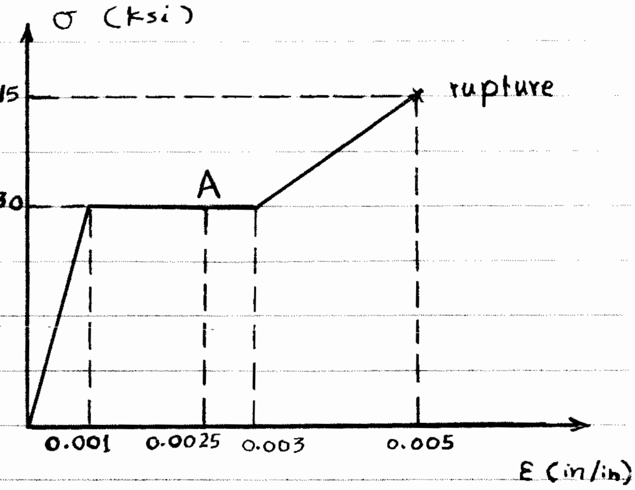
#2

Examples (σ-ε and Material Behavior)

Example 1:

Given:

The idealized stress-strain diagram shown



Req.d.:

a) the elastic modulus E
b) the yield point stress σ_{yp}
c) the ultimate strength σ_{ult}
d) the modulus of resilience MR
e) the toughness
f) the permanent (plastic) strain ε_p
if the material is loaded up to point A, then the load is released.

Soln.:

a) $E = \dfrac{\sigma}{\varepsilon}$ (in the elastic range)

$\Rightarrow E = 30/0.001 \Rightarrow$ $\underline{\underline{E = 30\,000 \text{ ksi}}}$

b) $\underline{\underline{\sigma_{yp} = 30 \text{ ksi}}}$

c) $\underline{\underline{\sigma_{ult.} = 45 \text{ ksi}}}$

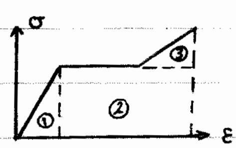
d) MR = area under σ-ε curve upto the proportional limit

$= \frac{1}{2}(30)(0.001) \Rightarrow$ $\underline{\underline{MR = 0.015 \text{ in-k/in}^3}}$ (ksi)

e) Toughness = total area under the σ-ε curve

$= 0.015 + 30(0.004) + \frac{1}{2}(15)(0.002)$

$\Rightarrow$ $\underline{\underline{\text{Toughness} = 0.15 \text{ in-k/in}^3}}$

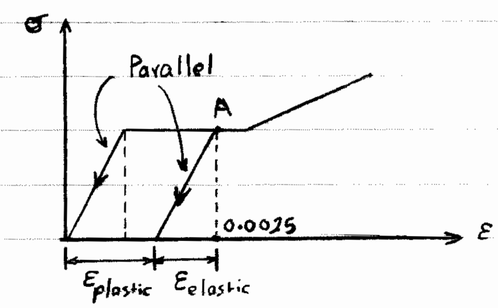


f) $\varepsilon_{total} = \varepsilon_{elastic} + \varepsilon_{plastic}$

$\varepsilon_{elastic} = \sigma_A / E = 30/30000 = 0.001$

$\Rightarrow 0.0025 = 0.001 + \varepsilon_{plastic}$

$\Rightarrow$ $\underline{\underline{\varepsilon_{plastic} = 0.0015 \text{ in/in}}}$

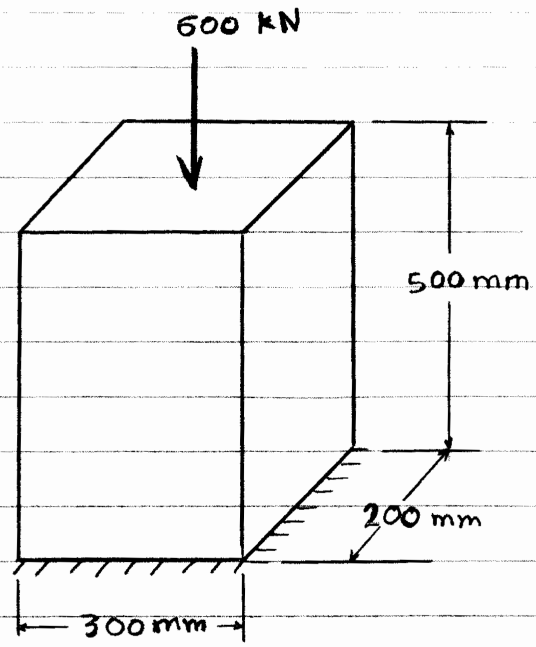


Example 2 :

Given :

The block shown

After applying the force, the length decreased by 0.0685 mm while the 300 mm-dimension increased by 0.0125 mm



Reqd. :

a) the modulus of elasticity

b) Poisson's ratio

c) the new value of the 200 mm-dimension

d) the change in the volume

e) the unit change in volume (dilatation)

f) the percent shrinkage (- elongation)

g) the percent increase in area

h) the material of the block

Soln. :

a) $E = \frac{\sigma}{\varepsilon}$ (assume linear elastic behavior)

$\Rightarrow \sigma = P/A = -600(10)^3/(300)(200) = -10$ MPa

$\varepsilon = \Delta\ell/\ell = -0.0685/500 = -1.37(10)^{-4}$ mm/mm

$\Rightarrow E = -10(10)^6/-1.37(10)^{-4} \approx 7.3(10)^{10}$ Pa $\Rightarrow$ $\underline{\underline{E = 73 \text{ GPa}}}$

b) $\nu = \left|\frac{\varepsilon_\ell}{\varepsilon_a}\right| \Rightarrow \varepsilon_\ell = 0.0125/300 = 4.167(10)^{-5}$ m/m

$\Rightarrow \nu = 4.167(10)^{-5}/1.37(10)^{-4} \Rightarrow$ $\underline{\underline{\nu \approx 0.3}}$

c) $\nu = \left|\frac{\varepsilon_\ell}{\varepsilon_a}\right|$ (Note there are two lateral strains in two directions with the same value)

$\Rightarrow \Delta_{200} = \varepsilon_\ell(200) = 4.167(10)^{-5}(200) = 0.00833$ mm $\Rightarrow$ $\underline{\underline{\text{new dimension} = 200.00833 \text{ mm}}}$

Note that this value is $\left(\frac{200}{300}\right)0.0125$

d) $\Delta V = V_f - V_o = (500-0.0685)(300.0125)(200.00833) - 500(300)(200) \Rightarrow$ $\underline{\underline{\Delta V = -1611 \text{ mm}^3}}$ (decrease)

e) dilatation $= e = \Delta V/V$ [note: $e \approx (1-2\nu)\sigma/E$]; $e = -1611/3(10)^7 \Rightarrow$ $\underline{\underline{e = -5.37(10)^{-5}}}$

f) % shrinkage $= \frac{\Delta\ell}{\ell}(100) = \frac{0.0685}{500}(100) \Rightarrow$ $\underline{\underline{\text{\% shrinkage} = 0.0137}}$ $\longleftrightarrow$ $\underline{\underline{\text{\% elongation} = -0.0137}}$

g) % increase in area $= \frac{A_f - A_o}{A_o}100 = [300.0125(200.00833) - 300(200)]100/300(200) \Rightarrow$ $\underline{\underline{\text{\% increase in } A = 0.008}}$

h) From E and ν values, the material is probably **Aluminum** [[Are the answers reasonable?