① #19

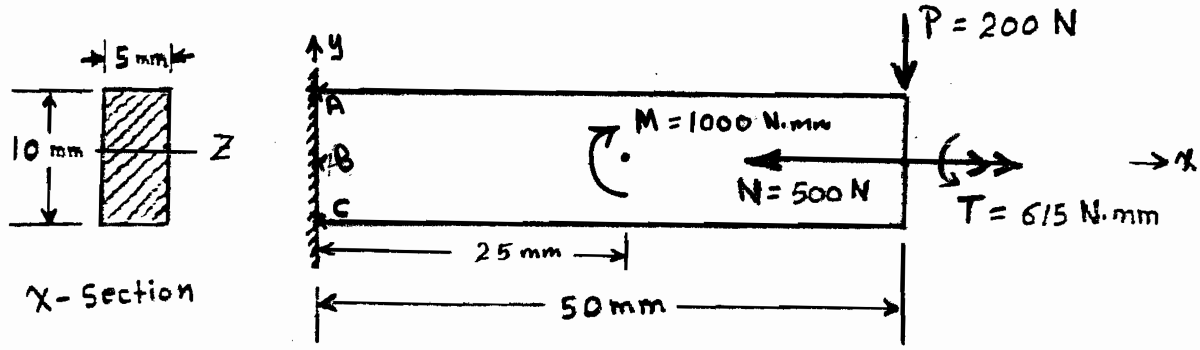
Compound Normal & Shearing Stresses
(Combined) Example 1

Given:

The beam shown below

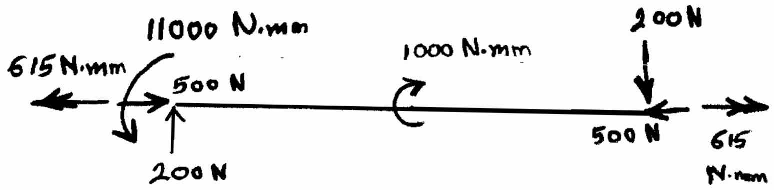
Req.d.:

The values & locations of the points of maximum normal & shearing stresses



Sol.n.:

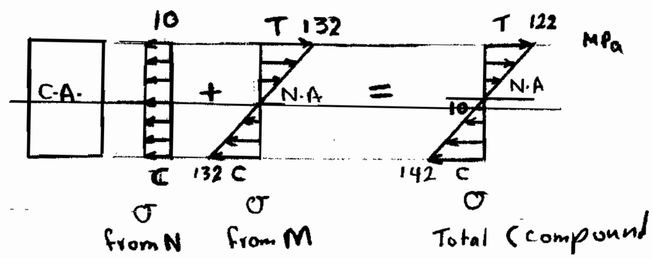
$$I_z = \frac{1}{12}(5)(10)^3 = 416.67 \ mm^4$$



$$\sigma_x = \pm\frac{N_x}{A} \pm \frac{M_z y}{I_z}$$

Max. N & M at the fixed end

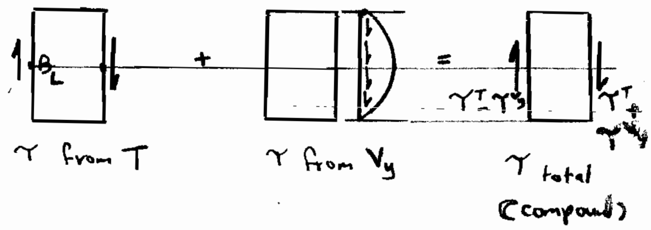
$$\sigma^C_{max} = \frac{500}{10(5)} + \frac{11000(5)}{416.67} = 10 + 132 \Rightarrow$$



$$\boxed{\sigma^C_{max} = 142 \ N/mm^2 = 142 \ MPa \ @ \ C}$$

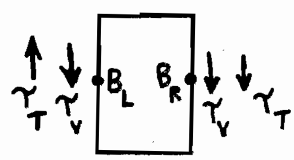
$$\Rightarrow \sigma^T_{max} = -10 + 132 \Rightarrow$$

