CE 203 (112)

KEY SOLUTION

EXAM 1

PROBLEM	FACULTY
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Problem # 1

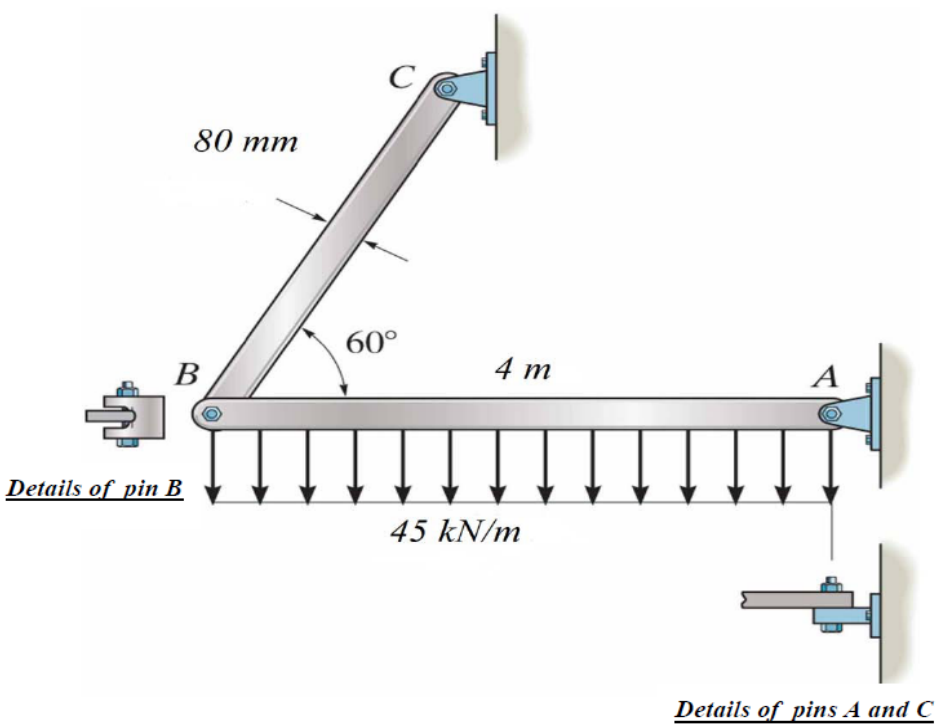
The beam is supported by a pin at A and link BC. Determine:

a) the average shear stresses in the pins at A and B,

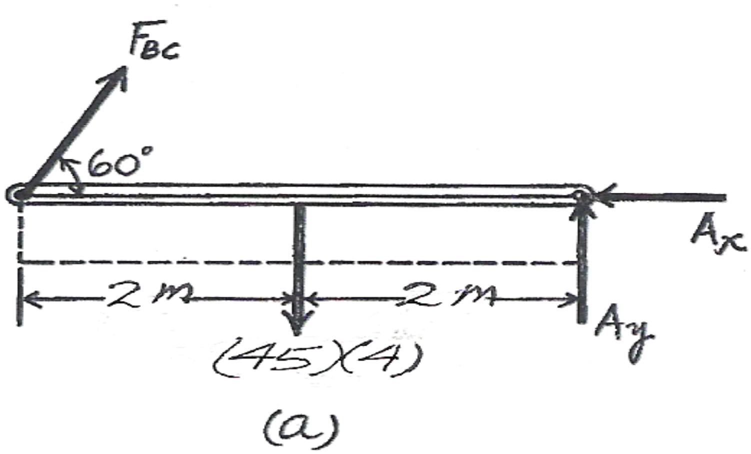
b) the average normal stress in link BC,

c) the bearing stress between pin C and the link.

All pins have a diameter of 20 mm
Thickness of link BC = 30 mm



Solution:



Referring to the FBD of member AB in Figure (*a*),

$$\circlearrowleft + \sum M_A = 0; \quad (45\,\text{kN/m} \times 4\,\text{m})(4\,\text{m}/2) - (F_{BC}\sin 60^\circ)(4\,\text{m}) = 0,$$

$$F_{BC} = 90/\sin 60^\circ = 103.923\ \text{kN}$$

$$\rightarrow + \sum F_x = 0; \quad -A_x + F_{BC}\cos 60^\circ = 0,$$

$$A_x = 51.962\ \text{kN}$$

$$\uparrow + \sum F_y = 0; \quad A_y + F_{BC}\cos 60^\circ - (45\,\text{kN/m})(4\,\text{m}) = 0,$$

