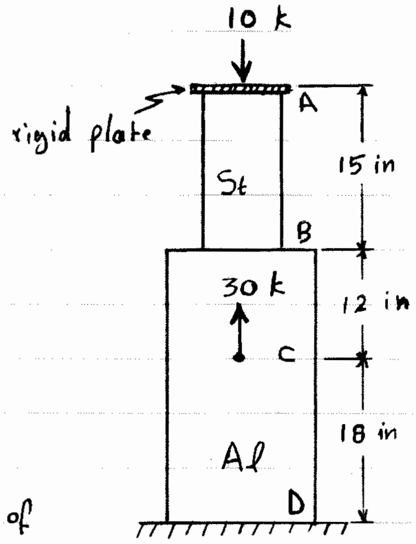
3

Examples of Statically-Determinate Axially-Loaded Members



Example 1 :

Given :

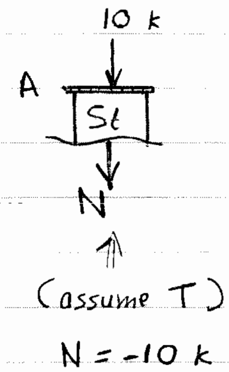
The figure shown

Steel : $E = 30\,000$ ksi ; $A = 2$ in^2

Aluminum : $E = 10\,000$ ksi ; $A = 6$ in^2

Req.d. :

1) The displacement of point A by the method of
   (a) integration
   (b) discrete element
   (c) superposition
2) σ_{max} (T & C) in St. and Al.
3) Justify the use of an elastic analysis



Solution :

1) (a) $e_{A/D}$ (↓ or δ)

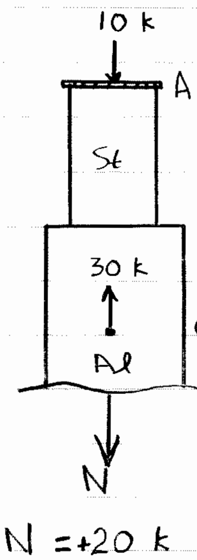
$$e_{A/D} = \int \frac{N}{EA}\,dx = \int_0^{15} \frac{-10}{30000(2)}\,dx + \int_{15}^{27} \frac{-10}{10000(6)}\,dx + \int_{27}^{45} \frac{20}{10000(6)}\,dx$$

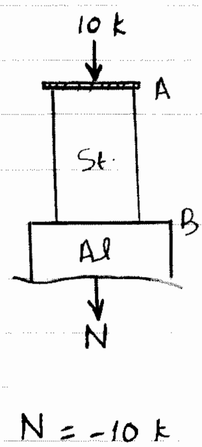
* "Guess" the answers ⇒ check!

$$= -\frac{150}{60000} - \frac{120}{60000} + \frac{360}{60000}$$

$$= -0.0025 - 0.002 + 0.006$$

⇒ $\boxed{e_{A/D} = 0.0015 \text{ in (up)}}$





Note that N is the internal force.

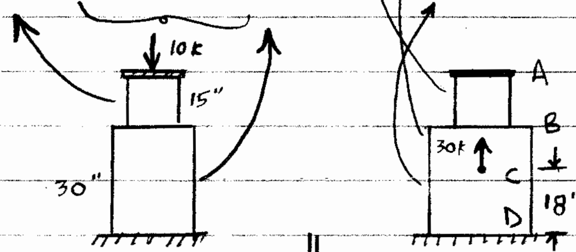
⇒ Get it from FBD

(b) $e_{A/D} = \frac{-10(15)}{30000(2)} + \frac{-10(12)}{10000(6)} + \frac{20(18)}{10000(6)}$ $\Leftarrow$ $e = \sum_{i=1}^{3} e_i$

$= -0.0025 - 0.002 + 0.006 = 0.0015$ in $(\uparrow)$

(c) $e_{A/D} = \frac{-10(15)}{30000(2)} + \frac{-10(\mathbf{30})}{10000(6)} + 0 + 0 + \frac{\mathbf{30}(18)}{10000(6)} = -0.0025 - 0.005 + 0.009$

$= 0.0015$ in $(\uparrow)$



$N_{AB} = N_{\mathbf{BD}} = 10k$ (C) ‖ note that N in AB and BC is zero; N in CD is 30k

2) $\sigma_{St} = \frac{N}{A} = \frac{-10}{2} \Rightarrow$ $\boxed{(\sigma_{St})_{max} = 5 \text{ ksi (C)}}$ $\Leftarrow$ in AB

$\sigma_{Al} = \frac{N}{A} \Rightarrow$

$\sigma_{al}^{C} = \frac{-10}{6} \Rightarrow$ $\boxed{(\sigma_{Al})_{max} = 1.67 \text{ ksi (C)}}$ $\Leftarrow$ in BC

$\sigma_{al}^{T} = \frac{20}{6} \Rightarrow$ $\boxed{(\sigma_{Al})_{max} = 3.33 \text{ ksi (T)}}$ $\Leftarrow$ in CD

3) Since $|(\sigma_{St})_{max}| = 5 \text{ ksi} << \sigma_{ys}^{St}$

and $|(\sigma_{Al})_{max}| = 3.33 \text{ ksi} << \sigma_{ys}^{Al}$

} See materials properties in the textbook.

the use of an elastic analysis is justified

Important: Note that internal forces are used in the formulas.

