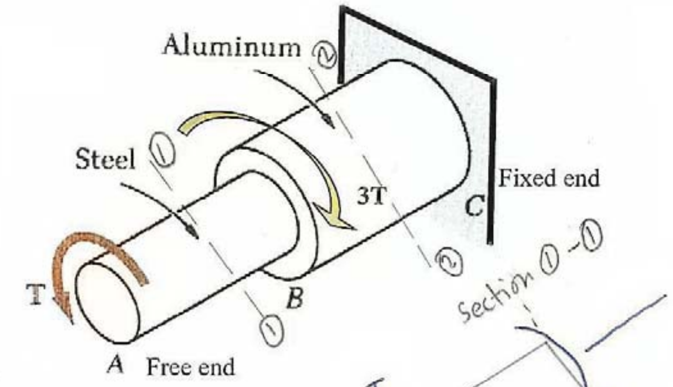
Problem 1:

Part – a:

A compound solid shaft ABC has a 30-mm diameter for segment AB and is made of steel with an allowable shear stress of 90 MPa, while the other segment of the solid shaft BC has 50-mm diameter and is made of an aluminum alloy with an allowable shear stress of 60 MPa. Determine the largest **absolute** torque T which may be *safely* applied if torques T and 3T are applied *respectively* at sections A and B (as shown in Fig. P-1(a)).

Fig. P-1(a): A compound shaft subjected to two torques T and 3T at A and B, respectively.



Solution:

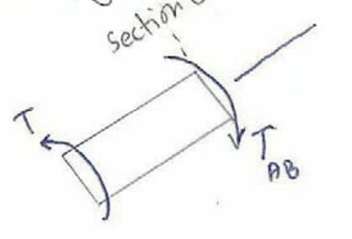
Segment AB

Internal Torque

$T_{AB} = 1T$

$J_{AB} = \frac{\pi}{2}(0.015)^4$

$= 7.952 \times 10^{-8}\ m^4$.



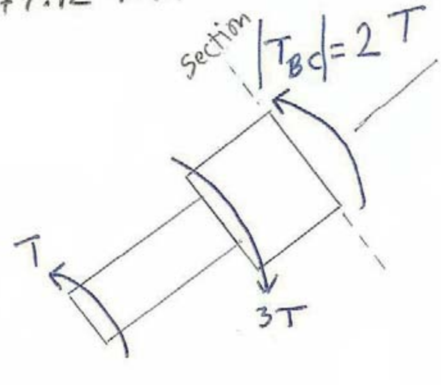
$(\tau_{max})_{AB} = (\tau_{allow}) = 90\ MPa = \frac{T \cdot r_{AB}}{J_{AB}}$

$\Rightarrow T = \frac{(90 \times 10^6\ Pa) \cdot (7.952 \times 10^{-8}\ m^4)}{0.015m} = 477.12\ N{\cdot}m$

Segment BC

Internal Torque, $|T_{BC}| = 2T$

$J_{BC} = \frac{\pi}{2}(0.025)^4 = 6.136 \times 10^{-7}\ m^4$.



$(\tau_{max})_{BC} = \frac{2T \cdot r_{BC}}{J_{BC}} = 60\ MPa$.

$\Rightarrow T = \frac{(60 \times 10^6\ Pa) \cdot (6.136 \times 10^{-7}\ m^4)}{2\ (0.025m)}$

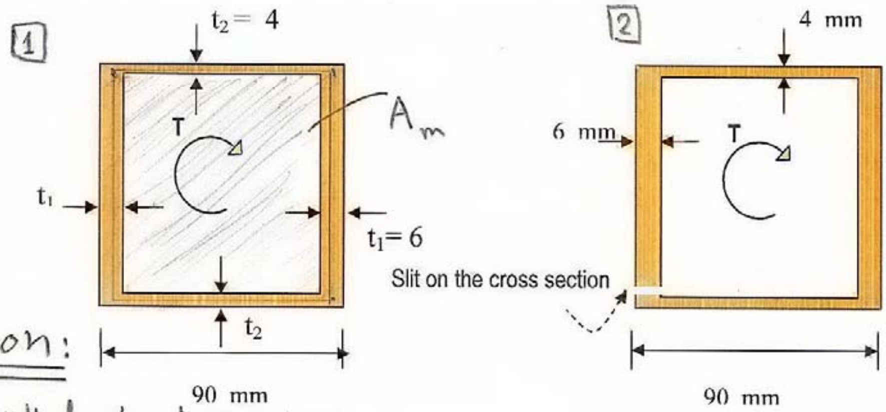
$= 736.29\ N{\cdot}m$

$\Rightarrow$ The largest torque $T = \underline{477.12\ N{\cdot}m}$.