$$\boxed{\sigma^T_{max} = 122 \ N/mm^2 = 122 \ MPa \ @ \ A}$$



$$\tau_{max} = \frac{VQ}{Ib} + \frac{T}{\alpha b a^2} = \frac{200(5\times5\times2.5)}{416.67(5)} + \frac{615}{0.246(10)(5)^2} = 6.0 + 10 \Rightarrow$$

$$\boxed{\tau_{max} = 16 \ N/mm^2 = 16 \ MPa \ @ \ B_R}$$



Unlike σ, the sign of τ is not very important at this stage.

Note that at B_L $\tau = 6 - 10 = \underline{-4 \ MPa}$

Also note the V & T are constant along the axis of the beam; thus τ_{max} is at all points on the level of B_R (along the x-axis).

Example 2

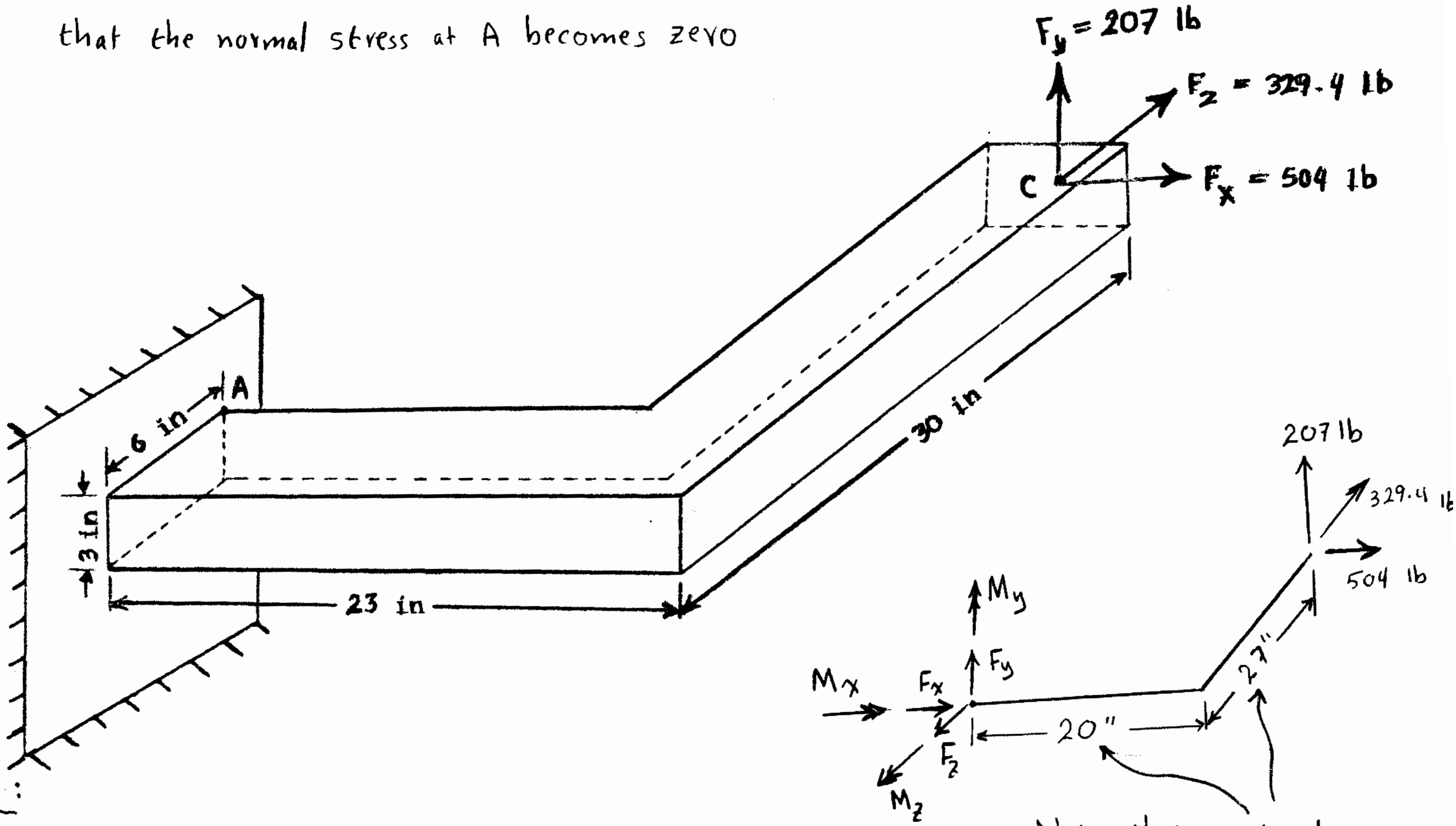
Given :

