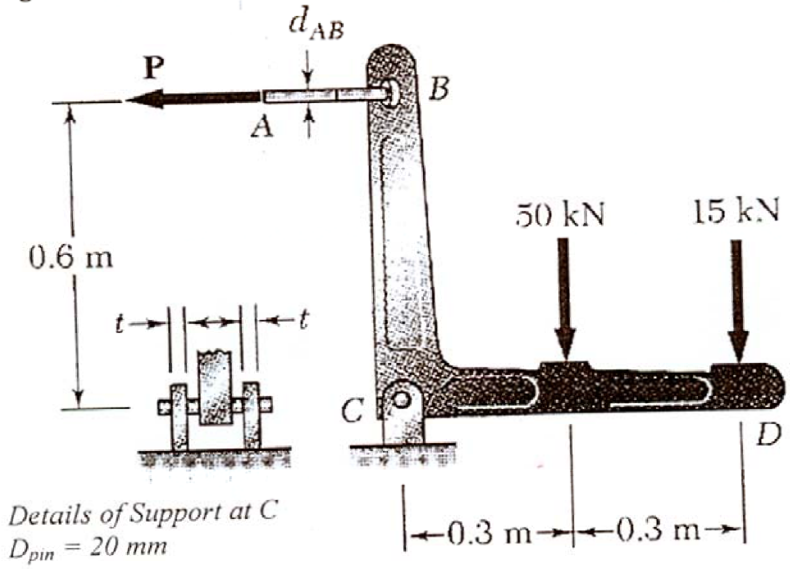
Key Solution_Exam 1

Problem 1:

The rigid link BCD is supported by pin C and cable BA, and carries two concentrated loads as shown in Fig. P-1. Knowing that ultimate strengths of material are $\sigma_{ult} = 400$ MPa and $\tau_{ult} = 250$ MPa and a factor of safety FS = 1.5 is to be used against all modes of material failure:

1. Determine the required diameter d_{AB} of the cable. (10)
2. Compute shear stress τ in the *pin* used at support C. (10)
3. Determine the required thickness t of the bearing support at C if the allowable bearing stress is 250 MPa. (05)

Fig. P-1: Link BCD supported with a pin-support and a cable BA.



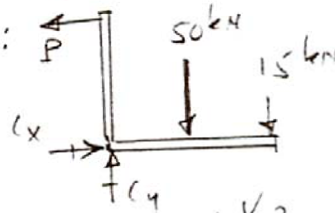
1) $\sigma_{AB} = N/A_{AB} = P/(\pi d_{AB}^2/4) \ngtr \frac{400}{1.5} \Rightarrow \frac{4P}{\pi d_{AB}^2} = \frac{400\times10^3}{1.5}$

$\circlearrowleft \sum M_c = 0$: $0.6P - 50\times0.3 - 15\times0.6 = 0 \Rightarrow P = 40$ kN

$\therefore d_{AB}^2 = \frac{4P}{\pi} \times 1.5/400\times10^3 = 4\times40/\pi \times 1.5/400\times10^3 = 1.9095\times10^{-4}$ m^2

$\therefore d_{AB} = 1.382\times10^{-2}$ m $\Rightarrow d_{AB} \simeq 13.82$ mm

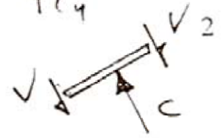
2) Compute Support C reactions C_x & C_y :



$C_x - P = 0 \Rightarrow C_x = 40$ kN

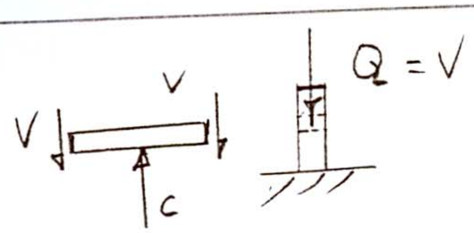
$C_y - 50 - 15 = 0 \Rightarrow C_y = 65$ kN

$C = \sqrt{C_x^2 + C_y^2} = 76.32$ kN



$2V - C = 0 \Rightarrow V = C/2 = 38.2$ kN

$\tau = V/A = 38.2/(\pi(0.02)^2/4) = 121.6$ MPa



$$\sigma_b = \frac{Q}{A_b} = \frac{V}{A_b} \ngtr 250 \text{ MPa}$$

$$A_b \nless \frac{\overset{(kN)}{V}}{250 \times 10^3}$$

$$\therefore A_b = \frac{38.2}{250 \times 10^3} = 1.528 \times 10^{-4} \text{ m}^2$$

$$A_b = t\, D_{pin} = t \times 0.02$$

$$t \times 0.02 = 1.528 \times 10^{-4}$$

$$\therefore \text{Req'd } t = \frac{1.528 \times 10^{-4}}{0.02} = 7.64 \times 10^{-3} \text{ m}$$