Problem 1 (cont'd):

Part – b:
A torque T = 200 kN.m is to be applied to a shaft with *either* one of the two thin-walled tubular cross sections (shown in Fig. P-1(b)) with identical dimensions *but* with right section having a slit that extend along the the bar length. Calculate *the maximum shear stress* developed in each one of the two cross sections.

Fig. P-1(b): Thin-walled cross-sections with identical dimensions



Solution:

1. Thin-walled closed section:

$$\tau_{max} = \frac{T}{2 A_m \cdot t}$$

$$A_m = (0.09-0.006)\cdot(0.09-0.004)$$
$$= 7.224 \times 10^{-3}\ m^2.$$

Max. torque will occur at $t_2 = 4mm$.

$$\tau_{max} = \frac{200 \times 10^3\ N \cdot m}{2\,(7.224 \times 10^{-3}\ m^2)(0.004 m)} = 3.46 \times 10^9\ \frac{N}{m^2} = 3.46\ GPa.$$

2. Thin-walled open section:

$$J = \frac{bt^3}{3} \Rightarrow J_{total} = 2 * \left[\frac{(0.09)(0.004)^3}{3}\right] + 2 * \left[\frac{(0.082)(0.006)^3}{3}\right]$$
$$= 15.648 \times 10^{-9}\ m^4.$$

Max. torque will occur at $t_1 = 6\ mm$.

$$\tau_{max} = \frac{T \cdot t_1}{J_{tot}} = \frac{(200 \times 10^3\ N \cdot m)(0.006 m)}{15.648 \times 10^{-9}\ m^4} = 76.69 \times 10^9\ Pa$$
$$= 76.69\ GPa.$$

Problem 2:

The 700 mm long metal shaft ABC (shown in Fig. P-2) is to carry a torque T_o applied on the cross section at B. The shaft has a circular cross section and is supportd by a *rigid support* at C, while the *suppport at end A allows an angular rotation* of 1 degree. If cross section r = 10 mm, and the material shear modulus G = 70 GPa:

a. determine the **maximum allowable torque** $T_o^{(max)}$ if the allowable shear stress is τ_{all} = 45 MPa;

b. determine the angular rotation ϕ_B of the cross-section at B, if *only* a torque value T_o = 42.6 N.m is applied at B.

Note:

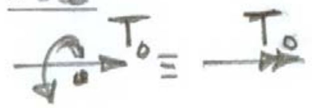
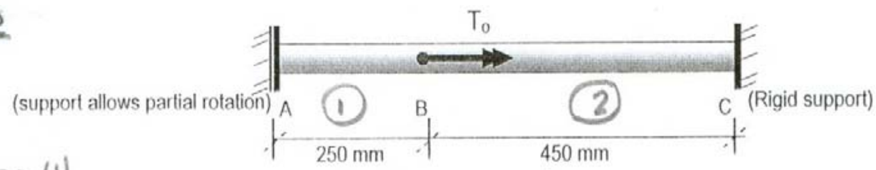
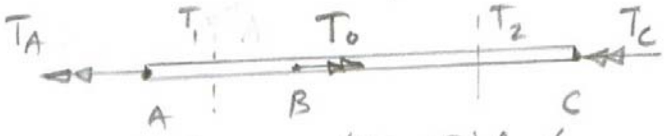


Fig. P-2 : Metal shaft ABC with torque T_o.



Part a:

$T_A + T_C = T_0$ --- (1)



$\phi_c = \phi_A + \sum Tl/GJ$

$\phi_c = 0 \Rightarrow -1(\pi)/180° = T_A l_1/GJ + (T_A - T_0)\, l_2/GJ$

$\because GJ$ = constant $\Rightarrow \frac{-\pi GJ}{180°} = 0.25\, T_A + 0.45\, T_A - 0.45\, T_0$

$T_0 = \frac{1}{0.45}\left(0.7\, T_A + \pi GJ/180\right) \Rightarrow T_0 = 1.56\, T_A + 0.0388\, GJ$

$GJ = 70\times10^9 \times \pi (0.01)^4/2 = 1099.6$ N·m²

$T_0 = 42.7 + 1.56\, T_A$ --- (2)

or $T_0 = 42.7 + 1.56\,(T_0 - T_c) \Rightarrow T_0 = 2.79\, T_c - 76.2$ ...(2)'

Assume: $\tau_2 = T_2 r/J = \tau_{all} \Rightarrow T_2 = \tau_{all} J/r = 70.69$ N·m