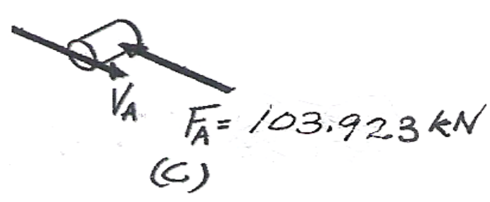
$$A_y = 90.000\ \text{kN}$$

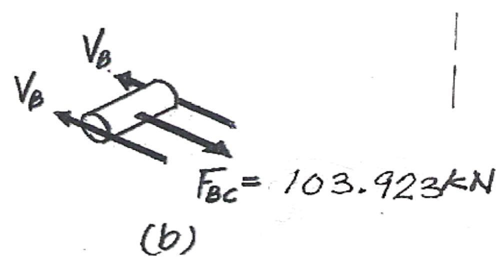
Thus the force acting on pin A is

$$F_A = \sqrt{(A_x)^2 + (A_y)^2} = \sqrt{(51.962\ \text{kN})^2 + (90.000\ \text{kN})^2}$$

$$F_A = 103.923\ \text{kN}$$

Pin *A* is subjected to *single shear*, Figure (*b*), while pin *B* is subjected to double shear, Figure (*c*),





$$V_A = F_A = 103.923\ \text{kN}, \quad \text{while} \quad V_B = F_{BC}/2 = 103.923/2 = 51.962\ \text{kN}$$

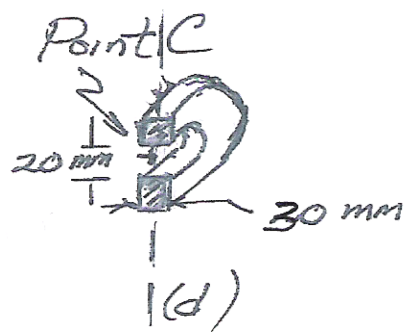
$$(\tau_A)_{average} = V_A/A = 103.923\ \text{kN} / (\pi x 0.01^2)\,\text{m}^2 = 330.797\ \text{MPa}$$

$(\tau_B)_{average} = V_B / A = 51.962 \text{ kN} / (\pi x 0.01^2) \text{m}^2 = 165.400 \text{ MPa}$

Cross sectional area of the link BC is 30mm x 80mm = 2400mm^2 = 2.4×10^{-3}m^2

$(\sigma_{BC})_{average} = F_{BC}/A = 103.923 \text{ kN} / 2.4 \times 10^{-3} \text{m}^2 = 43.301 \text{ MPa}$

Referring to the FBD of a section through point C in link BC, Figure (d),



$F_C = F_{BC} = 103.923 \text{ kN}$

$A_{bearing} = 30\text{mm} \times 20\text{mm} = 600\text{mm}^2 = 0.6 \times 10^{-3}\text{m}^2$

$(\sigma_{bearing})_{BC} = F_C / A_{bearing} = 103.923 \text{ kN} / 0.6 \times 10^{-3} \text{m}^2 = 173.205 \text{ MPa}$

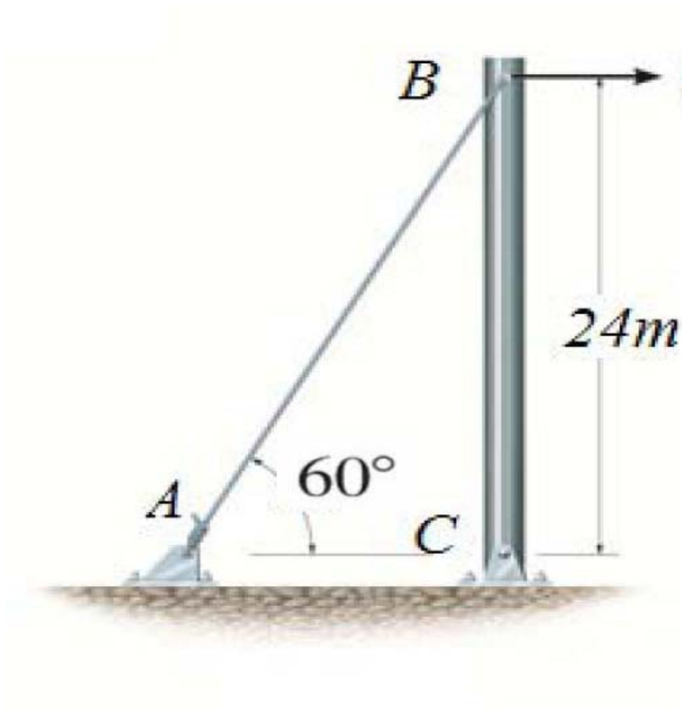
Problem # 2

The rigid pipe is supported by a pin at C and wire AB. The pin has a diameter of 20 mm while the wire has a diameter of 10 mm. If the allowable normal stress for the wire is $\sigma_{allow} = 255$ MPa and the allowable shear stress for the pin is $\tau_{allow} = 131$ MPa, determine:

a) the maximum P that can be applied to the assembly,

b) the increase in length and reduction in diameter of wire AB.