The member drawn below

Req'd. :

a) The normal stress at A

b) The required increase / decrease in the force F_x so that the normal stress at A becomes zero



Note that center-lines must be used

Sol'n :

$F_x = 504$ lb ←

$M_y = \ominus[329.4(20) - 504(27)] = 7020$ in-lb

$M_z = \ominus[504(0) + 207(20)] = -4140$ in-lb

⇑ on the FBD

$I_y = \frac{1}{12}(3)(6)^3 = 54 \text{ in}^4$

$I_z = \frac{1}{12}(6)(3)^3 = 13.5 \text{ in}^4$

* note $\ominus$ signs from Equil.

a) $\sigma = \pm\frac{N}{A} \pm \frac{M_y z}{I_y} \pm \frac{M_z y}{I_z} \Rightarrow$

$$\sigma_A = \frac{504}{18} + \frac{7020(3)}{54} - \frac{4140(1.5)}{13.5}$$

$$= 28 + 390 - 460 \Rightarrow \boxed{\sigma_A = 42 \text{ psi (C)}}$$

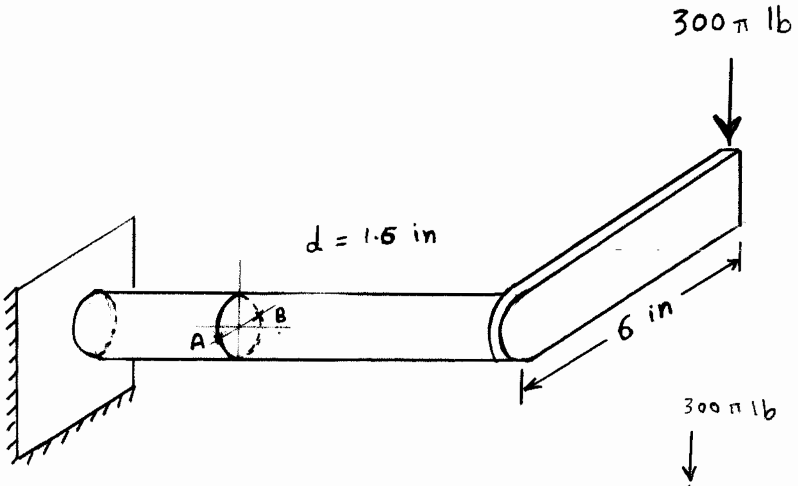
b) $\frac{\Delta F_x}{18} + \frac{\Delta F_x(27)(3)}{54} \equiv 42 \Rightarrow \boxed{\Delta F_x = 27 \text{ lb (increase)}}$

Note that the increase/decrease in F_x influences the normal force as well as M_y

Example 3

Given :

