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# Examples

## Shear Force Diagram (SFD) & Bending Moment Diagram (BMD) (FBD and Equations Method)

### Statics Review

#### Example 1:

Given:

The beam shown

Req'd.:

The shear force and bending moment equations and diagrams.

Soln.:

The beam can be taken as one segment  $\Rightarrow$  one section.

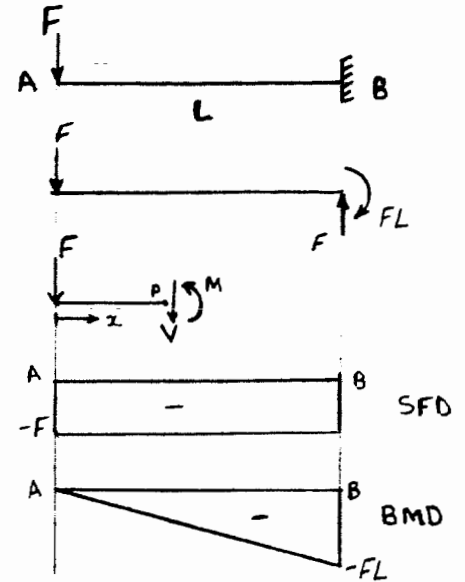
$$0 \leq x \leq L$$

In the FBD shown,  $\uparrow \Sigma F_y = 0 \Rightarrow$   $V = -F$

$\curvearrowright \Sigma M_p = 0 \Rightarrow$

$M + Fx = 0 \Rightarrow$   $M = -Fx$

The shear force diagram (SFD) and the bending moment diagram (BMD) are shown in the figure above.



#### Example 2:

Given:

The beam shown

Req'd.:

SFD & BMD

Soln.:

The reactions are shown.

We need two sections.

$$AB \left( 0 \leq x \leq \frac{L}{2} \right)$$

In the FBD shown,  $\uparrow \Sigma F_y = 0 \Rightarrow$   $V = \frac{P}{2}$

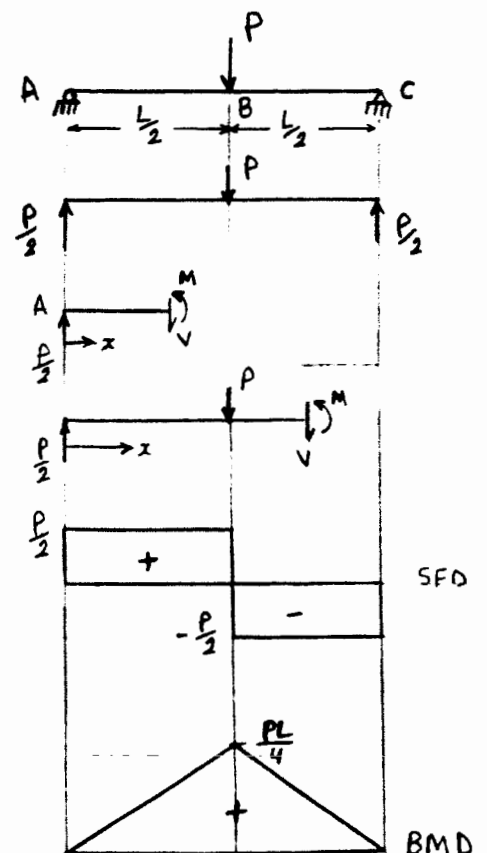
$\curvearrowright \Sigma M = 0 \Rightarrow$   $M = \frac{P}{2}x$

$$BC \left( \frac{L}{2} \leq x \leq L \right)$$

In the FBD shown,  $\uparrow \Sigma F_y = 0 \Rightarrow$   $V = -\frac{P}{2}$

$\curvearrowright \Sigma M = 0 \Rightarrow$   $M = \frac{P}{2}x - P(x - \frac{L}{2}) = \frac{P}{2}(L - x)$

The SFD & BMD are shown.