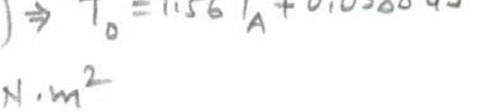
$T_c = T_2 = T_A - T_0 \Rightarrow T_0 = 2.79(70.69) - 76.2 = 121.03$ N·m

check $\tau_1 = T_1 r/J = T_A r/J = 50.43\,(0.01)/1.5708\times10^{-8} = 31.1$ MPa

$\therefore T_0^{(max)} = 121.03$ N·m

Part b: Now since $T_0^{(max)} = 121.03$ N·m (with $\phi_c = 0$ & $\phi_A = 1\pi/180°$)

Then since T_0 (applied) $\ll T_0^{max}$



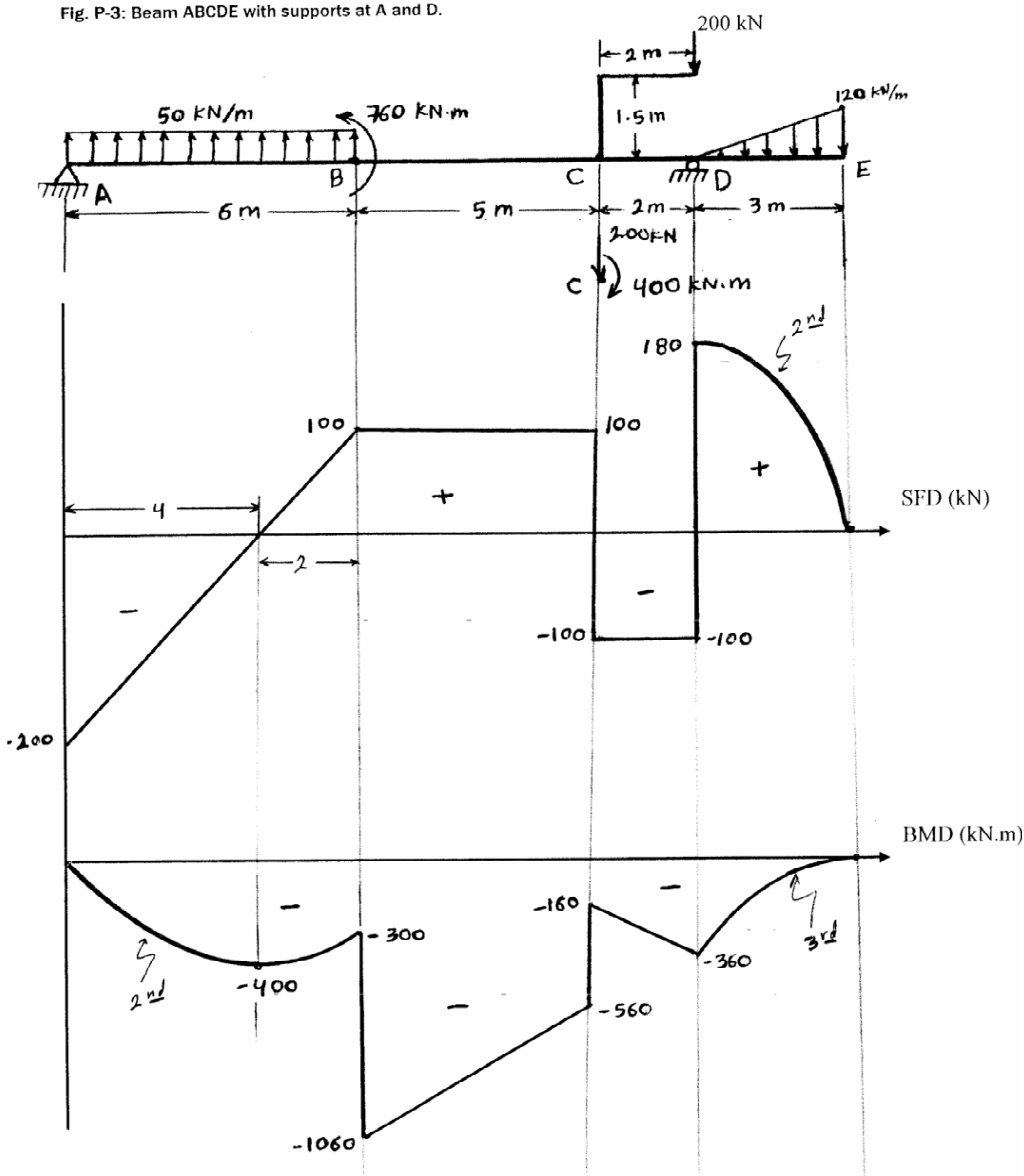
$\phi_{B/c} = T_2 l_2/GJ = \frac{42.6\,(0.45)}{1099.6}$

$= 0.0174$ rad $= 0.9969$ deg

Problem 3:

Draw **the shear force and bending moment diagrams** for the beam ABCDE shown below (Fig. P-3). *Write the degree of the curve on each one.* Use the **graphical (summation/area) method**; *no credit will be given if another method is used.* Use "appropriate" scale.
Note: *the reactions are given as* $R_A = 200\ kN$ (↓), and $R_D = 280\ kN$ (↑).

