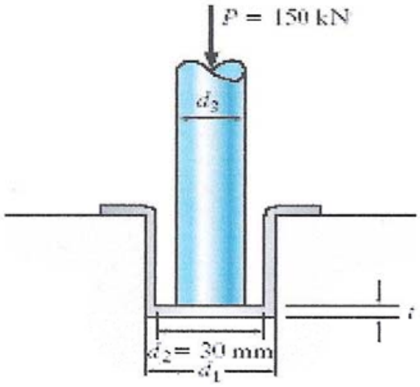
CE 203 Structural Mechanics - I

First Major Exam

Key Solution of the Problems

Solution of Problem - 1

Determine the smallest dimensions of the circular end cap (d_1 and t) and the circular shaft (d_3), if the load it is required to support is $P = 150$kN. The allowable stresses for tension, bearing compression of bottom of shaft on the circular end cap), and shear are $(\sigma_t)_{allow} = 475$MPa, $(\sigma_b)_{allow} = 475$MPa, and $\tau_{allow} = 115$MPa. Also, determine the safety factor in tension, if $(\sigma_t)_{failure} = 680$MPa.



$$F.S. = \frac{(\sigma_t)_{Failure}}{(\sigma_t)_{allow}} = \frac{680\ MPa}{475\ MPa} = \underline{\underline{1.432}}$$

Allowable normal (tensile) Stress: End cap outer diameter ; d_1

$$A = \frac{P}{(\sigma_t)_{allow}} \; ; \; \frac{\pi}{4}(d_1^2 - d_2^2) = \frac{150\times10^3 N}{475\times10^6 N/m^2}$$

$$d_1^2 - (0.03m)^2 = \frac{4\times150\times10^3 N}{\pi(475\times10^6 N/m^2)}$$

$$d_1 = \sqrt{4.021\times10^{-4} + 9\times10^{-4}}$$

$$= 3.6085\times10^{-2} m$$

$$= \underline{36.085\ mm}$$

Allowable bearing Stress: Shaft diameter ; d_3

$$A = \frac{P}{(\sigma_b)_{allow}} \; ; \; \frac{\pi}{4} d_3^2 = \frac{150\times10^3 N}{475\times10^6 N/m^2}$$

$$d_3 = \sqrt{\frac{4\times150\times10^3}{\pi(475\times10^6 N/m^2)}} = \underline{\underline{20.05}}\ mm$$

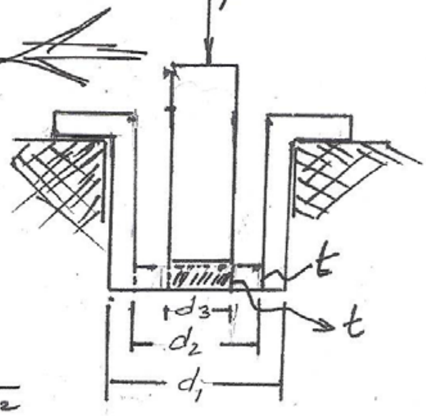
Allowable Shear Stress; End Cap thickness, t

$$A_{shear} = \frac{P}{\tau_{allow}} ; \quad (\pi d_2)t = \frac{150 \times 10^3 N}{115 \times 10^6 N/m^2}$$