Example 3:

Given:

The beam shown

Req'd.:

SFD &amp; BMD

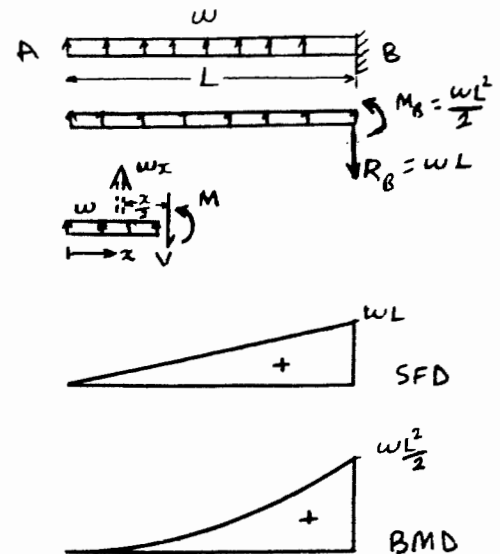
Soln.:

Only one section is needed. Reactions are shown  
AB ( $0 \leq x \leq L$ )

In the FBD shown,  $\uparrow \Sigma F_y = 0 \Rightarrow V = wx$

$\curvearrowright \Sigma M = 0 \Rightarrow M = wx \left(\frac{x}{2}\right) = \frac{wx^2}{2}$

SFD & BMD are shown.

Example 4:

Given:

The beam shown

Req'd.:

SFD &amp; BMD

Soln.:

The reactions are first calculated from FBD ①

$\curvearrowright \Sigma M_c = 0 = 60 - 50(2) + 10(4)(2) - 4R_A \Rightarrow R_A = 10 \text{ N } \uparrow$

$\uparrow \Sigma F_y = 0 = R_C + 50 - 10(4) + 10 \Rightarrow R_C = -20 \text{ N} = 20 \text{ N } \downarrow$

Two sections are needed:

AB ( $0 \leq x \leq 2 \text{ m}$ )

In FBD ②,  $\uparrow \Sigma F_y = 0 \Rightarrow$

$10 - 10x - V = 0 \Rightarrow V = 10 - 10x = 10(1-x)$

$\curvearrowright \Sigma M = 0 = M - 10x + 10x\left(\frac{x}{2}\right) + 60 \Rightarrow M = -60 + 10x - 5x^2$

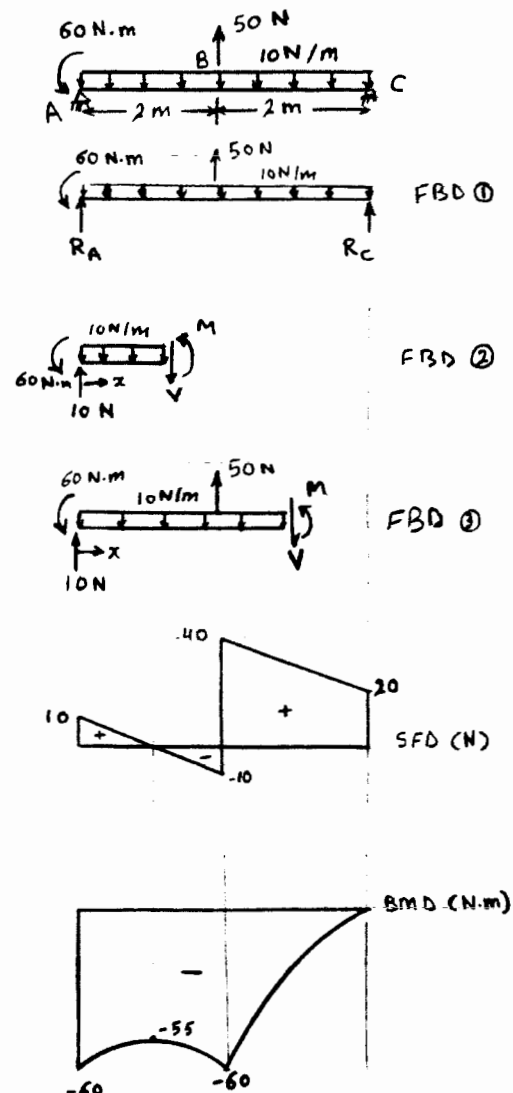
BC ( $2 \leq x \leq 4 \text{ m}$ )

