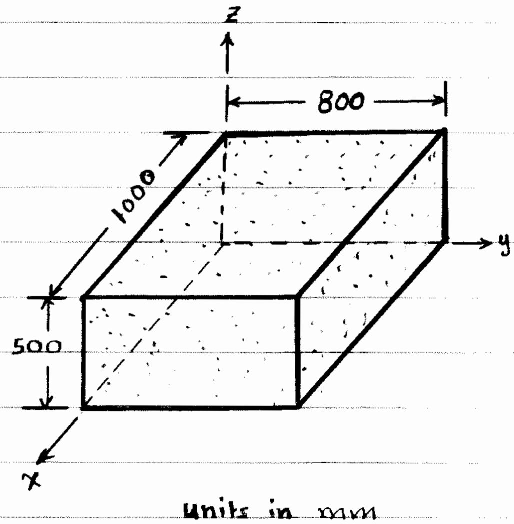
#7

Examples on Multi-Dimensional States of Stress and Strain (Generalized Hooke's Law)

Example 1:



Given:

The block shown,
After applying forces in the x, y, and z directions, the changes in dimensions are

$\delta_x = 0.3$ mm

$\delta_y = -0.16$ mm

$\delta_z = 0.2$ mm

$E = 70$ GPa ; $\nu = 0.3$

Req'd.:

The normal strains and stresses

Sol'n.:

$$\varepsilon_x = \frac{\delta_x}{l_x} = \frac{0.3}{1000} = 300\,\underbrace{(10)^{-6}}_{\mu}\ \text{mm/mm} \Rightarrow \boxed{\varepsilon_x = 300\ \mu\ \text{mm/mm}}$$

$$\varepsilon_y = \frac{\delta_y}{l_y} = \frac{-0.16}{800} \Rightarrow \boxed{\varepsilon_y = -200\ \mu\ \text{mm/mm}}$$

$$\varepsilon_z = \frac{\delta_z}{l_z} = \frac{0.2}{500} \Rightarrow \boxed{\varepsilon_z = 400\ \mu\ \text{mm/mm}}$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu(\varepsilon_y + \varepsilon_z)\right] = \frac{70\,(10)^9}{(1+0.3)(1-0.6)}\left[(1-0.3)300 + 0.3(-200+400)\right](10)^{-6}$$

$$= 1.3462\,(10)^{11}\,(270)\,(10)^{-6} \Rightarrow \boxed{\begin{aligned}\sigma_x &= 36.3\,(10)^6\ \text{Pa}\\ &= 36.3\ \text{MPa (T)}\end{aligned}}$$

$$\sigma_y = \text{"}\ \left[(1-\nu)\varepsilon_y + \nu(\varepsilon_x + \varepsilon_z)\right] \Rightarrow \boxed{\sigma_y = 9.42\ \text{MPa (T) !!}}$$

$$\sigma_z = \text{"}\ \left[(1-\nu)\varepsilon_z + \nu(\varepsilon_x + \varepsilon_y)\right] \Rightarrow \boxed{\sigma_z = 41.7\ \text{MPa (T)}}$$

Example 2:

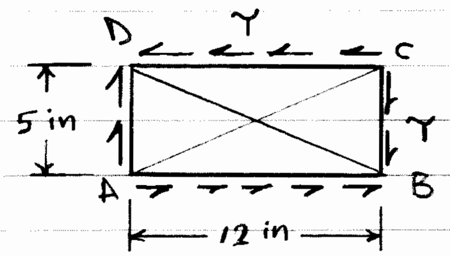
Given:

The plate shown

$\tau = 20$ ksi

$E = 10\,000$ ksi

$\nu = 0.3$

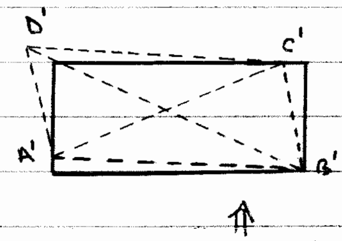


Reqd.:

a) The shearing strains between AB & AD and between AB & BC

b) The change in lengths of AC & BD

c) The normal strains in the diagonals



exaggerated

Soln.:

$$G = \frac{E}{2(1+\nu)} = \frac{10\,000}{2(1+0.3)} = 3846.2 \text{ ksi}$$

a) $\gamma_{DAB} = \dfrac{\tau}{G} = \dfrac{-20}{3846} \Rightarrow \boxed{\gamma_{DAB} = -0.0052 \text{ rad } (\text{in/in})}$

Note that $-$ sign means the angle DAB became $> 90^\circ$ as shown.

$\gamma_{ABC} = \dfrac{+20}{3846} \Rightarrow \boxed{\gamma_{ABC} = 0.0052 \text{ rad } (\text{in/in})}$ ← You may drop the units.

b) & c) Note that the shearing strain is an angle measured in radian.

$AC = BD = \sqrt{(5)^2 + (12)^2} = 13$ in

$\gamma = 0.0052$ rad $= 0.2979^\circ$

$\Rightarrow D'A'B' = 90 + 0.2979 = 90.2979^\circ$

$A'B'C' = 90 - 0.2979 = 89.7021^\circ$

$(B'D')^2 = (A'B')^2 + (A'D')^2 - 2A'B'(A'D')\cos 90.2979^\circ$

$= (12)^2 + (5)^2 - 2(12)(5)\cos 90.2979^\circ = 169.624 \Rightarrow B'D' = 13.0240$ in

Note that $A'B' = AB$, $B'C' = BC$, $C'D' = CD$, $D'A' = DA$ because shearing stress does not change the lengths of the sides of the rectangle.

$\Delta l_{BD} = 13.024 - 13 \Rightarrow \boxed{\Delta l_{BD} = 0.024 \text{ in}} \Rightarrow \varepsilon_{BD} = \dfrac{\Delta l}{l} = \dfrac{0.024}{13} \Rightarrow \boxed{\varepsilon_{BD} = 1846\ \mu \text{ in/in}}$

$(A'C')^2 = (12)^2 + (5)^2 - 2(12)(5)\cos 89.7021 \Rightarrow A'C' = 12.9760$ in

$\Rightarrow \Delta l_{AC} = 12.976 - 13 \Rightarrow \boxed{\Delta l_{AC} = -0.02402} \Rightarrow \varepsilon_{AC} = \dfrac{-0.024}{13} \Rightarrow \boxed{\varepsilon_{AC} = -1846\ \mu \text{ in/in}}$