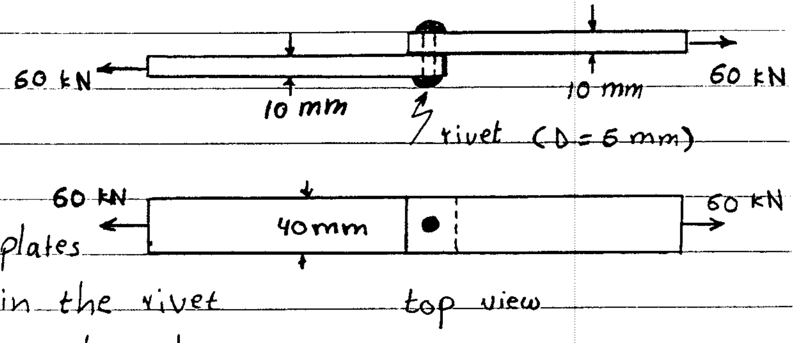
Very important note: Do NOT depend only on the handouts given to you.
You MUST read/understand the textbook and DO the H.W.

Examples (Ch. 1: Average Stresses)

Example 1:



Given:

The two plates shown

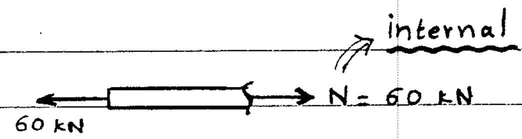
Req.d.:

The normal stress in the plates

The average shearing stress in the rivet

The average bearing stress in the plates

Soln.:



normal stress $= \sigma = \dfrac{N}{A}$

$\Rightarrow \quad \sigma = \dfrac{60\,(10)^3}{10\,(40)} = 150 \text{ N/mm}^2 = 150\,(1000)^2 \text{ N/m}^2$

$\Rightarrow \quad \boxed{\sigma = 150\,(10)^6 \text{ N/m}^2 = 150\,(10)^6 \text{ Pa} = 150 \text{ MPa (T)}}$

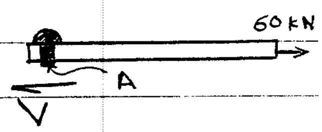
Note that total (gross) area is used

* Be careful about the units!
* The internal force must be used in the stress formula

Average shearing stress in the rivet $= \tau_{ave} = \dfrac{V}{A}$

$V = 60$ kN

$A = \pi r^2 = \dfrac{\pi}{4} D^2 = \dfrac{\pi}{4}(6)^2 = 9\pi \text{ mm}^2$



Note: there is only one rivet, and it is in single shear.

$\Rightarrow \quad \boxed{\tau_{ave} = \dfrac{60\,(10)^3}{9\pi} \Rightarrow \tau_{ave} = 2122 \text{ N/mm}^2 = 2122 \text{ MPa}}$

Average bearing stress in the plates $= \sigma_b = \dfrac{P}{A}$

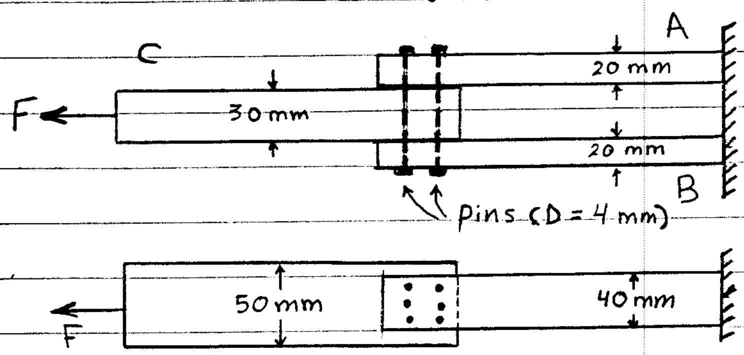
P = normal force $= 60$ kN

A = projected area $= Dt = 6\,(10) = 60 \text{ mm}^2$

$\Rightarrow \quad \sigma_b = \dfrac{60\,(10)^3}{60} \quad \Rightarrow \quad \boxed{\sigma_b = 1000 \text{ N/mm}^2 = 1000 \text{ MPa}}$

Example 2 :

Given:



- The figure shown
- The maximum allowable normal stress in the plates is 12 MPa
- The maximum allowable shearing stress in the pins is 110 MPa
- The maximum allowable bearing stress in the plates is 25 MPa

Req'd.:

The maximum allowable force F which can be applied.