In FBD ①,  $\uparrow \Sigma F_y = 0 \Rightarrow$

$10 - 10x + 50 - V = 0 \Rightarrow V = 60 - 10x = 10(6-x)$

$\curvearrowright \Sigma M = 0 \Rightarrow$

$M + 60 - 10x - 50(x-2) + 10x\left(\frac{x}{2}\right) = 0 \Rightarrow M = -160 + 60x - 5x^2$



### Example 5:

Given:

The beam shown

Req'd.:

SFD & BMD

Soln.:

First, reactions are needed.

$$\text{In FBD ①, } \sum M_c = 0 = 75(5) - 3R_B + 90(1)$$

$$\Rightarrow R_B = -95 \text{ N} = 95 \text{ N} \downarrow$$

$$\text{In FBD ②, } \uparrow \sum F_y = 0 \Rightarrow R_E = 110 \text{ N} \uparrow$$

$$\sum M_E = 0 = -75(10) + 95(8) + 90(6) - 200 + M_E$$

$$\Rightarrow M_E = 350 \text{ N}\cdot\text{m}$$

AB ( $0 \leq x \leq 2 \text{ m}$ ): FBD ③

$$\uparrow \sum F_y = 0 \Rightarrow V = 75 \text{ N}$$

$$\sum M = 0 \Rightarrow M = 75x$$

BC ( $2 \leq x \leq 5 \text{ m}$ ): FBD ④

Note that if there is a distributed load on the segment, then one should first "cut"; after that an equivalent concentrated load can be placed at the centroid.

$$\uparrow \sum F_y = 0 = 75 - 95 - 20(x-2)\left(\frac{x-2}{2}\right) - V$$

$$\Rightarrow V = -20 - 10(x-2)^2$$

$$\sum M = 0 = -75x + 95(x-2) + 20(x-2)\left(\frac{x-2}{2}\right)\left(\frac{x-2}{3}\right) + M$$

$$\Rightarrow M = 190 - 20x - \frac{10}{3}(x-2)^3$$

CD ( $5 \leq x \leq 9 \text{ m}$ ): FBD ⑤

$$\uparrow \sum F_y = 0 \Rightarrow V = -110 \text{ N}$$

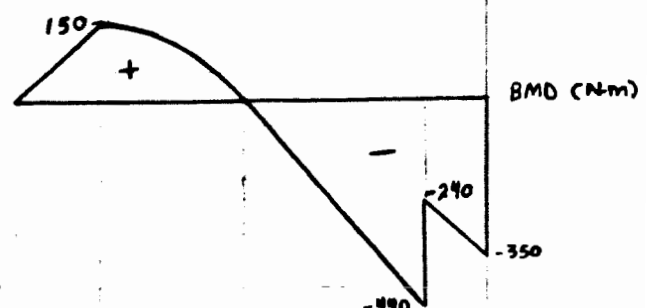
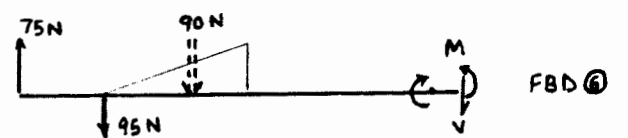
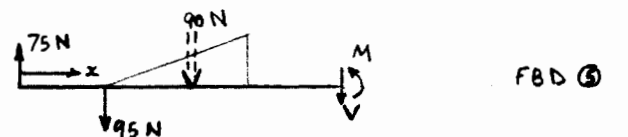
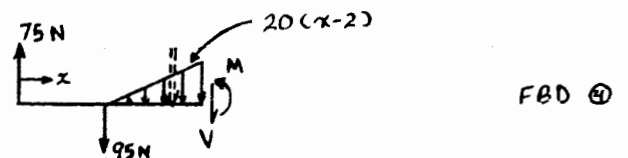
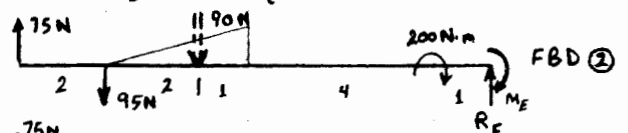
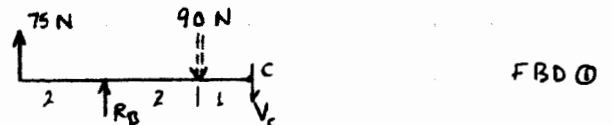
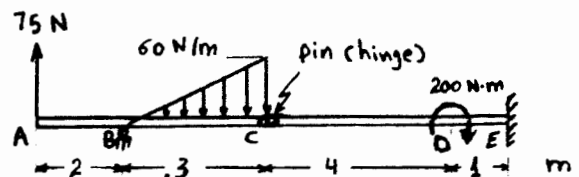
$$\sum M = 0 \Rightarrow M = 550 - 110x$$