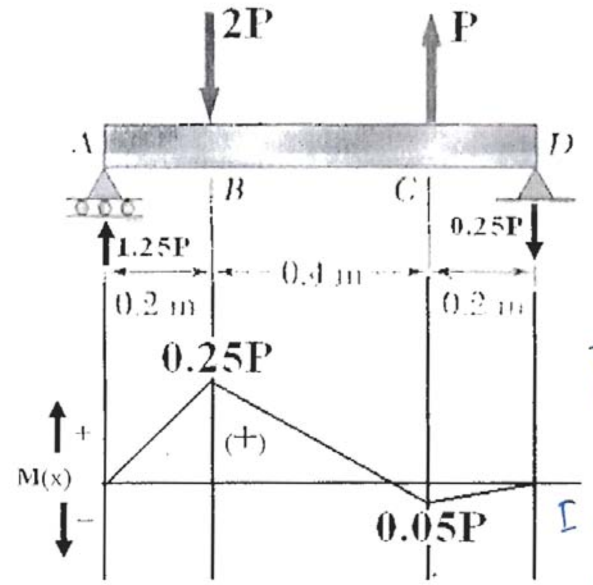
Fig. P-3: Beam ABCDE with supports at A and D.



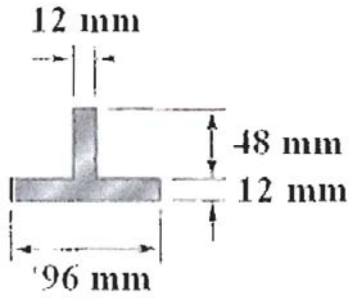
Problem 4:

A simply-supported beam ABCD having an inverted **T** cross-section is to carry two concentrated loads as is shown in Fig. P-4. Determine the *largest permissible* value of P which can be applied if the allowable stresses are $\sigma_{all(t)} = 80$ MPa (in *tension*), and $\sigma_{all(c)} = 140$ MPa (in *compression*).

Fig. P-4: Beam ABCD with two concentrated loads.



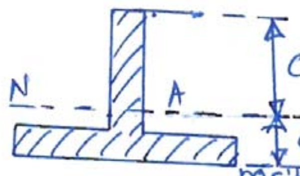
BMD (N.m)



Cross section

$$\bar{y} = \frac{96\times12\times6 + 48\times12\times36}{96\times12 + 48\times12} = \underline{16\,mm}$$

$$I = \frac{96\times12^3}{12} + 96\times12\times(16-6)^2 + \frac{12\times48^3}{12} + 12\times48\times(36-16)^2 = \underline{470016\ mm^4}$$



$C_{top} = 60 - 16 = 44\,mm$

$C_{bot} = 16\,mm$

Maximum positive bending moment is at B

$M_{max} = +0.25P\ N\text{-}m = 250P\ N\text{-}mm$

(This moment will induce comp. stress above N.A. and tensile stress below N.A.)

$$(\sigma_{max})_{comp.} = \frac{M_{max}\,C_{top}}{I} = \sigma_{allow(c)} = 140$$

$$\Rightarrow \frac{250P\times44}{470016} = 140 \Rightarrow \underline{P = 5982\,N}$$

$$(\sigma_{max})_{ten.} = \frac{M_{max}\,C_{bot}}{I} = \sigma_{allow(t)} = 80$$

$$\Rightarrow \frac{250P\times16}{470016} = 80 \Rightarrow \underline{P = 9400N}$$

Maximum negative moment is at C, $M_{max} = -0.05P$ N.m $= -50$ N-mm

(This moment will induce tensile stress above N.A. and comp. stress below N.A.) $\Longrightarrow$

$$(\sigma_{max})_{comp} = \frac{M_{max}\, C_{bot}}{I} = \sigma_{allow(c)} = 140$$

$$\Rightarrow \frac{50P \times 16}{470016} = 140 \Rightarrow \underline{P = 82253\text{ N}}$$

$$(\sigma_{max})_{ten} = \frac{M_{max}\, C_{top}}{I} = \sigma_{allow(t)} = 80$$

$$\Rightarrow \frac{50P \times 44}{470016} = 80 \Rightarrow P = \underline{17091\text{ N}}$$

$\therefore$ Largest permissible value of P

= least of above four values of P

= $\underline{\underline{5982\text{ N}}}$ Answer