The shaft shown

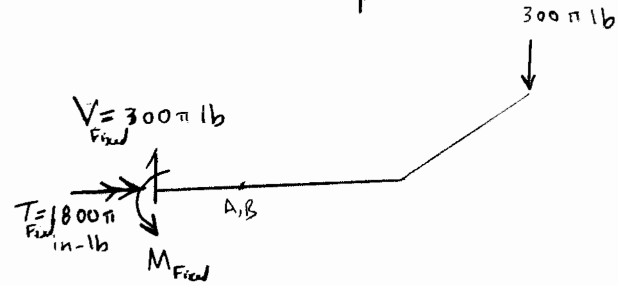


Req.d. :

τ at A & B (at the end of a horizontal diameter)

Solution :

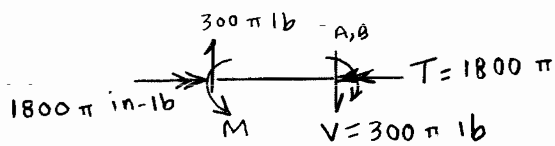
$$T = M_{xx} = -300\pi\,(6) = -1800\pi \quad \text{in-lb}$$



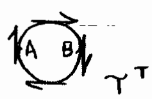
$$\tau^T = \frac{Tr}{J}$$

$$J = \frac{\pi}{32} d^4 = \frac{\pi}{32}(1.6)^4 = 0.2048\,\pi \ \text{in}^4$$

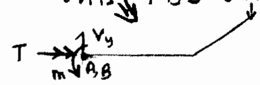
$$\tau_A^T = \frac{-1800\pi\,(0.8)}{0.2048\,\pi} = -7.031 \text{ ksi } (\uparrow)$$



$$\tau_B^T = +7.031 \text{ ksi } (\downarrow)$$



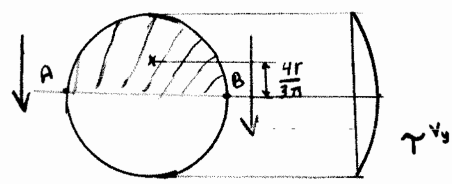
Note: You may take this FBD directly



$$V_y = +0.3\pi\ k \quad (\downarrow)$$

$$\tau^V = \frac{VQ}{Ib}$$

$$I = \frac{\pi}{64} d^4 = 0.1024\,\pi \ \text{in}^4$$



$$Q = A\bar{y} \Rightarrow$$

$$Q_A = Q_B = \frac{\pi}{2} r^2 \left(\frac{4}{3\pi} r\right) = \frac{2}{3} r^3 = \frac{2}{3}(0.8)^3 = 0.3413 \ \text{in}^3$$

$$b = d = 1.6 \ \text{in}$$

$$\tau_A^V = \tau_B^V = \frac{0.3\pi(0.3413)}{0.1024\,\pi\,(1.6)} = +0.625 \text{ ksi } (\downarrow)$$

$$\tau_A = \tau_A^T + \tau_A^V = -7.031 + 0.625 \Rightarrow \boxed{\tau_A = -6.41 \text{ ksi}}$$

$$\tau_B = \tau_B^T + \tau_B^V = 7.031 + 0.625 \Rightarrow \boxed{\tau_B = +7.66 \text{ ksi}}$$

note that the relative sign of τ is important but not the absolute (or actual) sign itself. ⇒ you may write