$$t = \frac{150 \times 10^3 N}{\pi (0.03 m)(115 \times 10^6 N/m)}$$

$$= 13.84 \times 10^{-3} m$$

$$= \underline{\underline{13.84 \ mm}}$$



Also,

$$A_{shear} = \frac{P}{\tau_{all}} ;$$

$$(\pi d_3)t = \frac{150 \times 10^3 N}{115 \times 10^6 N/m^2}$$

$$t = \frac{150 \times 10^3 N}{\pi (0.02005 m)(115 \times 10^6 N/m)}$$

$$= 20.708 \times 10^{-3} m$$

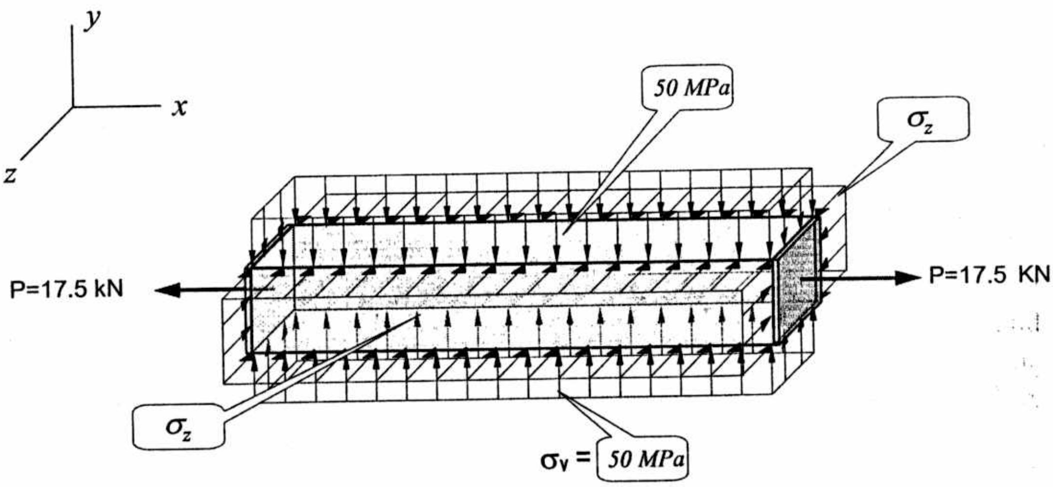
$$= \underline{\underline{20.708 \ mm}}$$

$\therefore$ t for the end cap is $\underline{\underline{20.708 \ mm}}$

Problem 2:

2-a A block of a material (with: E = 200 GPa; $\nu = 0.23$) is subjected simultaneously to an axial force P = 17.5 kN in the x-direction (applied through an attached plate of area 200 mm^2), and a pressure in the y-direction of magnitude 50 MPa as shown in Fig. P-2(a). Determine the required stress that should be applied to the block in the z-direction so that the dilatation (*i.e.* change in material-volume) is zero.

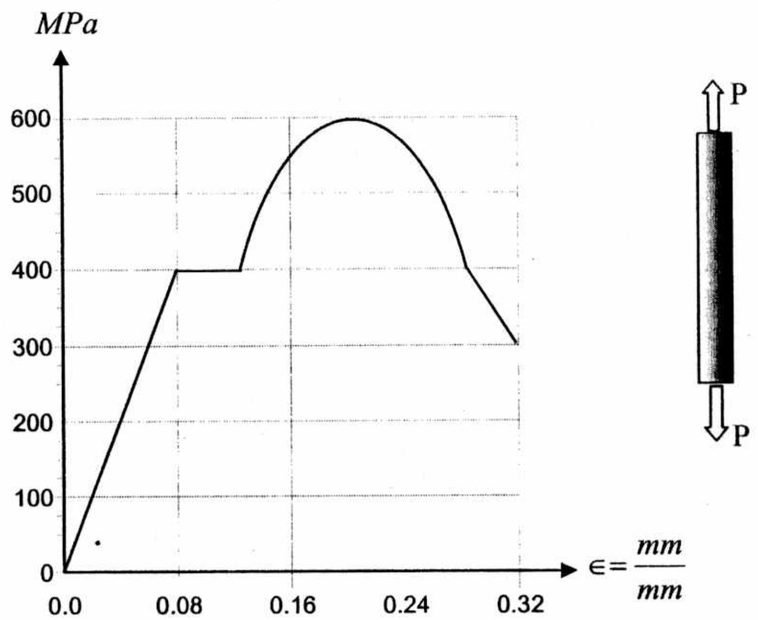
Fig. P-2(a): A block of material under combined load-cases



2-b A rod of diameter 200 mm is subjected to an *axial force* P =16 MN. If the material behavior is according to the stress-strain curve given in Fig. P-2(b) below, find the following:

- The strain corrseponding to the applied force.
- The ultimate stress σ_{ult}, the failure stress σ_f, the yield stress σ_y and modulus of elasticity E.
- The Modulus of Resilience of the given material.

