بسم الله الرحمن الرحيم

King Fahd University of Petroleum & Minerals

DEPARTMENT OF CIVIL ENGINEERING

Second Semester 1431-32 / 2010-11 (102)

CE 203 STRUCTURAL MECHANICS I

Major Exam I

Tuesday, March 29, 2011 7:00-9:30 P.M.

Student Name	Family				First			
ID No. (9 Digits)								

CIRCLE YOUR COURSE--SECTION NO.						
Section #	1 & 2	3	4	5	6 & 7	8
Instructor	Altayyib	Dulaijan	Ghamdi	Suwaiyan	Khathlan	Ahmad

Summary of Scores

Problem	Full Mark	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	
Remarks		

Notes:

1. A sheet that includes selected Basic Formulae and definitions is provided with this examination.
2. Write clearly and show all calculations, FBDs, and units.

Problem # 1

Rigid member *ABC*, which is supported as shown, is subjected to a load *P*. If the diameter of the pin at *C* is 20 mm and the diameter of cable BD is 10 mm, determine the *largest load P* that can be applied.

Given: τ_{fail} in the pin at C = 240 MPa; σ_{fail} in the cable BD = 300 MPa,
factor of safety (F.S.) for both types of stresses is 3.0
The pin at C is in *double shear*

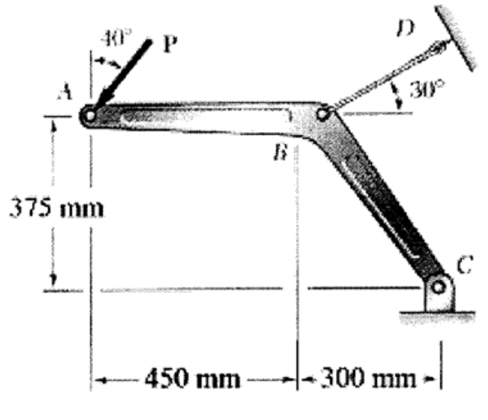
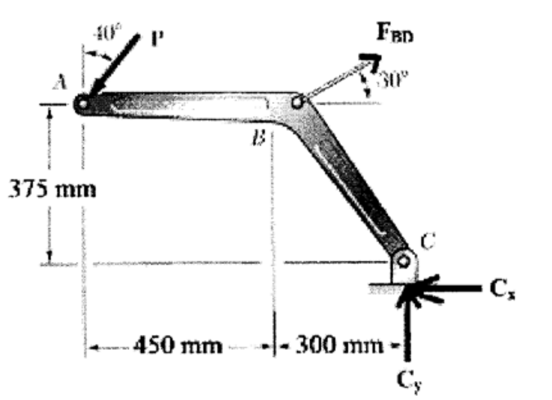


Figure 1



Free-body diagram

Applying equilibrium conditions to the above F.B.D, we have:

$$\Sigma M_{about\ C} = P\cos 40 \times (450+300) + P\sin 40 \times 375 - F_{BD}\sin 30 \times 300 - F_{BD}\cos 30 \times 375 = 0$$

$$\Rightarrow \underline{F_{BD} = 1.718P}$$

$$\Sigma F_x = 0 \Rightarrow -P\sin 40 + F_{BD}\cos 30 - C_x = 0$$

$$\Rightarrow \underline{C_x} = P\sin 40 - 1.718P \times \cos 30 = \underline{0.845P}$$

$$\Sigma F_y = 0 \Rightarrow -P\cos 40 + F_{BD}\sin 30 + C_y = 0$$

$$\Rightarrow \underline{C_y} = P\cos 40 - 1.718P \times \sin 30 = -0.093P$$

$$= \underline{0.093P\ (\downarrow)}$$

Shear force in pin at C,

$$\underline{V_C} = \frac{1}{2}\left[\sqrt{(0.845P)^2 + (0.093P)^2}\right] = \underline{0.425P}$$

P.T.O. $\Rightarrow$

$$\sigma_{allow} = \frac{\sigma_{fail}}{F.S.} = \frac{300}{3} = 100\text{ MPa}$$

$$\tau_{allow} = \frac{\tau_{fail}}{F.S.} = \frac{240}{3} = 80\text{ MPa}$$

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{1.718P}{\frac{\pi}{4}\left[10^2\right]} = 0.02187P = \sigma_{allow}$$

$$\Rightarrow \quad 0.02187P = 100$$

$$\Rightarrow \quad \underline{P = 4572\text{ N}}$$

$$\tau_c = \frac{V_c}{A_c} = \frac{0.425P}{\frac{\pi}{4}\left[20^2\right]} = 1.3528\times10^{-3}P = \tau_{allow}$$