$\tau_A = 7.031 - 0.625 = +6.41$ ksi

$\tau_B = -7.031 - 0.625 = -7.66$ ksi

Example 4

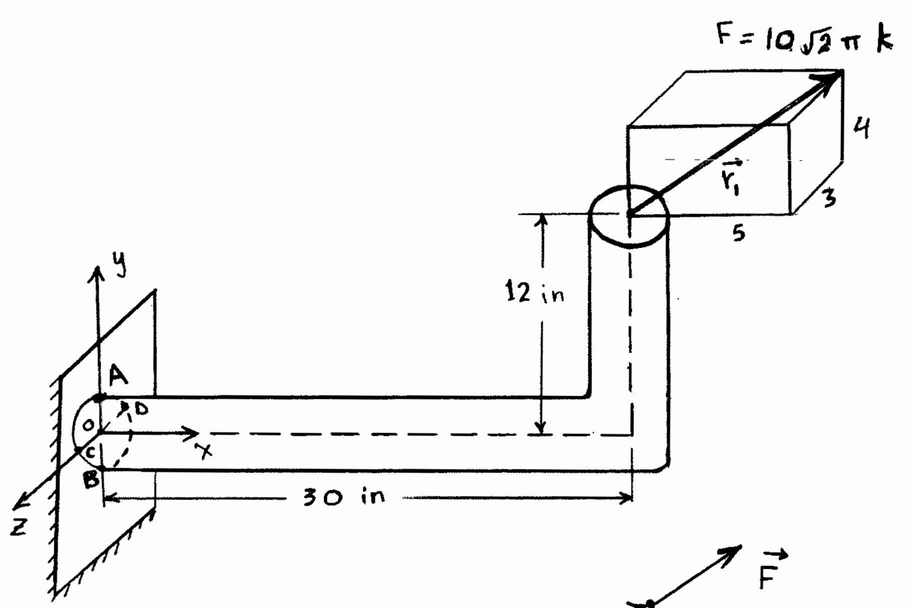
Given:

The shaft shown

$d = 4$ in

Req.d.:

The stress components at A, B, C, D and O



Solution:

$\vec{r}_1 = 5\vec{i} + 4\vec{j} - 3\vec{k}$

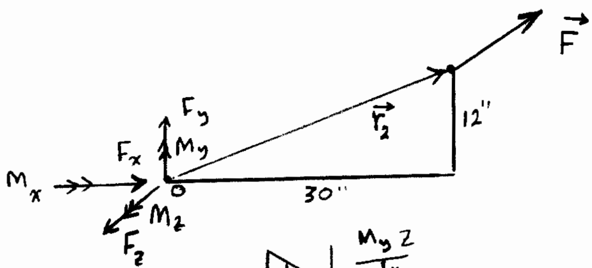
$|\vec{r}_1| = \sqrt{5^2 + 4^2 + 3^2} = 5\sqrt{2}$

$\vec{F} = \dfrac{F}{|\vec{r}_1|}\vec{r}_1 = F\vec{u}$

note that $\vec{u} = \dfrac{\vec{r}_1}{|\vec{r}_1|}$

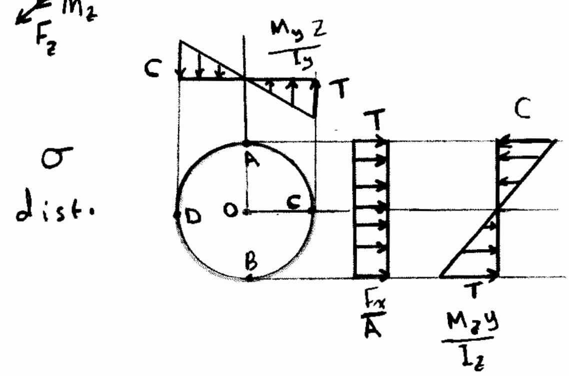
$\vec{F} = \dfrac{10\sqrt{2}\,\pi}{5\sqrt{2}}(5\vec{i} + 4\vec{j} - 3\vec{k})$

$= 10\pi\vec{i} + 8\pi\vec{j} - 6\pi\vec{k}$ (k)



$\vec{M}_o = \vec{r}_2 \times \vec{F}$

$\vec{r}_2 = 30\vec{i} + 12\vec{j} + 0\vec{k}$



$$\vec{M}_o = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 30 & 12 & 0 \\ 10\pi & 8\pi & -6\pi \end{vmatrix} = -72\pi\vec{i} + 180\pi\vec{j} + 120\pi\vec{k} \quad \text{(in-k)}$$