Fig. P-2(b): A rod under axial loading



Solution of Problem - 2

Q2(a):

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$e = 0 \Rightarrow 3 \quad \sigma_x + \sigma_y + \sigma_z = 0 \quad \left(\text{as } \frac{1-2\nu}{E} \neq 0\right)$$

3 $\sigma_x = \frac{P}{A} = 87.5\ MPa$

3 $\sigma_y = -50\ MPa$

$\sigma_z = \sigma_z$

$\therefore$ 1 $87.5 - 50 + \sigma_z = 0 \Rightarrow \boxed{\sigma_z = -37.5\ MPa}$

Q2(b)

2 $\sigma = \frac{P}{A} = \frac{16\times10^6}{\frac{\pi(200)^2}{4}} = 509\ MPa$

$\Rightarrow$ 2 $e = 0.14 - 0.15$

2 $\sigma_{ult} = 600\ MPa$

2 $\sigma_f = 300\ MPa$

2 $\sigma_y = 400\ MPa$

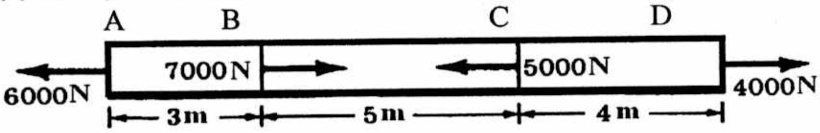
2 $E = 5\ GPa$

3 $MR = \frac{1}{2}(.08)(400) = 16\ MPa$

Problem 3:

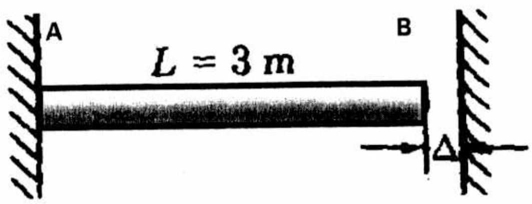
3-a An aluminum (with $E = 7 \times 10^4$ MPa) bar ABCD has a cross-sectional area of 325 mm^2, and carries four axial loads applied at the locations shown in Fig. P-3(a). Determine the total change in length ΔL of the bar.

Fig. P-3(a): Axially loaded Aluminum bar ABCD



3-b A bronze (with: $E = 80 \times 10^3$ MPa, and $\alpha = 18 \times 10^{-6}/°C$) bar AB is 3 meters long, has a cross-sectional area of 320 mm^2, and is placed between two rigid walls (as shown in Fig. P-3(b)). If the gap size is Δ = 2.5 mm at a temperature of −20°C, *determine* the final temperature at which the compressive stress in the bar will be 35 MPa.

Fig. P-3(b): Bar AB between two rigid walls



Solution of Problem – 3 (a)

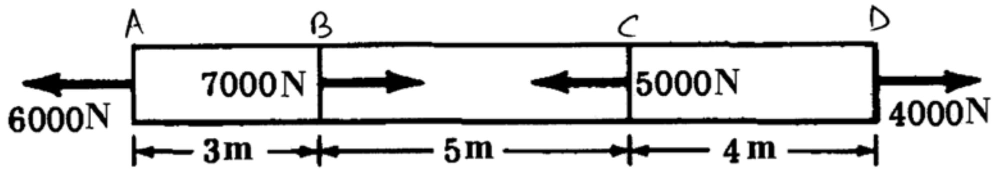
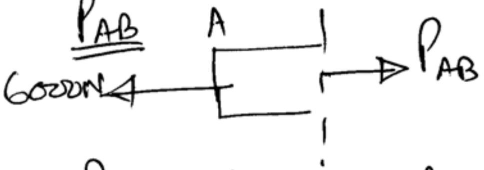


Fig. 3a

Solution

Internal forces

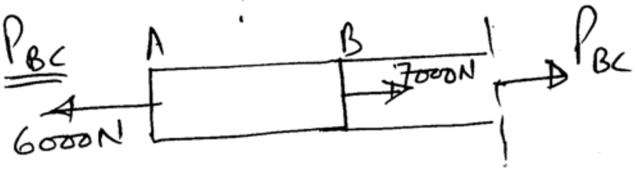
P_{AB}



$\Sigma F_x = 0 \Rightarrow -6000 + P_{AB} = 0$

$\Rightarrow P_{AB} = 6000\,N\ (T)$

P_{BC}



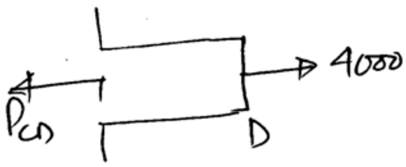
$\Sigma F_x = 0$

$\Rightarrow -6000 + 7000 + P_{BC} = 0$

$\Rightarrow P_{BC} = -1000\,N$

$\Rightarrow P_{BC} = 1000\,N\ (C)$

P_{CD}



$\Sigma F_x = 0$

$\Rightarrow -P_{CD} + 4000 = 0$

$\Rightarrow P_{CD} = 4000\,N\ (T)$

$$\delta_{A/D} = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

$$= \frac{P_{AB} L_{AB}}{E_{AB} A_{AB}} + \frac{P_{BC} L_{BC}}{E_{BC} A_{BC}} + \frac{P_{CD} L_{CD}}{E_{CD} A_{CD}}$$

