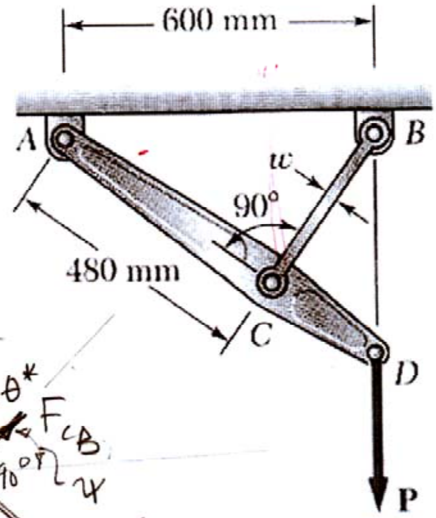
Problem 1: (Make-up)

The rigid bar ACD is supported by a link CB (with rectangle area) and a pin-support at A in a state of single shear, and it carries a load P = 50 kN as shown in Fig. P-1. Knowing that ultimate strengths of material are σ_{ult} = **600** MPa and τ_{ult} = 300 MPa and a factor of safety FS=1.5 is to be used against all modes of material failure:

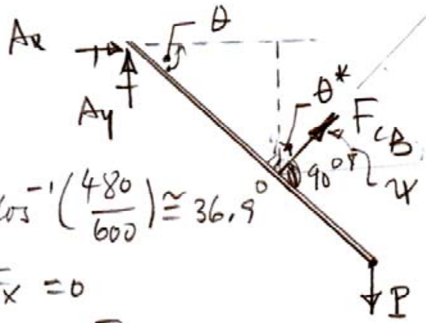
1. Determine the required width *w* of the link CB if its thickness is 10 mm. (15)
2. Compute shear stress τ in the pin at support A if diameter of the pin is 4 cm. (10)

Fig. P-1: Rigid bar ACD supported with a link CB and pin-support at A.



1) Compute w:

FBD of ACD



$\theta = \cos^{-1}\left(\frac{480}{600}\right) \cong 36.9^\circ$

$\overset{+}{\rightarrow} \sum F_x = 0$

$A_x + F_{CB}\cos\psi = 0$

$\circlearrowleft \sum M_A = 0$: $F_{CB} = 50 \times 600/480 = 62.5$ kN

$\therefore A_x = -F_{CB}\sin\theta^* = -F_{CB}\sin\theta = -37.5$

$+\uparrow \sum F_y = 0$: $A_y = P - F_{CB}\cos\theta = 50 - 62.5\cos 36.5^\circ = 0.0197$ kN

$\sigma_{CB} = \frac{N_{CB}}{A_{CB}} = \frac{F_{CB}}{wt} \not> \sigma_{all} = \frac{600}{1.5} = 400$ MPa

$w \not< \frac{62.5 \times 10^3}{400 \times 10^6 \times 0.01} \Rightarrow w \geq 15.6$ mm

2) Compute τ_A:

$\tau_A = V_A/A_c = \frac{\sqrt{A_x^2 + A_y^2}}{\pi(0.04)^2/4} \cong \frac{37.53 \text{ kN}}{1.257 \times 10^{-3}}$

$\therefore \tau_A \cong 29.9$ MPa

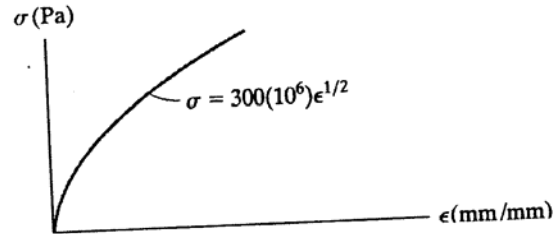
Problem 2:

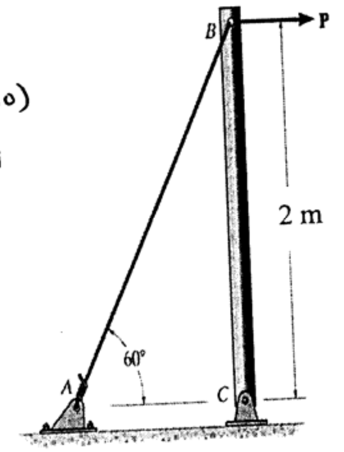
A vertical rigid post CB hinged at C and is supported by a cable BA and caries a force P (as shown in Fig. P-2). When the force P, was applied the length of cable AB increased by 25 mm. If the cable is made from material having the given stress-strain and has an original diameter of 10 mm, calculate:

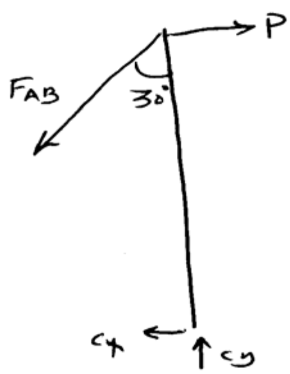
1. The magnitude of force P.
2. The final diameter of the cable.

Assume Possion's ratio ν =0.25.

Fig. P-2 Vertical post CB.







$L_{AB} = \frac{2000}{\sin 60} = 2309.4$ mm

$\varepsilon_{AB} = \frac{25}{2309.4} = .011$

from σ-ε relation

$\sigma = 31.2$ MPa

∵ $F_{AB} = \sigma A = 31.2 * \frac{\pi}{4}(10)$

$F_{AB} = 2451.5$ N

$P = 1225$ N

$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$

$\varepsilon_{lat} = -0.00275$

∵ $\frac{\Delta dia}{d} = -.00275$

$\Delta dia = -.0275$ mm

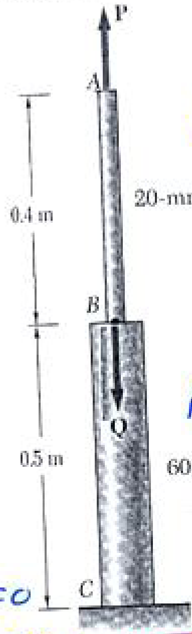
Find diameter = 9.9725 mm

Problem 3:

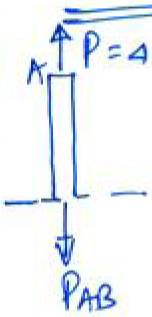
Both parts of the post ABC are made of an aluminum for which $E = 70$ GPa. It is subjected to forces P and Q as shown in Fig. P-3. Taking force $\boldsymbol{P} = 4$ kN, determine:

1. the value of the force $\boldsymbol{Q}$ applied at B so that the deflection at A is zero.
2. the corresponding deflection of B.

Fig. P-3: Vertical post ABC (with solid circular cross sections) carries two forces P and Q.



Internal forces (P_{AB} & P_{BC})

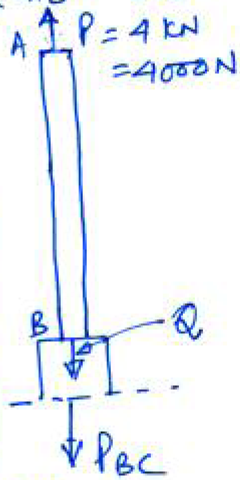


$\Sigma F_y = 0$

$\Rightarrow 4 - P_{AB} = 0$

$\Rightarrow P_{AB} = 4$ kN $= 4000$ N

[03]



$\Sigma F_y = 0$

$\Rightarrow 4000 - Q - P_{BC} = 0$

$\Rightarrow P_{BC} = (4000 - Q)$ N [05]

$A_{AB} = \frac{\pi}{4}(20)^2 = 314.16\ mm^2$

$A_{BC} = \frac{\pi}{4}(60)^2 = 2827.43\ mm^2$

1. Q

$$\delta_{A/C} = \frac{P_{AB} L_{AB}}{A_{AB} E} + \frac{P_{BC} L_{BC}}{A_{BC} E} = 0 \quad [05]$$

$$\Rightarrow \frac{4000 \times 400}{314.16\,E} + \frac{(4000 - Q) \times 500}{2827.43\,E} = 0$$

$$\Rightarrow 50.93 + 707.36 - 0.17684\,Q = 0$$

$$\Rightarrow Q = 32800 \text{ N} = 32.8 \text{ kN} \quad \text{Ans}$$

[05]

