بسم الله الرحمن الرحيم

King Fahd University of Petroleum & Minerals

DEPARTMENT OF CIVIL ENGINEERING

First Semester 1432-33 / 2011-112 (111)

CE 203 STRUCTURAL MECHANICS I

Major Exam I

Tuesday, October 26, 2011 (**7:30-9:30** P.M.)

KEY SOLUTION

Note to Students:

Even though the course is not "standard grading", being around the average does not indicate C performance, since there is a minimum amount of course comprehension needed to pass the course satisfactorily, irrespective of the exam average and the performance of other students.
Therefore, students who did poorly in this exam should do double effort in the remaining of the semester to avoid disappointing grade.
After reviewing the key solution and still having a concern about your mark, you may consult with the faculty members who prepared each problem, who are:

Problem # 1 Dr. A.A. AL-Khathlan
Problem # 2 Dr. M.M. AL-Zahrani
Problem # 3 Dr. H.N. AL-Ghamedy
Problem # 4 Dr. S.A. AL-Ghamdi

The deadline for review is Wednesday November 16, 2011.

Problem # 1

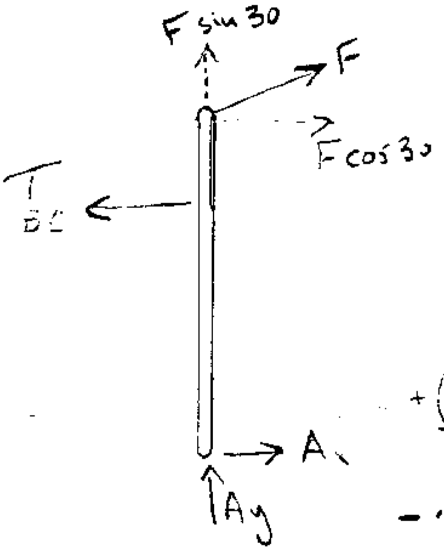
Rod AD is rigid and is supported using a pin at A and a cable at B. Determine the largest value of $\boldsymbol{F}$ that can be safely applied.

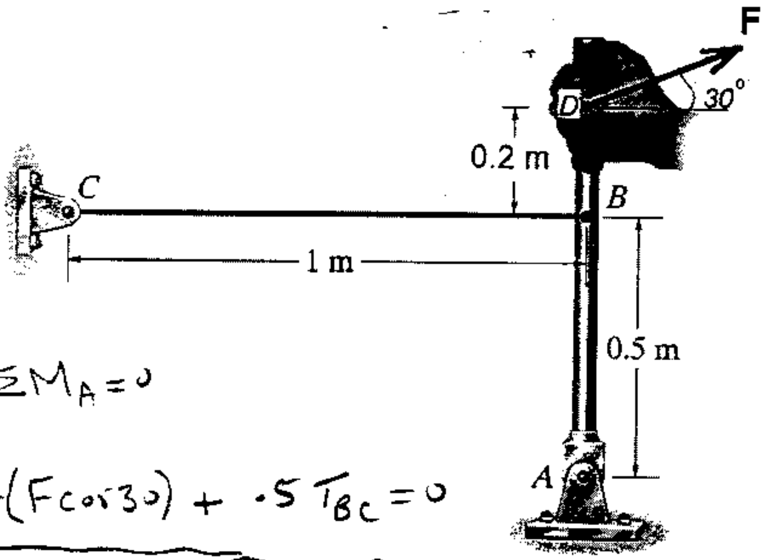
For all the pins and the cable use: $\sigma_{ult} = 40$ MPa, $\tau_{ult} = 20$ MPa, Safety Factor = 4

For cable BC, area = 80 mm^2,

For pin at A, area = 100 mm^2, in double shear

For pin at C, area = 120 mm^2, in single shear





$+\circlearrowleft \Sigma M_A = 0$

$-.7(F\cos 30) + .5\,T_{BC} = 0$

$\boxed{T_{BC} = 1.4F\cos 30}$ ... (1)

$\Sigma F_x = 0 \qquad A_x = T_{BC} - F\cos 30 = \boxed{0.4F\cos 30}$ (2), $\Sigma F_y = 0$ $\boxed{A_y = -F\sin 30}$ (3)

Using (2) & (3) $\quad R_A = \sqrt{A_x^2 + A_y^2} = \boxed{0.608\,F}$ (4)