DE ( $9 \leq x \leq 10 \text{ m}$ ): FBD ⑥

$$\uparrow \sum F_y = 0 \Rightarrow V = -110 \text{ N}$$

$$\sum M = 0 \Rightarrow M = 750 - 110x$$

Note that in this problem 4 sections are needed.



Example 6 :

Given :

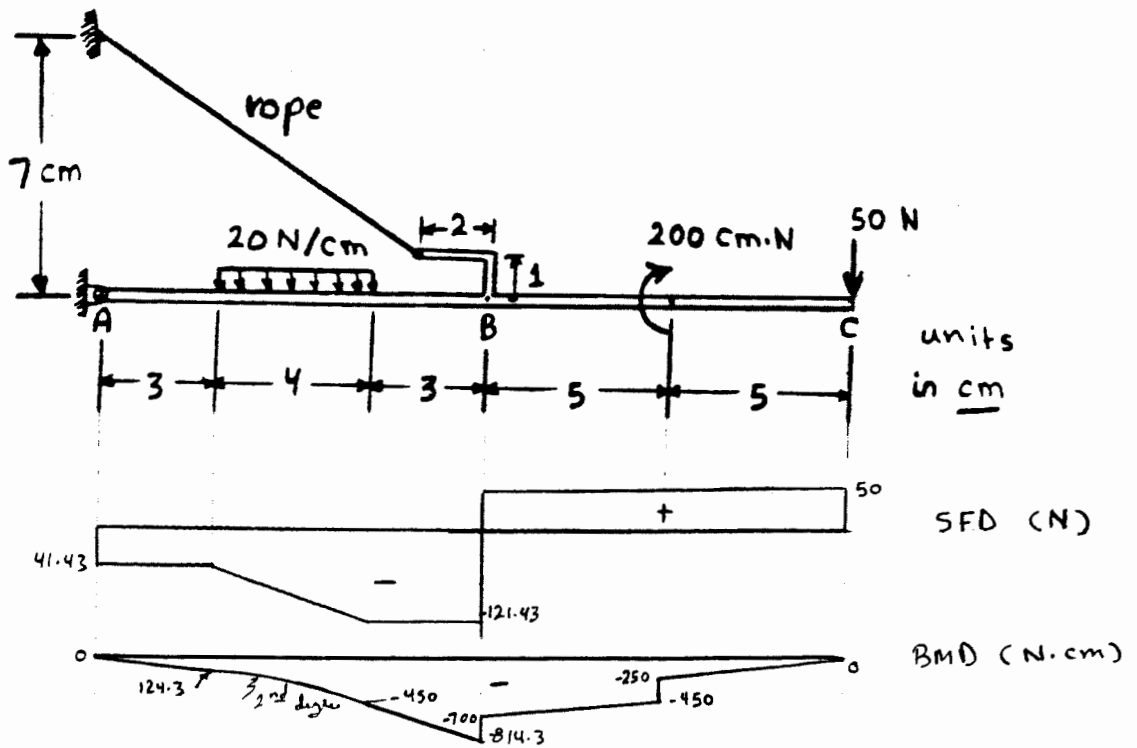
The beam shown below

Reqd. :

SFD and BMD

Soln. :

The diagrams are drawn below. Verify that by drawing FBD's and writing eqs.



Example 7 :

As in example 6