2. δ_B

$$\delta_B = \delta_{B/C} = \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{(4000 - 32800) \times 500}{2827.43 \times 70 \times 10^3}$$

$$= -0.0727 \text{ mm} = +0.0727 \text{ mm} (\downarrow) \quad \text{Ans}$$

[07]

Problem 4:

A vertical compound post AB is composed of a hollow steel cylinder with cross sectional area $A_{ST} = 2000\ mm^2$, and solid aluminum cylinder with cross sectional area $A_{AL} = 4000\ mm^2$. The post carries a mass of weight $W = 200$ kN (just resting symmetrically on both parts of the post) as shown in Fig. P-4.

10% 1. Determine the stress σ_{ST} in the steel cylinder.

15% 2. Determine the required minimum temperture change so that all the load of the weight W is carried only by the aluminum cylinder (*knowing that the two parts of the post AB can expand freely at end A*).

Given:

$E_{ST} = 200$ GPa; $\alpha_{ST} = 12 \times 10^{-6} / {}^{0}C$ — 10 points

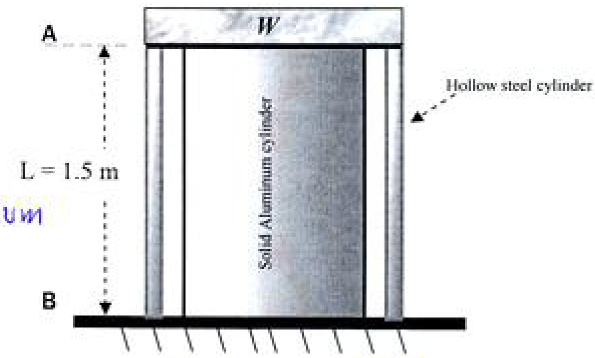
$E_{AL} = 70$ GPa; $\alpha_{AL} = 24 \times 10^{-6} / {}^{0}C$ — 15 points

Fig. P-4: Compound post AB composed of solid aluminum cylinder and hollow steel cylinder..

Solution:

P_{ST} = force in Steel

P_{AL} = ″ ″ Aluminum



From statics:

$$P_{ST} + P_{AL} = 200000 \quad \text{---- (1)}$$

(4)

From Compatability :-

(4)

$$\Delta_{ST} = \Delta_{AL}$$

$$\frac{P_{ST} L_{ST}}{A_{ST} E_{ST}} = \frac{P_{AL} L_{AL}}{A_{AL} E_{AL}} \Rightarrow \frac{P_{ST}}{2000 \times 10^{-6} \times 200 \times 10^{9}} = \frac{P_{AL}}{4000 \times 10^{-6} \times 70 \times 10^{9}}$$

$$280\, P_{ST} = 400\, P_{AL} \Rightarrow P_{AL} = 0.7\, P_{ST} \quad \text{------ (2)}$$

subst. in (1) to get $P_{ST} = 117{,}647.1$ N

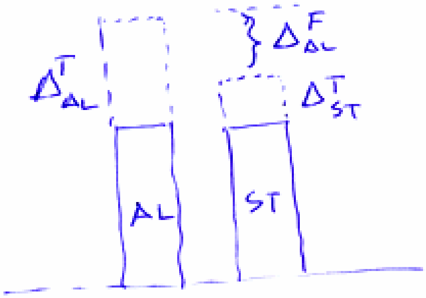
$$\sigma_{ST} = \frac{P_{ST}}{A_{ST}} = \frac{117647.1}{2000 \times 10^{-6}} = 58.823 \text{ MPa}$$

(2)

Part (2) :-

let change in temp. = T (°C)

Compatability :-



(8)

$$\Delta_{AL}^{T} = \Delta_{AL}^{F} + \Delta_{ST}^{T}$$

$$24 \times 10^{-6} (T) \times 1.5 = \frac{200{,}000 \times 1.5}{4000 \times 10^{-6} \times 70 \times 10^{9}} + 12 \times 10^{-6} (T) \times 1.5$$

$$12 \times 10^{-6} (T) = 0.7143 \times 10^{-3}$$

$$T = 59.524\ ^{\circ}C$$

(7)