Sol'n.:

First, consider the normal stresses in the plates:

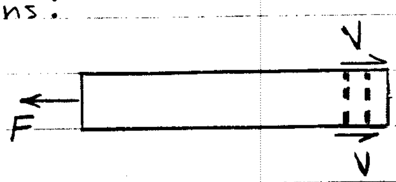
Plates A and B : $\sigma = \frac{F}{A} \Rightarrow 12 \equiv F / \underbrace{2}_{\text{two plates A and B}}(20)(40) \qquad \underline{F_1 = 19.2 \text{ kN}}$

* Note that total (gross) area is used. Take care of units!

Plate C : $\sigma = \frac{F}{A} \Rightarrow 12 \equiv \frac{F}{30(50)} \Rightarrow \underline{F_2 = 18 \text{ kN}}$

Second, consider the shearing stress in the pins:

There are 6 pins which act in double shear as shown.



$\tau_{ave} = \frac{V}{A} \Rightarrow 110 \equiv \frac{F}{\underbrace{2}_{\text{double shear}}\underbrace{(6)}_{\text{six pins}}\underbrace{(\frac{\pi}{4})(4)^2}_{\text{area of each pin}}}$

$\Rightarrow \underline{F_3 = 16.59 \text{ kN}}$

Third, consider the bearing stresses in the plates:

Plates A and B : $\sigma_b = \frac{P}{A} = \frac{\frac{1}{2}F}{6(4)20} \equiv 25 \Rightarrow \underline{F_4 = 24 \text{ kN}}$

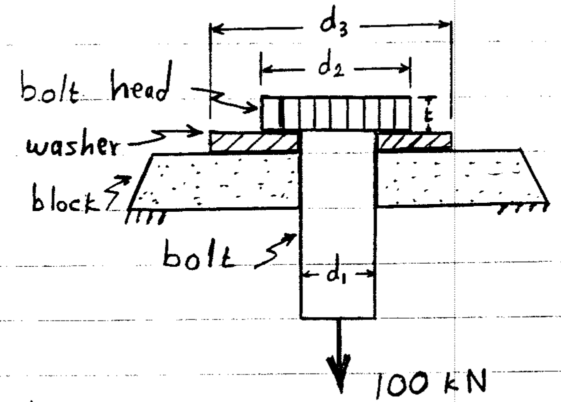
Plate C : $25 \equiv F / 6(4)30 \Rightarrow \underline{F_5 = 18 \text{ kN}}$

From $F_1, \ldots, F_5$, choose the smallest force $\Rightarrow \boxed{F_{max} = F_3 = 16.59 \text{ kN}}$

Example 3:

Given:

The figure shown



Req.d.:

a) The normal stress in the bolt

b) The sheering stress in the bolt head

c) The bearing stress between the bolt head and the washer

d) The bearing stress between the washer and the block

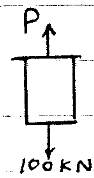
$d_1 = 20$ mm

$d_2 = 40$ mm

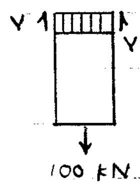
$d_3 = 70$ mm

$t = 10$ mm

Sol.n.:



a) $\sigma = \frac{P}{A} = \frac{100(10)^3}{\frac{\pi}{4}(20)^2} \Rightarrow \boxed{\sigma = 318.3 \text{ MPa (T)}}$



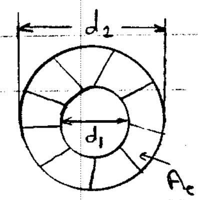
V is around the cylindrical surface

b) $\tau = \frac{V}{A}$

$A = \pi (d_1) t = \pi(20)10 = 200\pi \text{ mm}^2$

$V = 100$ kN

$\Rightarrow \tau = \frac{100(10)^3}{200\pi} \Rightarrow \boxed{\tau = 159.2 \text{ MPa}}$

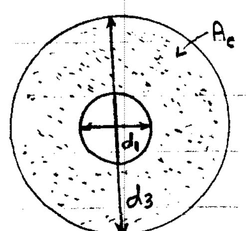


c) $\sigma_b = \frac{P}{A_c}$

$A_c = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}[(40)^2 - (20)^2] = 942.48 \text{ mm}^2$

$P = 100$ kN

$\Rightarrow \sigma_b = \frac{100(10)^3}{942.48} \Rightarrow \boxed{\sigma_b = 106.1 \text{ MPa}}$ between bolt head and washer



d) $\sigma_b = \frac{P}{A_c}$

$A_c = \frac{\pi}{4}(d_3^2 - d_1^2) = \frac{\pi}{4}[(70)^2 - (20)^2] = 3534.3 \text{ mm}^2$

$P = 100$ kN

$\sigma_b = \frac{100(10)^3}{3534.3} \Rightarrow \boxed{\sigma_b = 28.29 \text{ MPa}}$ between washer and block

Note that the bearing stress is reduced a lot by adding a washer (28.29 vs. 106.1 MPa).