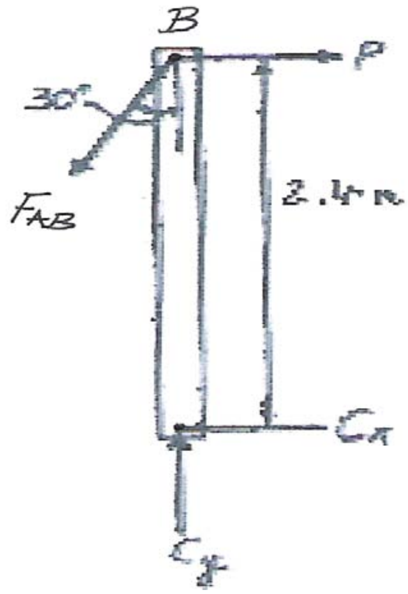
Use E = 70 GPa and ν = 0.35.



Detail of connection at C

Solution:

Referring to the FBD of the rigid pipe BC, Figure below;



$\curvearrowleft + \sum M_A = 0;$

$(F_{AB} \sin 30^o)(2.4\text{m}) - P(2.4\text{m}) = 0,$

$F_{AB} = P / \sin 30^o = \underline{2P}$

$\rightarrow + \sum F_x = 0;$

$- F_{AB} \sin 30^o + P - C_x = 0,$

$C_x = P - F_{AB} \sin 30^o$

$C_x = P - (P / \sin 30^o) \sin 30^o = \underline{0}$

$+ \sum F_y = 0;$

$Cy - F_{AB} \cos 30^o = 0,$

$Cy - (P / \sin 30^o) \cos 30^o = 0$

$Cy = \underline{1.732\ P}$

The stress in the guy wire $\boldsymbol{\sigma}_{AB} = \boldsymbol{\sigma}_{allow} = 255$ MPa $= F_{AB} / A_{\text{guy wire}} = 2P / \pi(0.005m)^2$

$$P = (255 \times 10^6\ \text{N/m}^2)[\pi(0.005m)^2] / 2$$

$= \underline{10.014}$ kN

Pin C is subjected to a *double- shear* action by the reaction at C;

$$F_C = C_y/2 = 1.732P/2$$

$$\tau_{pin\ C} = \tau_{allow} = 131 \text{ MPa} = F_C / A_{pin\ C} = 1.732P/2[\pi(0.01m)^2]$$

$$P = (131 \times 10^6 \text{ N/m}^2)(2)[\pi(0.01m)^2]/1.732$$

$$= \underline{47.523 \text{ kN}}$$

Therefore, $\boldsymbol{P}_{max}$ should not exceed 10.014 kN

$$\boldsymbol{P}_{max} = \underline{10.014 \text{ kN}}$$

The axial force in the guy wire AB is $F_{AB} = 2P = 20.028$ kN, but $\boldsymbol{\sigma}_{AB} = E\,\varepsilon_{AB}$, where $\varepsilon_{AB} = \Delta_{AB}/L_{AB}$, and $\boldsymbol{\sigma}_{AB} = F_{AB}/A_{guy\ wire}$ or

$$F_{AB}/A_{guy\ wire} = E(\Delta_{AB}/L_{AB})$$

The stretch in the guy wire AB is

$$\Delta_{AB} = (F_{AB} L_{AB}/A_{guy\ wire} E) = [2P/(A_{guy\ wire} \times E)] L_{AB}$$

$$\Delta_{AB} = \{20.028\text{kN}/[\pi(0.005m)^2 \times 70\ x\ (10)^6\text{kN}/m^2\}(2.4\text{m}/\cos 30^o)$$

$$\underline{\Delta_{AB} = 10.42 \times 10^{-3} \text{ m}}$$

Poissonâ€™s Ratio; $\boldsymbol{\nu} = \varepsilon_{Radial} / \boldsymbol{\varepsilon}_{AB} = 0.35$

$$\boldsymbol{\varepsilon}_{Radial} = -0.35 \times \boldsymbol{\varepsilon}_{AB} = 0.35 \times 3.643x10^{-3} = -1.2754x10^{-3}$$

The reduction in the diameter is therefore.

$$\Delta_{Radia} = (-1.275x10^{-3})(10\text{mm}) = -12.75x10^{-3}\ mm$$

Problem # 3 Key Solution:

Rod ABC has a negligible mass and only supports two axial loads P and 21 P as shown. If *only* part AB is subjected to a temperature change $\Delta T = 40\ ^{o}C$, determine:

a) the required value of **P** if the total length ABC remains constant,
b) the displacement δ_B of point B,
c) the relative displacement $\delta_{B/C}$.
d) the final length of rod AB.