Req'd: $t = 7.64$ mm

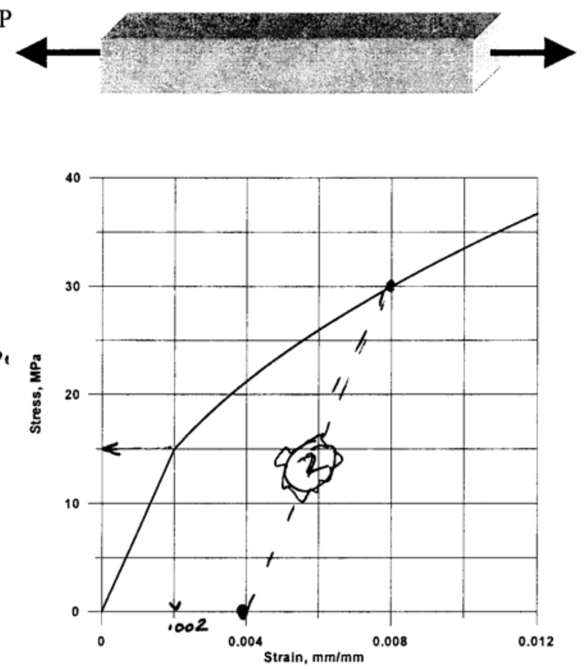
Problem 2:

A bar made from a given alloy, with the stress-strain diagram shown in Fig. P-2 and with a Poisson's ratio $\nu = 0.25$, was originally 1 m long and had a square cross-section 100 mm x100 mm.

With the axial force P applied, the square cross-section became 99.95 mm x 99.95 mm, determine:

a) The magnitude of the applied force. 10
b) The final length of the bar with the force applied. 10
c) If the force is increased to 300 kN then *released completely*, what would be the final length of the bar. (5)

Fig. P-2 Axial loading of a bar with known stress-strain diagram



(2) $\Delta_{lat} = -0.05$ mm

(3) $\varepsilon_{lat} = \dfrac{-0.05}{100} = -5 \times 10^{-4}$

(2) $\nu = -\dfrac{\varepsilon_{lat}}{\varepsilon_{long}}$

(3) $\therefore \varepsilon_{long} = 0.002$

(3) from diagram $\sigma = 15$ MPa

(2) $\therefore \Delta_{long} = \varepsilon_{long} * L = 2$ mm

$\therefore$ Applied force

(3) $P = \sigma A = 150$ KN

(b) $L_{final} = L_o + \Delta_{long}$

(2) $= 1002$ mm

(c) $\sigma_{becomes} = 30$ MPa ε will be .008

the bar will recover elastically // to linear portion

which results in a permenant strain of .004

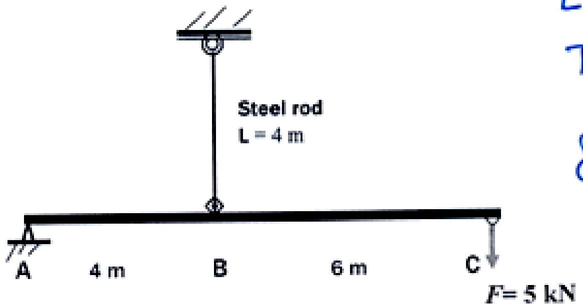
$\therefore$ permenant deformation $= 4$ mm (3)

$\therefore$ Final length $= 1004$ mm

Problem 3:

The rigid bar **ABC** (shown in Fig. P-3) is hinged at *A*, supported by a steel rod at *B*, and it is required that the vertical movement of the end **C** must not exceed 3 mm. Determine the required diameter of the steel rod for a load F = 5 kN applied at end **C**. Take E for steel rod as 2×10^5 N/mm^2.

Fig. P-3: Rigid bar ABC.



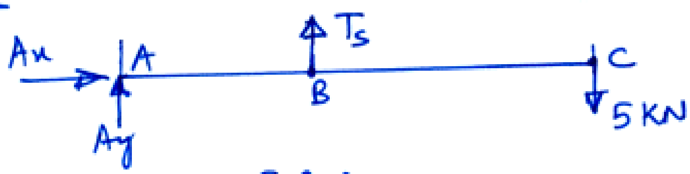
Let:

T_s = force in steel rod

δ_s = Deformation in steel rod.

d = diameter of steel rod

Force in steel rod



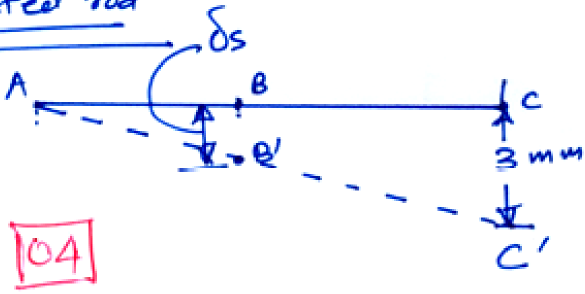
$\boxed{03}$

F.B.D.

