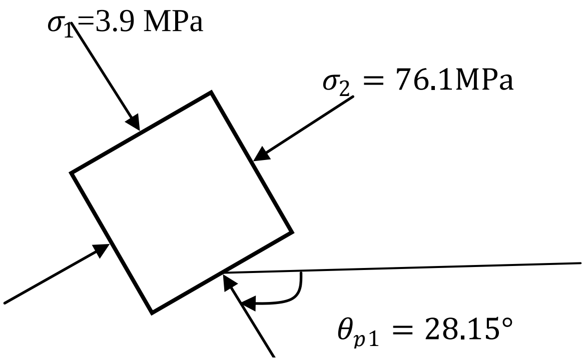
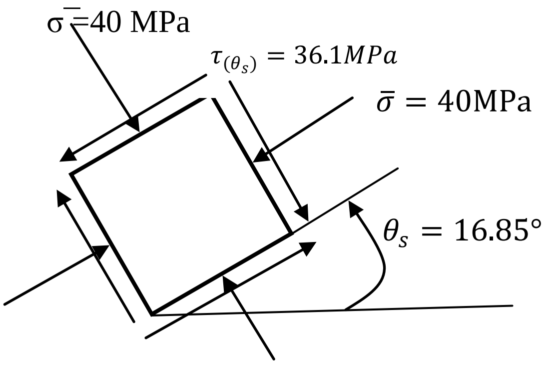


The results on the element (the element rotated by 50° clockwise**)**



The results on the element (principle normal stresses and orientation)

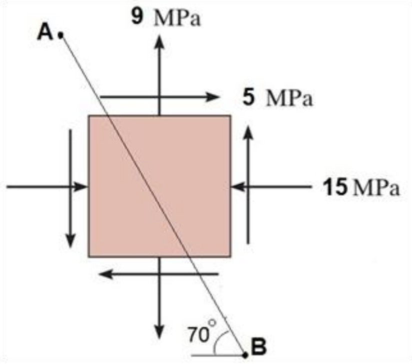


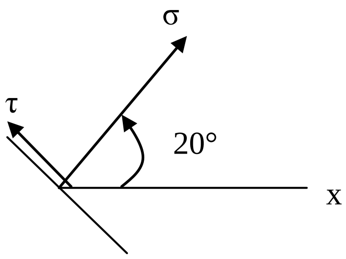
The results on the element (maximum shear stress and orientation)

Problem # 5 :

Given Data:

- The shown element
- Using Mohr's circle for the solution.





Required:

i. Determining the equivalent state of stress normal and tangent to Line AB.
ii. Determining the principal normal stresses and orientation.
iii. Determining the maximum shear stress and orientation.
iv. Showing the result of each part on a properly-oriented element

Solution:

$\sigma_x = -15MPa, \quad \sigma_y = 9MPa, \qquad and\ \tau_{xy} = 5MPa$

$$\sigma_{av} = \frac{\sigma_x + \sigma_y}{2} = \frac{-15+9}{2} = -3MPa$$

Now we can draw Mohr's Circle as follows:-

$R = \sqrt{12^2 + 5^2} = 13\ MPa.$

$\sigma_1 = 13 - 3 = 10\ MPa\ \boldsymbol{Ans}.$

$\sigma_2 = -13 - 3 = -16\ MPa\ \boldsymbol{Ans}.$

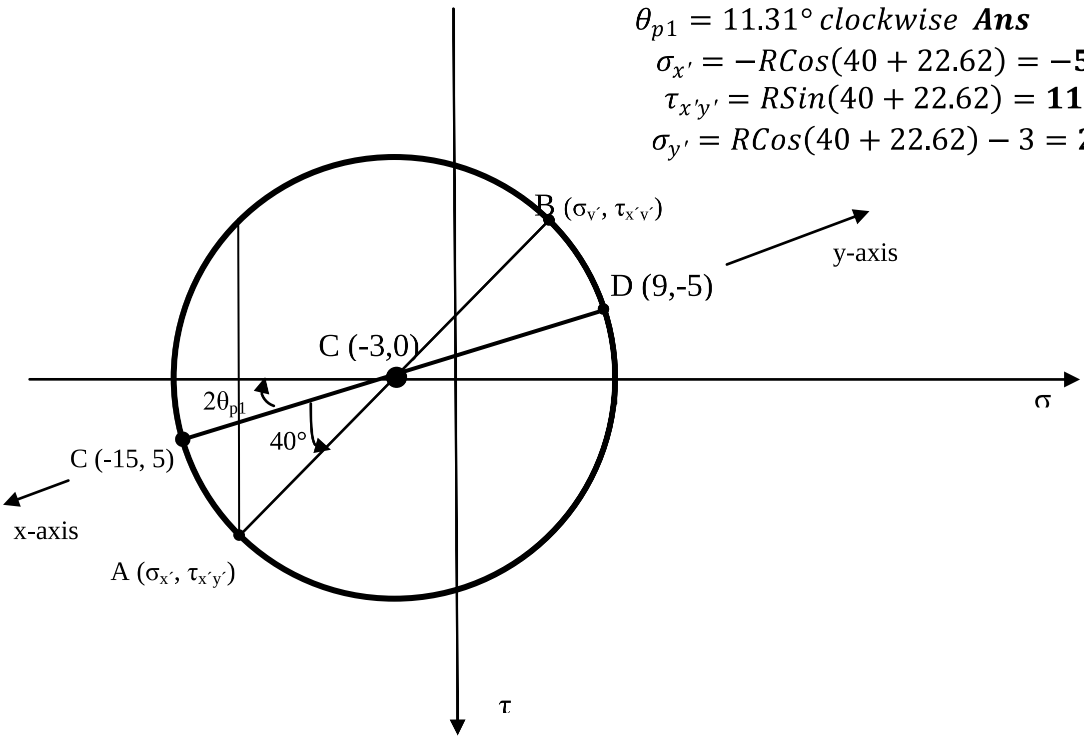
$2\theta_{p1} = \tan^{-1}\left(\frac{5}{-12}\right) = -22.62°$

$\theta_{p1} = 11.31°\ clockwise\ \ \boldsymbol{Ans}$

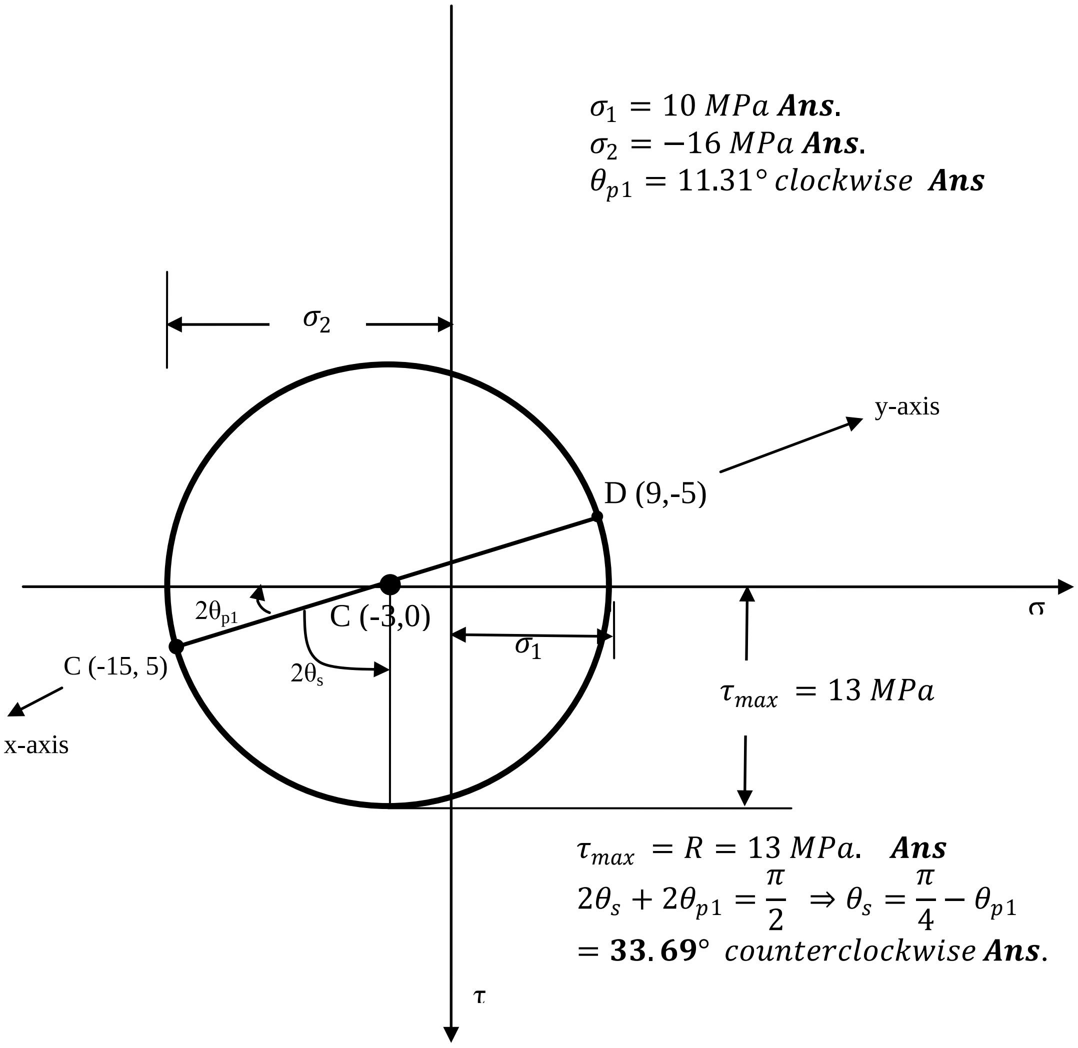
$\sigma_{x'} = -RCos(40 + 22.62) = \mathbf{-5.98}\boldsymbol{MPa}$