We have, $A_{AB} = A_{BC} = A_{CD} = 325\ mm^2$, $E_{AB} = E_{BC} = E_{CD} = 0.7\times10^5\ N/mm^2$

$$\delta_{A/D} = \frac{+6000\times3\times10^3}{0.7\times10^5\times325} + \frac{(-1000\times5\times10^3)}{0.7\times10^5\times325} + \frac{4000\times4\times10^3}{0.7\times10^5\times325}$$

$$= 1.2747\ mm \quad \text{Ans.}$$

Solution of Problem 3 (b)

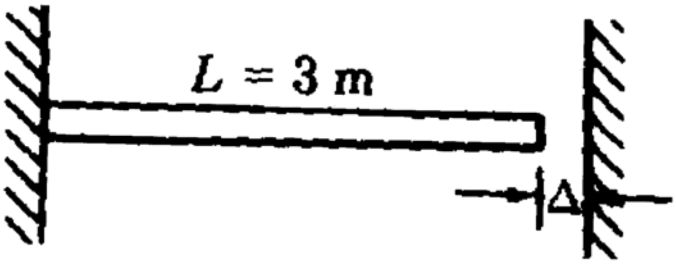


Fig. 3b

Solution

$L = 3\ m = 3000\ mm$, $A = 320\ mm^2$

$E = 80\times10^3\ N/mm^2$, $\alpha = 18\times10^{-6}/°C$

$\sigma_T = 35\ N/mm^2$

Let, T = temperature at which the $\sigma_T = 35\ N/mm^2$

$\therefore\ \Delta T = T - (-20) = (T+20)$

$\delta_T = \alpha \Delta T L = 18\times10^{-6}\times(T+20)\times3000$

$= 0.054(T+20) = (0.054T + 1.08)\ mm$

$\delta_{T,\,allowed} = 2.5\ mm$

$\delta_{T,\,prevented} = \delta_T - \delta_{T,\,allowed} = 0.054T + 1.08 - 2.5$

$= (0.054T - 1.42)\ mm$

$$\epsilon_T = \frac{\delta_{T,\,prev.}}{L} = \frac{0.054T - 1.42}{3\times1000}$$

$$\sigma_T = \epsilon_T E = \frac{(0.054T - 1.42)\times 80\times10^3}{3000} = 35$$

$\Rightarrow 0.054T - 1.42 = 1.3125$

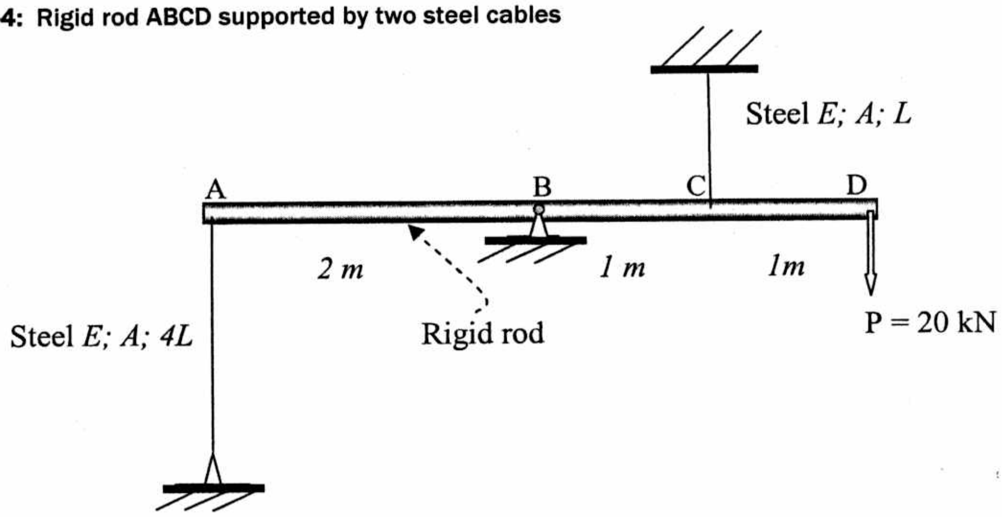
$$\Rightarrow T = \frac{1.3125 + 1.42}{0.054} = \underline{\underline{50.6\ °C}}$$

Ans

Problem 4:

The rigid rod ABCD (shown in Fig. P-4) is supported using two steel cables (at points A and C) and a pin support at B.