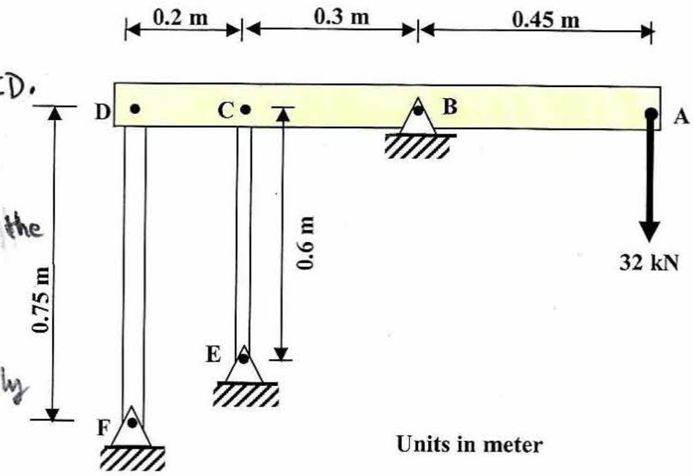
$\sigma_{all} = \dfrac{\sigma_{ult}}{S.F.} = \dfrac{40}{4} = 10$ MPa $\quad , \quad \tau_{all} = \dfrac{\tau_{ult}}{S.F.} = \dfrac{20}{4} = 5$ MPa

- Check cable BC $\quad \sigma_{all} = 10 = \dfrac{1.4F\cos 30}{80} \quad \boxed{F = 659.8\text{ N}}$
- Check pin C $\quad \tau_{all} = 5 = \dfrac{1.4F\cos 30}{120} \quad \boxed{F = 494.9\text{ N}}$
- Check pin A $\quad \tau_{all} = 5 = \dfrac{.608F}{(2)(100)} \quad \boxed{F = 1645\text{ N}}$

$\therefore$ $\boxed{F_{max} = 494.9\text{ N}, \text{ controlled by pin at C}}$ Answer

Problem # 2

The 10-mm diameter rod CE and 15-mm diameter rod DF are attached to the rigid bar ABCD as shown. Knowing that the two rods are made of aluminum with E = 70 GPa, **determine the stress in rod CE and the corresponding deflection at point A**.



Solution

Consider the F.B.D of bar ABCD.

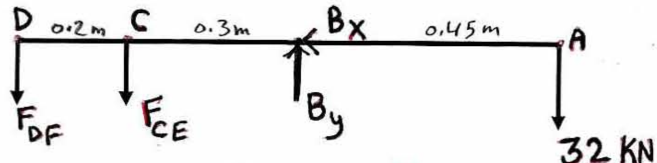
This is statically indeterminate since there are more unknowns than the available equilibrium equations.

We are looking for the forces in rods CE and DF, therefore we apply the equilibrium equ.:

$\circlearrowleft + \sum M_B = 0;$

5 $+32\,\text{KN}\,(0.45) - F_{CE}\,(0.3) - F_{DE}\,(0.5) = 0$

$\Rightarrow 14.4\times10^3\,\text{N} = 0.3\,F_{CE} + 0.5\,F_{DE}$ — ①

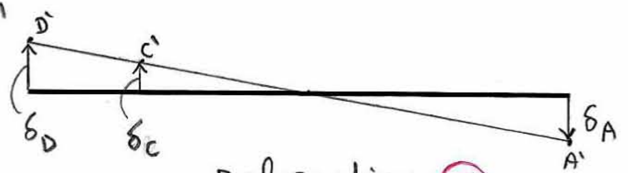


F.B.D ②

Compatibility Condition

After applying the 32KN load, the position of the rigid bar is A'BC'D'.

From similar triangles we have



Deformations ③

3 $\dfrac{\delta_c}{0.3} = \dfrac{\delta_D}{0.5} \Rightarrow \delta_c = 0.6\,\delta_D$ — ②

2 $\dfrac{\delta_A}{0.45} = \dfrac{\delta_D}{0.5} \Rightarrow \delta_A = 0.9\,\delta_D$ — ③

Deformations. $\Rightarrow$ from equ ② $\Rightarrow \dfrac{F_{CE}\cdot L_{CE}}{A_{CE}\cdot \cancel{E}} = 0.6\,\dfrac{F_{DF}\cdot L_{DF}}{A_{DF}\cdot \cancel{E}} \Rightarrow F_{CE} = 0.6\left[\dfrac{A_{CE}\cdot L_{DF}}{A_{DF}\cdot L_{CE}}\right]\cdot F_{DF}$