$\tau_{x'y'} = RSin(40 + 22.62) = \mathbf{11.54}\boldsymbol{MPa}$

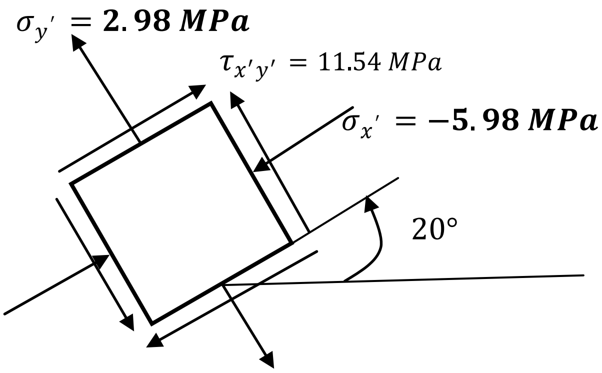
$\sigma_{y'} = RCos(40 + 22.62) - 3 = \mathbf{2.98}\boldsymbol{MPa}$



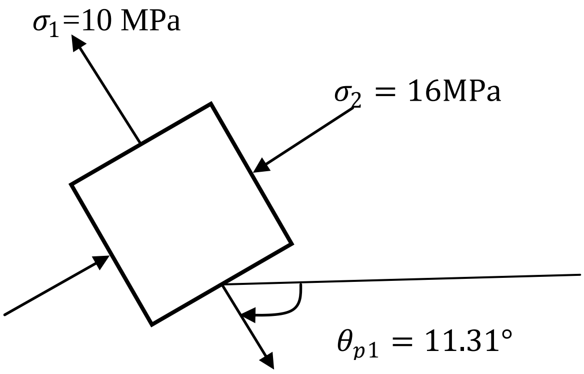
$\sigma_1 = 10\ MPa$ ***Ans.***
$\sigma_2 = -16\ MPa$ ***Ans.***
$\theta_{p1} = 11.31°\ clockwise$ ***Ans***



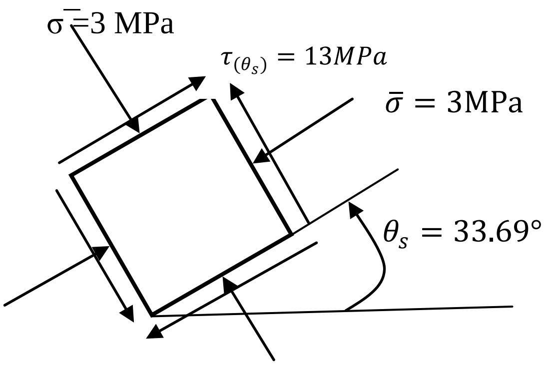
$\tau_{max} = R = 13\ MPa.$ ***Ans***
$2\theta_s + 2\theta_{p1} = \frac{\pi}{2} \Rightarrow \theta_s = \frac{\pi}{4} - \theta_{p1}$
$= \mathbf{33.69°}\ counterclockwise$ ***Ans.***



The results on the element (the element rotated by 20° counterclockwise**)**



The results on the element (principle normal stresses and orientation)



The results on the element (maximum shear stress and orientation)