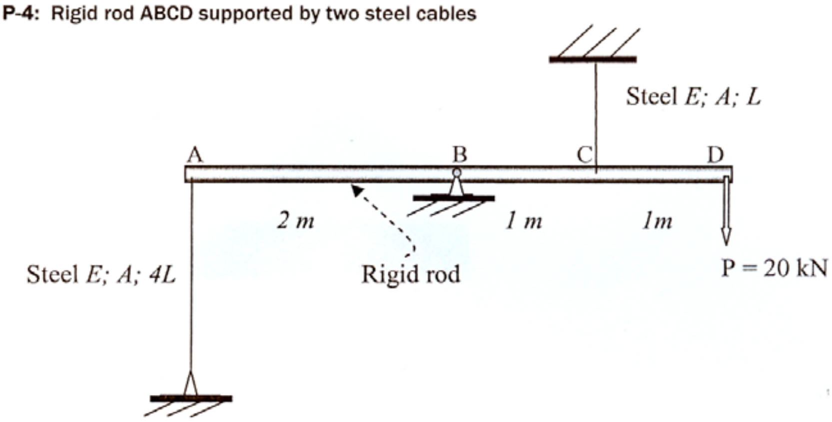
Fig. P-4: Rigid rod ABCD supported by two steel cables



4-a Determine the **magnitude and direction** of the support reactions at the pin B.
4-b Determine the **magnitude and direction** of the vertical displacements at points A and D.
(Assume: $E = 200$ GPA; $L = 0.8$ m; and $A = 200\ \text{mm}^2$).

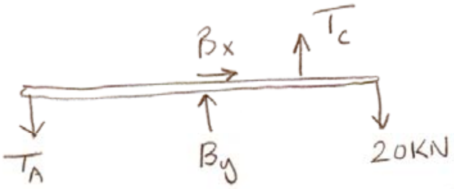
Solution of Problem - 4

Fig. P-4: Rigid rod ABCD supported by two steel cables



4-a Determine the **magnitude and direction** of the support reactions at the pin B.
4-b Determine the **magnitude and direction** of the vertical displacements at points A and D.
(Assume: $E = 200$ GPA; $L = 0.8$ m; and $A = 200$ mm^2).

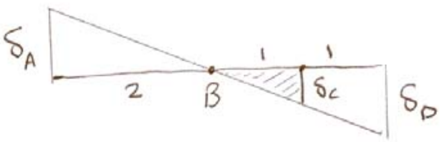
a)



$\Sigma F_x = 0$ $\boxed{B_x = 0}$

$+\uparrow \Sigma F_y = 0$ $B_y + T_c - T_A - 20 = 0$ (1)

$+\circlearrowleft \Sigma M_B = 0$ $2T_A + (1)T_c - (2)(20) = 0$ (2)



Using the 2 triangles $\delta_A = 2\delta_c$

using $\delta = \dfrac{Tl}{EA}$

$$\frac{T_A(4l)}{EA} = \frac{2T_c l}{EA} \quad \therefore \boxed{T_c = 2T_A}$$

Sub. in (2) $4T_A = +40$ $\boxed{T_A = +10 \text{ kN}}$, Sub in (1) $B_y = +10$ kN

$\therefore \boxed{B_x = 0}$ Ans. & $\boxed{B_y = 10 \text{ kN} \uparrow}$ Ans.

b) Using given E, A, L

$$\delta_A = \frac{(T_A)(4l)}{EA} = \frac{(10{,}000)(4)(800)}{(200{,}000)(200)} = +0.8 \text{ mm}$$

using the triangles $\delta_D = \delta_A = 0.8$ mm

Displ. at A is 0.8 mm $\uparrow$
Displ at D is 0.8 mm $\downarrow$ Ans