$\Rightarrow F_{CE} = 0.6\left[\dfrac{\frac{\pi}{4}(0.01)^2\cdot(0.75)}{\frac{\pi}{4}(0.015)^2\cdot(0.6)}\right]F_{DF} \Rightarrow \boxed{F_{CE} = \dfrac{F_{DF}}{3}}$ — ②'

Substitute into equ ①:

5 $0.3\left[\dfrac{F_{DF}}{3}\right] + 0.5\,F_{DF} = 14.4\times10^3\,\text{N} \Rightarrow \boxed{\begin{array}{l} F_{DF} = 24.0\times10^3\,\text{N} \\ F_{CE} = 8.0\times10^3\,\text{N}\end{array}}$

The stress in rod CE, $\sigma_{CE} = \dfrac{8\times10^3\,\text{N}}{\frac{\pi}{4}(0.01\text{m})^2} = 101.86\times10^6\,\text{Pa} = \boxed{101.86\,\text{MPa}.}$

Deformation at D, $\delta_D = \dfrac{(24\times10^3\,\text{N})(0.75\text{m})}{\frac{\pi}{4}(0.015\text{m})^2\cdot(70\times10^9\,\text{Pa})} = 1.455\times10^{-3}\,\text{m} = 1.455\,\text{mm}$

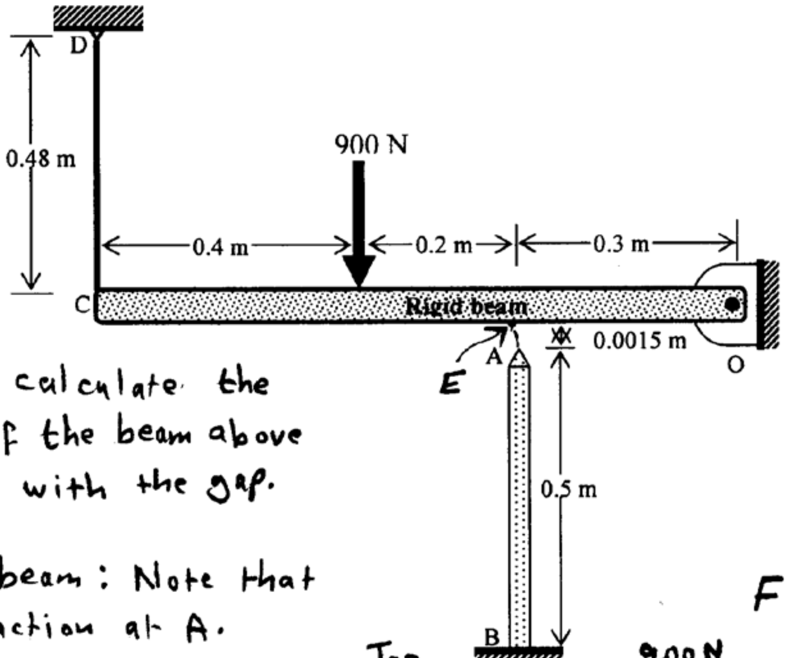
5 Deformation at A (from equ. ③), $\boxed{\delta_A = 0.9\,(\delta_D) = 1.31\times10^{-3}\,\text{m} = 1.31\,\text{mm}}\downarrow$

Problem # 3

Determine the required **change in the temperature** of rod **AB** if the problem is to **remain statically determinate**. Note that the gap shown is before applying the load and temperature.

	L(m)	A (m^2)	E (N/m^2)	α (/°C)
rod AB	0.5	$1\,(10)^{-6}$	$100\,(10)^9$	$20\,(10)^{-6}$
cable CD	0.48	$2\,(10)^{-7}$	$200\,(10)^9$	

Solution :

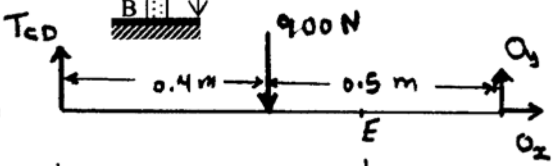


(Point) To keep the problem stat. det., the rigid beam needs to just touch AB at A.

We need to calculate the displacement of the beam above A and compare with the gap.

(3) FBD of the beam : Note that no force/reaction at A.