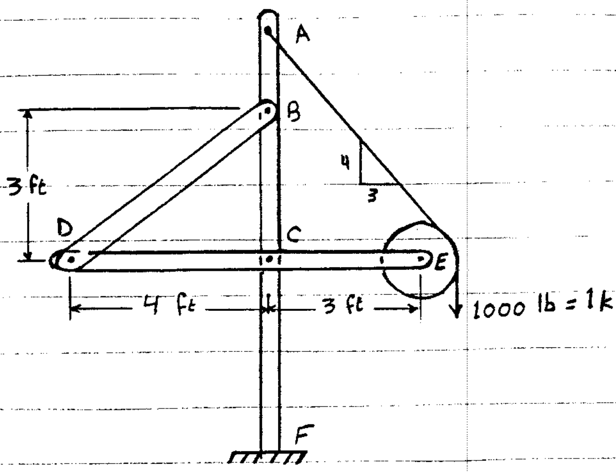
Example 4 :

Given :

The frame shown

$A_{BD} = 0.25 \text{ in}^2$

$\frac{1}{2}$ in.-diameter pin at C



Reqd. :

The normal stress in member BD

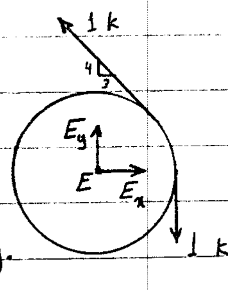
The shearing stress in the pin at C

Connection at C

Soln. :

In order to calculate the stresses, the forces have to be found first.

* Note that we have to start with the pulley to analyze the frame ; we can not start with any of the members. (Why ?!) However, member DCE can be taken with the pulley.

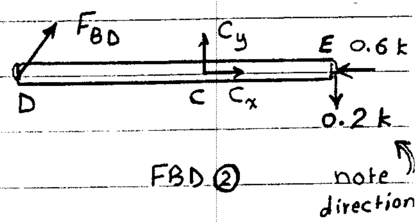


FBD ①

From FBD ①,

$\Sigma F_x = 0 \Rightarrow E_x - 1(\frac{3}{5}) = 0 \Rightarrow E_x = 0.6 \text{ k}$

$\Sigma F_y = 0 \Rightarrow E_y + 1(\frac{4}{5}) - 1 = 0 \Rightarrow E_y = 0.2 \text{ k}$



FBD ②

From FBD ②,

$\circlearrowleft\!+ \ \Sigma M_C = 0$ (Do not use $\Sigma F_x = 0$ or $\Sigma F_y = 0$! Why?!)

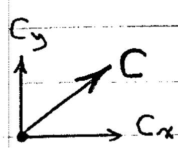
$\Rightarrow -0.2(3) - F_{BD}(\frac{3}{5})4 = 0 \Rightarrow F_{BD} = -0.25 \text{ k}$

$\Rightarrow F_{BD} = 0.25 \text{ k (C)}$ ← Note that BD is a two-force member (review statics!)

$$\sigma_{BD} = \frac{F}{A} = \frac{0.25}{0.25} \Rightarrow \boxed{\sigma_{BD} = 1 \text{ k/in}^2 = 1 \text{ ksi} \quad (C)}$$

$\Sigma F_x = 0 \Rightarrow C_x = 0.8 \text{ k}$

$\Sigma F_y = 0 \Rightarrow C_y = 0.35 \text{ k}$ ⇐ Check !



$\tau = V/A$; $V = C$ (resultant)

$C = \sqrt{(0.8)^2 + (0.35)^2} = 0.8732 \text{ k}$

$$\Rightarrow \tau_c = \frac{C}{A} = \frac{0.8732}{2(\frac{\pi}{4})(0.5)^2} \Rightarrow \boxed{\tau_c = 2.22 \text{ ksi}}$$

Note that the pin at C is in double shear (see figure).