Example 2 :

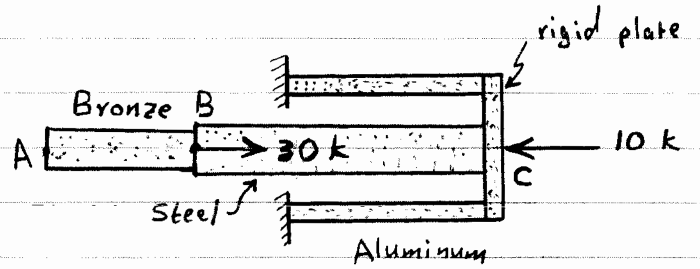
Given :

The figure shown

$\Delta T = 100\ ^\circ F$ in bronze

$= 50\ ^\circ F$ in aluminum

$= 0\ ^\circ F$ in steel



The materials properties are given in the table below:

Material	L (in)	A (in²)	E (ksi)	α (in/in/°F)
Bronze	15	1.5	12 000	$10 (10)^{-6}$
Steel	30	3	30 000	$6.5 (10)^{-6}$
Aluminum	20	2	10 000	$13 (10)^{-6}$

Req'd:

a) The displacement of point A

b) The stresses in the bronze, steel and aluminum

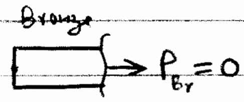
c) The required change in steel temperature in order to keep A in its original position

Soln. :

a) Since the problem is statically determinate, the temperature does not introduce any stress; however, the lengths of the members are affected.

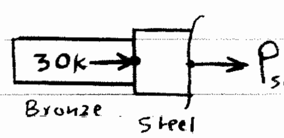
(1) displacement of A due to loads only

$$e_{bronze} = \left(\frac{PL}{EA}\right)_{Br} = 0 \qquad (P \to 0)$$



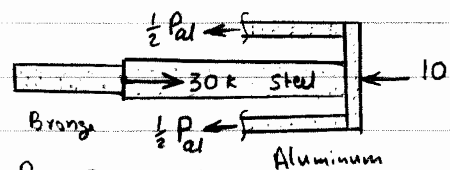
$$e_{steel} = \left(\frac{PL}{EA}\right)_{St} = \frac{30\,(30)}{30000\,(3)} = 0.01 \text{ in}$$

Do not worry about the sign at this stage



$P_{St} = -30\text{ k} = 30\text{ k (C)}$

$$e_{Alum.} = \left(\frac{PL}{EA}\right)_{Al} = \frac{20\,(20)}{10000\,(2)} = 0.02 \text{ in}$$



$P_{al} = 30 - 10 = 20\text{ k}$

$$(e_A)_{\text{due to load}} = e_{Br} + e_{St} + e_{Al} = 0 + 0.01 + 0.02 = 0.03 \text{ in } (\rightarrow)$$

⇑ Get the directions & signs by physical inspection

(2) displacement of A due to temperature increase

$$\delta = \alpha \Delta T L$$

$$\delta_{Br} = 10(10)^{-6}(100)(15) = 0.015 \text{ in}$$

$$\delta_{Al} = 13(10)^{-6}(50)(20) = 0.013 \text{ in}$$

$$(e_A)_{\text{due to } \Delta T} = -0.015 + 0.013 = -0.002 \text{ in } (\leftarrow)$$

⇑ Take care of the signs (as above)

$$(e_A)_{Total} = (e_A)_{\text{due to } P} + (e_A)_{\text{due to } \Delta T} = 0.03 - 0.002$$

$$\Rightarrow \boxed{e_A = 0.028 \text{ in } (\rightarrow)}$$

b) $\sigma = \frac{P}{A}$

$$\Rightarrow \sigma_{Br} = \frac{0}{1.5} \Rightarrow \boxed{\sigma_{Br} = 0}$$

$$\sigma_{St} = \frac{-30}{3} \Rightarrow \boxed{\sigma_{St} = 10 \text{ ksi } (C)}$$

$$\sigma_{Al} = \frac{20}{2} \Rightarrow \boxed{\sigma_{al} = 10 \text{ ksi } (T)}$$

c) Point A moved to the right. ⇒ In order to bring A back to its original position, the temperature in the steel rod has to increase. ⇒

$$(\alpha \Delta T L)_{St} \equiv 0.028$$

$$6.5(10)^{-6}(30)\Delta T = 0.028 \Rightarrow \boxed{\Delta T_{St} = 143.6\ °F \text{ (increase)}}$$

Are the answers above "reasonable"? THINK !!