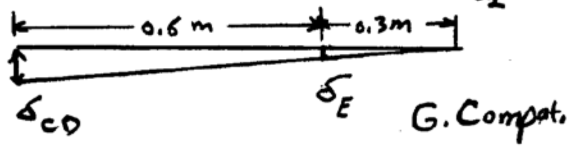
FBD



$\circlearrowleft{+}\ \Sigma M_o = 0 \Rightarrow$

(3) $900(0.5) - T_{CD}(0.9) = 0 \Rightarrow T_{CD} = 500\,N$

(3) Geom. Compatibility drawing $\Rightarrow$



(4) $\dfrac{\delta_{CD}}{0.9} = \dfrac{\delta_E}{0.3} \Rightarrow \delta_E = \frac{1}{3}\delta_{CD}$

(2) $\delta_{CD} = (TL/AE)_{CD} = 500(0.48)/2(10^{-7})(200)(10^9) = 0.006\,m$

$\Rightarrow \delta_E = 0.006/3 = 0.002\,m$

(2) $0.002 > 0.0015 = \text{gap} \Rightarrow$ need to decrease T_{AB} to eliminate the difference (δ_{net})

(4) $\delta_{net} = 0.0015 - 0.002 = -0.0005\,m$ ($-5\times10^{-4}\,m$)

(4) $-0.0005 = (\alpha\,\Delta T L)_{AB} = 20(10^{-6})\,\Delta T(0.5)$

$\Rightarrow \boxed{\Delta T_{AB} = -50\,°C \text{ [decrease]}}$

Problem # 4

The thin *square plate* ABCD (with L = 10 cm; t = 1 cm; E = 80 GPa; ν = 0.30) is subjected to a bi-axial state of stress with known σ_x, and a value of σ_y to be specified such that side length AB remains *unchanged.* For the given sate of stress:

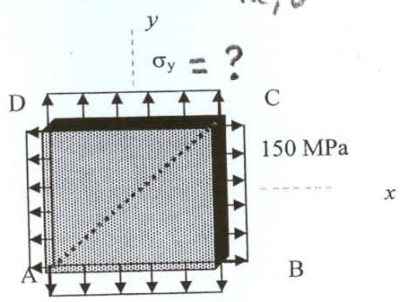
a) Determine the corresponding value of stress σ_y.
b) Determine the strain ε_{AC} along line AC.

$\sigma_x = 150\ MPa\ ;\ \sigma_y = ?\ ;\ \sigma_z = 0\ ;\ L_{AC,0} = 10\sqrt{2}\ cm$ (1)

a)
$\varepsilon_x = 1/E\,[\sigma_x - \nu(\sigma_y + \sigma_z)]$ (2)

$\varepsilon_y = 1/E\,[\sigma_y - \nu(\sigma_x + \sigma_z)]$ (2)

$\varepsilon_z = 1/E\,[\sigma_z - \nu(\sigma_x + \sigma_y)]$



$\varepsilon_x = \frac{\Delta L_x}{L_x} = 0$ (2)

$\ell_{AB} = L_x$

$0 = \sigma_x - \nu(\sigma_y + \cancel{\sigma_z})$ $\cancel{\sigma_z} = 0$ (2)

$\therefore\ \sigma_y = \frac{\sigma_x}{\nu} = 150/0.3$ (2)

$\Rightarrow \sigma_y = 500\ MPa\ (T).$ (1½)

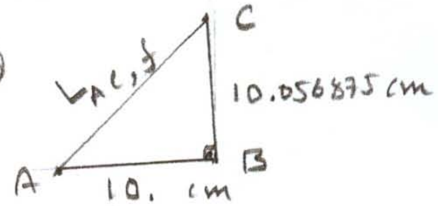
b)
$L_{xf} = L_{x0}$ since $\varepsilon_x = 0$ (2)

$L_{yf} = L_{y0}(1 + \varepsilon_y)$

$\varepsilon_y = 1/E\,[500. - 0.3(150 + 0)] = 455./E$ (2)

$\therefore\ \varepsilon_y = 5.69 \times 10^{-3}$ mm/mm (2)

$\therefore\ L_{yf} = 10 \times (1 + 5.69 \times 10^{-3}) = 10.056875$ cm



$L_{AC,f} = \sqrt{(10.)^2 + (10.056875)^2}$ (2)

$= 14.1824\,0934$ cm

$\Delta L_{AC} = L_{AC,f} - L_{AC,0}$ (3)

$= 0.04027372$ cm

$\therefore\ \varepsilon_{AC} = \Delta L_{AC}/L_{AC,0} = 2.8478 \times 10^{-3}$ mm/mm

(1½)