$$\Sigma M_{about\,A} = 0 \Rightarrow -5(6+4) + T_s(4) = 0 \quad \boxed{05}$$

$$\Rightarrow T_s = \frac{50}{4} = 12.5 \text{ KN}$$

Deformation in steel rod



AB = 4m
BC = 6m

$\boxed{05}$

We have

$$\frac{3}{6+4} = \frac{\delta_s}{4} \quad \boxed{04}$$

$$\Rightarrow \delta_s = \frac{12}{10} = 1.2 \text{ mm}$$

Diameter of steel rod

$$\delta_s = \frac{T_s L_s}{A_s E_s} \Rightarrow 1.2 = \frac{12.5\times10^3 \times 4\times10^3}{A_s \times 2\times10^5} \quad \boxed{05}$$

$$\Rightarrow A_s = 208.33 \text{ mm}^2 = \frac{\pi}{4} d^2 \quad \boxed{03}$$

$$\Rightarrow d = \sqrt{\frac{4\times208.33}{\pi}} = 16.28 \text{ mm}$$

Ans

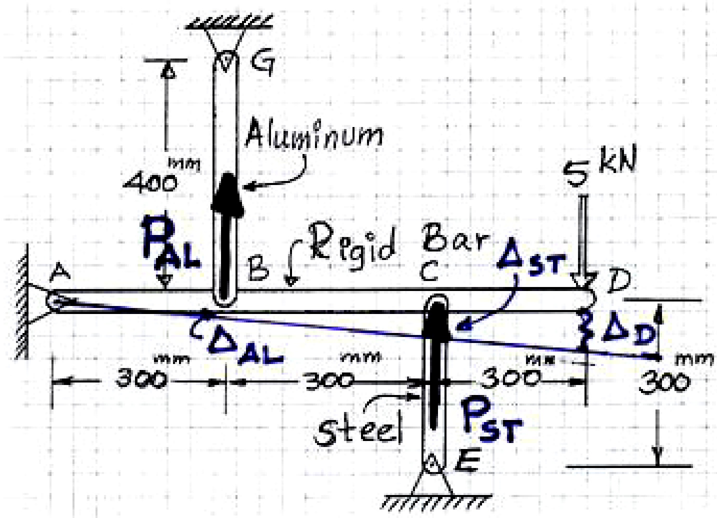
Problem 4:

A rigid bar ABCD (shown in Fig. P-4) is assumed of negligible weight and is held in position by an aluminum bar BG and by a steel bar CE. Given that E_{AL} = 70 GPa, A_{AL} = 120 mm^2, E_{ST} = 210 GPa, and A_{ST} = 50 mm^2:

1. Determine the vertical displacement at end D.
2. Determine the axial stress in steel bar CE

Fig. P-4: Rigid bar ABCD held in position by pin-support A and two metalic bars BG and CE.

$$\Delta = \frac{P \cdot L}{A \cdot E}$$



$\sum M_A = 0$ (+)

$$P_{AL} \times 0.3 + 0.6 \times P_{ST} - 5000 \times 0.9 = 0$$

$$\boxed{P_{AL} + 2 P_{ST} = 15000} \text{ - - - - } (1)$$

5

Compatability Equation :-

$$\left(\frac{\Delta_{AL}}{0.3} = \frac{\Delta_{ST}}{0.6} \right) \Longrightarrow 2 \Delta_{AL} = \Delta_{ST}$$

4

$$\frac{2 P_{AL} \times 0.4}{120 \times 10^{-6} \times 70 \times 10^{9}} = \frac{P_{ST} \times 0.3}{50 \times 10^{-6} \times 210 \times 10^{9}}$$

$$8400 P_{AL} = 2520 P_{ST}$$

$$\boxed{P_{ST} = 3.333 P_{AL}} \text{ - - - - - - } (2)$$

3

Solve (1) & (2) Simulatanously :

$$P_{AL} = 1956.5^{N} \qquad P_{ST} = 6521.7^{N}$$ (3)

(1) Evaluation Δ_D :-

from geometry $\Delta_D = 3\,\Delta_{AL}$

$$\Delta_{AL} = \frac{P_{AL}\,L_{AL}}{A_{AL}\,E_{AL}} = \frac{1956.5 \times 0.4}{120 \times 10^{-6} \times 70 \times 10^{9}}$$

$$= 0.0932 \times 10^{-3}\ m = 0.0932\ mm$$

$$\boxed{\Delta_D = 3\,\Delta_{AL} = 0.28\ mm}$$ (5)

(2)

$$\sigma_{ST} = \frac{P_{ST}}{A_{ST}}$$

$$= \frac{6521.7}{50 \times 10^{-6}} = 130.43\ MPa$$ (5)