$\Sigma\vec{F} = \vec{0} \Rightarrow \vec{F}_o = -\vec{F} = -10\pi\vec{i} - 8\pi\vec{j} + 6\pi\vec{k}$; $\Sigma\vec{M} = \vec{0} \Rightarrow \vec{M}_{fixed} = -\vec{M}_o = 72\pi\vec{i} - 180\pi\vec{j} - 120\pi\vec{k}$

[Note that these are Statics (Coordinates) signs]

$$\sigma_x = \pm\frac{F_x}{A} \pm \frac{M_z y}{I_z} \pm \frac{M_y z}{I_y}$$

$$\sigma_x^A = \frac{+10\pi}{4\pi} + \frac{+120\pi(2)}{\frac{\pi}{4}(2)^4} + \frac{+180\pi(0)}{\frac{\pi}{4}(2)^4} = 2.5 - 60 - 0 \Rightarrow$$

$$\sigma_B = \frac{+10\pi}{4\pi} - \frac{+120\pi(-2)}{(\pi/4)2^4} + \frac{+180\pi(0)}{(\pi/4)2^4} = 2.5 + 60 + 0 \Rightarrow$$

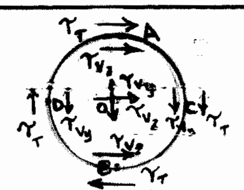
$$\sigma_C = \frac{+10\pi}{4\pi} - \frac{+120\pi(0)}{(\frac{\pi}{4})2^4} + \frac{+180\pi(2)}{(\pi/4)2^4} = 2.5 - 0 + 90 \Rightarrow$$

$$\sigma_D = \frac{+10\pi}{4\pi} - \frac{+120\pi(0)}{(\frac{\pi}{4})2^4} + \frac{+180\pi(-2)}{(\pi/4)2^4} = 2.5 - 0 - 90 \Rightarrow$$

$$\sigma_O = \frac{+10\pi}{4\pi} - \frac{+120\pi(0)}{(\pi/4)2^4} + \frac{+180\pi(0)}{(\pi/4)2^4} = 2.5 - 0 + 0 \Rightarrow$$

$\sigma_A = -57.5$ ksi $= 57.5$ ksi (C)
$\sigma_B = 62.5$ ksi (T)
$\sigma_C = 92.5$ ksi (T)
$\sigma_D = -87.5$ ksi $= 87.5$ ksi (C)
$\sigma_O = 2.5$ ksi (T)

$$\vec{\tau} = \vec{\tau}_{xy} + \vec{\tau}_{xz} + \vec{\tau}_{xs} = \frac{\vec{V}_y Q_z}{I_z b_z} + \frac{\vec{V}_z Q_y}{I_y b_y} + \frac{\vec{T} r}{J}$$



Point A: $V_y Q_z^{\,\to 0} / I_z b_z = 0$; $V_z Q_y / I_y b_y = 6\pi(\frac{\pi}{2} \times 2^2 \times \frac{4}{3\pi} \times 2) / (\frac{\pi}{4}2^4)4 = 2$ ksi

$Tr/J = 72\pi(2)/\frac{\pi}{2}(2)^4 = 18$ ksi $\Rightarrow \tau_A = 2 + 18 \Rightarrow \boxed{\tau_A = 20 \text{ ksi}}$

Similarly$_{B,C,D}$, $\tau_B = 18 - 2 \Rightarrow \boxed{\tau_B = 16 \text{ ksi}}$; $\tau_C = 2.67 + 18 \Rightarrow \boxed{\tau_C = 20.67 \text{ ksi}}$; $\tau_D = 18 - 2.67 \Rightarrow \boxed{\tau_D = 15.33 \text{ ksi}}$

Point O: $\vec{\tau}_O = 2.67\vec{j} + 2\vec{k} + 0^{\to \text{from } T}$; $\tau = \sqrt{(2.67)^2 + 2^2} \Rightarrow \boxed{\tau_O = 3.33 \text{ ksi}}$