$$\Rightarrow 1.3528\times10^{-3}P = 80$$

$$\Rightarrow \underline{P = 59136\text{ N}}$$

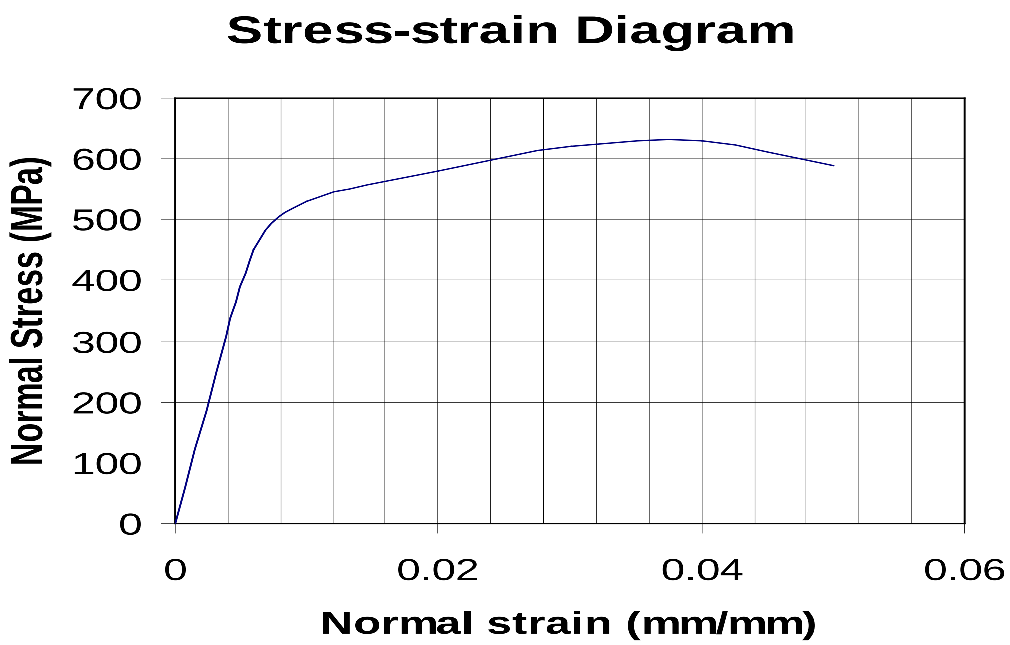
Therefore, the largest load that can be safely applied at A = 4572N

Answer

Problem # 2

The stress-strain diagram for an aluminum alloy that is used for making aircraft parts is shown below. A specimen having a gauge length of 300 mm and a diameter of 25 mm is stressed to 600 MPa. If Poisson's ratio, ***v*,** for this material is 0.35, determine the following:

1- The modulus of elasticity and the shear modulus.
2- The new length when the specimen is loaded.
3- The new diameter when the specimen is loaded.
4- The final length if the load is removed.



<u>Solution:</u>

1. Modulus of elasticity:

   $E = \sigma/\varepsilon$ from the linear portion of the stress strain diagram.
   Accordingly, $E = 300\text{x } 10^6/\ 0.004 = 75$ GPa.

   Shear modulus $G = E/\ 2(1+\upsilon)$, $G = 75\text{x}10^9/2\ (1+0.35) = 27.78$ GPa

2. If the material is loaded, the corresponding longitudinal strain is 0.024 mm/mm.
   Change in length $\delta = 0.024\text{x } 300 = 7.2$ mm.
   New length = 300 + 7.2 = 307.2 mm.

3. ε_{lat} = - (0.35x 0.024) = -8.4 x 10^{-3} mm/mm.
   Change in diameter = -8.4 x 10^{-3} x 25 = 0.21 mm.
   New diameter = 25-0.21 = 24.79 mm.

4. If the load is removed, $\varepsilon_{recovered}$ = 600x10^6/75 x 10^6= 8 x 10^{-3} mm/mm.

   $\varepsilon_{permanent}$ = 0.024 - 8 x 10^{-3} = 0.016 mm/mm.

   Change in length δ = 0.016x 300 = 4.8 mm.
   New length = 300 + 4.8 = 304.8 mm.

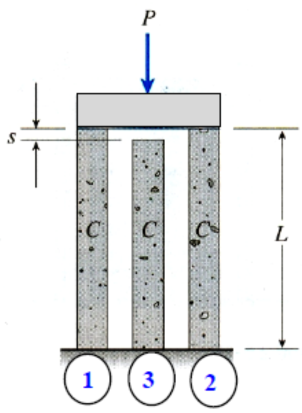
   Or from the graph by drawing a line parallel to linear portion of the stress-strain curve, this line will intersect the strain axis at approximately a strain ($\varepsilon_{permanent}$) 0.014 mm/mm. Accordingly,
   Change in length δ = 0.014x 300 = 4.2 mm.
   New length = 300 + 4.2 = 304.2 mm.

Problem # 3

A rigid plate (placed symmetrically atop three identical concrete posts) carries a load ***P*** as shown in the set-up given. If with an *initial* gap s = 1 mm, the set-up is also subjected to a temperature change $\Delta T = -40\,^{o}C$:

1. Determine the load ***P*** *just* to close the gap.
2. Determine the axial strain in middle post (for the loading from part 1 and temperature conditions specified).
3. Determine the normal stresses in *middle* and *right* posts (for the loading from part 1 and temperature conditions specified).

Assume: L = 3 m; $A_{post} = 40 \text{ x}10^{3}\ mm^{2}$; E = 30 GPa; $\alpha = 12 \text{ x } 10^{-6}\ /^{o}C$.



The three posts are designated as: **1, 3, and 2** (in the order shown on the Figure).

1) Force P (just to close gap):

$\Delta L_3 = \alpha\ \Delta T\ L_3 = 12 \text{ x } 10^{-6}\ (-40^{o}\text{ C})\ [3.0 - 1 \text{ x } 10^{-3}]$

$= -1.4395 \text{ x } 10^{-3}$ m (contraction).

Modified $s = s' = s + \Delta L_3 = 2.4395$ mm.

To *just* close gap s': the followings two conditions are to be satisfied

i) $\Delta L_1 = \Delta L_2 = s'$; and ii) $N_3 = 0$.

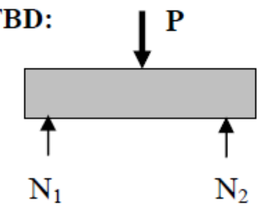
Then: $s' = \alpha\ \Delta T\ L_1 + NL_1/AE$ (all with same signs).

And $N = AE/L\ [s' - \alpha\ \Delta T\ L_1]$

$\because$ From **FBD**: $P = N_1 + N_2 = 2\ N$ (due to symmetry).

P = 2 N

FBD:



$$= \frac{2\times 30\times 10^{9}\times 40\times 10^{-3}[2.4395\times 10^{-3} - 1.44\times 10^{-3}]}{3.0} = 799.6\times 10^{3}\ N$$

$\cong 0.800\ \boldsymbol{MN}$

$\therefore$ **P** = 800. kN

2) Strain in middle post 3: ε_3 = only $\varepsilon_{thermal} = \alpha\ \Delta T$

$\therefore \varepsilon_3 = 12 \text{ x } 10^{-6}\ (-40\ ^{o}C = -4.8 \text{ x } 10^{-4}$ m/m.

3) Stresses in the three posts:

Since only posts 1 and 2 carry the load P (for the *specified* conditions with $N_3 = 0$), ***then*** the stresses induced are:

$$\boldsymbol{\sigma_1 = \sigma_2 = 0.5\ N / A_{post}} = \frac{0.5\times 800\text{ kN}}{(40\times 10^{-3})\text{ m}^{2}} = 10\text{ MPa (comp.)},\ \textbf{and } \boldsymbol{\sigma_3 = 0.}$$

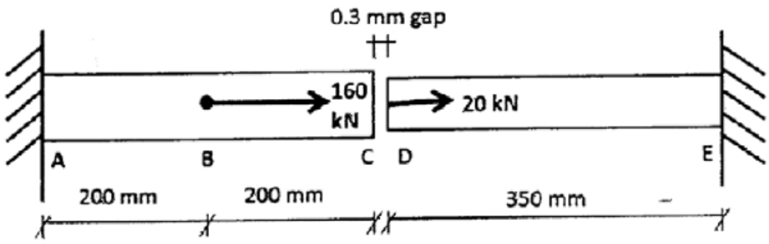
Problem # 4

The two rods have an initial gap of 0.3 mm before the application of the given loads.

a) Show that the given problem is statically indeterminate.
b) Determine the support reaction at point E.
c) Determine the final length of rod DE.

For rod AC : *E* = 20 GPa , and *A* = 800 mm^2
For rod DE : *E* = 40 GPa , and *A* = 400 mm^2

a) Check if the gap will close or not



$$(\Delta l)_{AC} = \frac{(160{,}000)(200)}{(20{,}000)(800)} = +2 \text{ mm}$$

$$(\Delta l)_{DE} = \frac{(-20{,}000)(350)}{(40{,}000)(400)} = -0.438 \text{ mm}$$

$\therefore (\Delta l)_{tot} = +1.56 > \text{gap}$

– S.I. problem

b) 160,000 N

R_E

R_A

20,000

(AB) R_E N $\quad N = 180{,}000 - R_E$

(BC) R_E N $\quad N = 20{,}000 - R_E$

(DE) R_E N $\quad N = -R_E$

$R_A + R_E = 180{,}000$ (1)

we need another eqn.

$(\Delta l)_{tot} = 0.3$, or $(\Delta l)_{AB} + (\Delta l)_{BC} + (\Delta l)_{DE} = 0.3$ mm

$$\left[\frac{(180{,}000 - R_E)(200)}{(20{,}000)(800)}\right] + \left[\frac{(20{,}000 - R_E)(200)}{(20{,}000)(800)}\right] + \left[\frac{-R_E\,(350)}{(40{,}000)(400)}\right] = 0.3 \text{ mm}$$

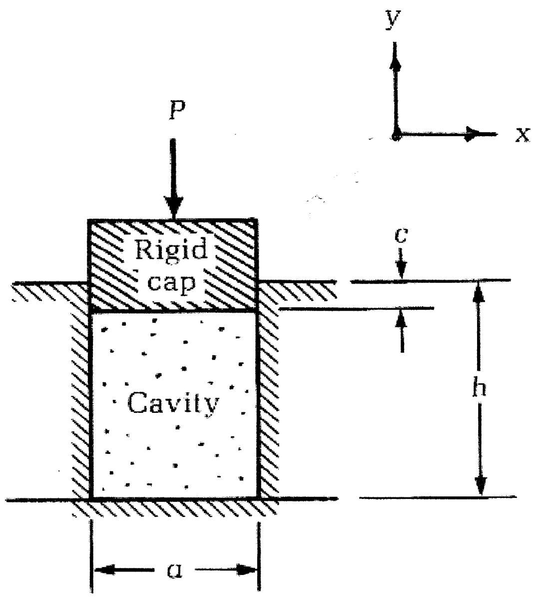
Solve for R_E , $\boxed{R_E = +46933 \text{ N}}$

c) $(\Delta l)_{DE} = \dfrac{NL}{EA} = \dfrac{(-46933)(350)}{(40{,}000)(400)} = -1.0267$ mm

Final $L_{DE} = 350 + \Delta l = \boxed{348.97 \text{ mm}}$

Problem # 5

A rigid material has a smooth rectangular cavity of dimensions (***a*** x ***b*** x ***h***), *25* mm x *30* mm x *90* mm engraved in it as shown below. The cavity is filled with a linearly elastic, isotropic material with modulus of elasticity, E = *2.5* GPa, and Poisson's ratio, ν = *0.40*, and compressed as shown in the figure by a rigid cap with a force ***P*** acting on it. If ***P*** = *70.8* kN, determine the decrease ***c*** in the height ***h***, and the change in volume ΔV of the material.



Solution:

This is a 3-D problem; however, no shearing strains occur in the material. Moreover, the only nonzero normal strain component is ϵ_y. The other strain components, ϵ_x and ϵ_z are zero because the rigid medium surrounding the cavity does not allow the expansion of the material in these directions. Thus

$$\epsilon_y = \frac{c}{h}, \text{ and } \epsilon_x = \epsilon_z = \gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0$$

Continue Solution:

Consider the normal strain equations of the generalized Hook's Law;

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$
$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$
$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

where, $\sigma_y = P/A_{cap} = -70.8\,kN/(25\,mm \times 30\,mm)(10^{-6}\,m^2/mm^2)$

$= \underline{-94.4\,MPa}$

and $\epsilon_x = \epsilon_z = 0$

Substituting;

$$0 = \frac{1}{2.5\,GPa}\left[\sigma_x - 0.4(-94.4 + \sigma_z)\right] \quad (1)$$
$$-\frac{c}{0.09\,m} = \frac{1}{2.5\,GPa}\left[-94.4 - 0.4(\sigma_x + \sigma_z)\right] \quad (2)$$
$$0 = \frac{1}{2.5\,GPa}\left[\sigma_z - 0.4(\sigma_x - 94.4)\right] \quad (3)$$

Subtracting Eq (3) from Eq (1);

We get; $\sigma_x = \sigma_z$

and solving for $\sigma_x = \sigma_z = -62.933\,MPa$

Substituting in Eq (2) and solving for $\underline{c}$;

$$-c = \frac{0.09\,m}{2.5\,GPa}\left[-94.4 + 0.4(62.933 + 62.933)\right]MPa$$

$$c = 1.586 \times 10^{-3}\,m$$

$$c = \underline{1.586\,mm}$$

The change in Volume of the material;

$$\Delta V = -(25 \times 30 \times 90) + (25 \times 30)(90 - 1.586)$$

$$= \underline{-1{,}189.5\,mm^3}$$