Given E = 70 GPa and α = 24 x 10^{-6} / °C.

$N_1 = +P$ (T) ; $N_2 = A_y$; $P - 21P - A_y = 0 \Rightarrow N_2 = 20P$ (C)

(8) a) $(\Delta \ell)^{tot}_{AB} + (\Delta \ell)^{tot}_{BC} = 0$

$N_1 \ell_1 / EA_1 + N_2 \ell_2 / EA_2 + \alpha \ell_2 \Delta T = 0$

$\frac{P(0.40)}{A_1} - \frac{20P(0.60)}{A_2} + \alpha E \ell_2 \Delta T = 0$

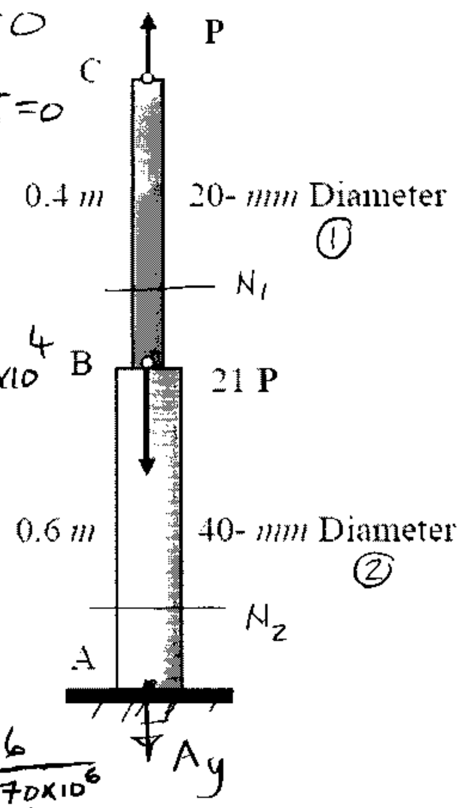
$A_1 = 3.1416 \times 10^{-4}\ m^2$

$A_2 = 1.2566 \times 10^{-3}\ m^2$

$\alpha E \ell_2 \Delta T = 4.032 \times 10^{7}\ N/m$

$1.2732 \times 10^{3} P - 9.5496 \times 10^{3} P = -4.032 \times 10^{4}$

$\therefore P = 4.872$ kN



(5) b) $\delta_B = \delta_A + (\delta_{B/A})^{tot}$

$= 0 + \alpha \ell_2 \Delta T + \frac{N_2 \ell_2}{A_2 E}$

$= 5.76 \times 10^{-4} - \frac{20 \times 4.872 \times 0.6}{1.2566 \times 10^{-3} \times 70 \times 10^{6}}$

$= 5.76 \times 10^{-4} - 6.6465 \times 10^{-4}$

$= -8.8651 \times 10^{-5}$ m $\Rightarrow \therefore \delta_B \cong 0.0887$ mm (↓)

(4) c) $\delta_{B/C} = (\Delta \ell)^{tot}_{BC} = N_1 \ell_1 / A_1 E = \frac{1.2732 \times 10^{3} P}{E}$

$= 8.862 \times 10^{-5}$ m

(3) d) $(L_{AB})_f = (L_{AB})_o + (\Delta \ell)^{tot}_{AB} = 0.6 + \delta_{B/A}$

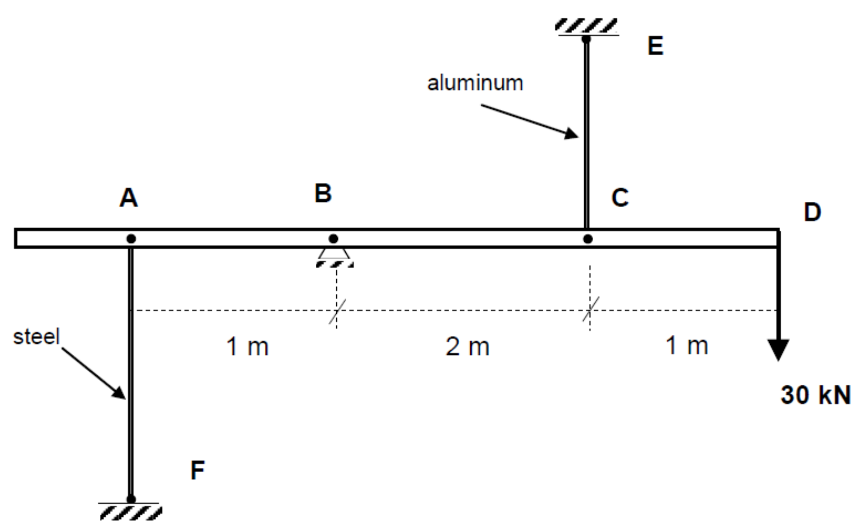
$= 0.59991$ m

Problem # 4

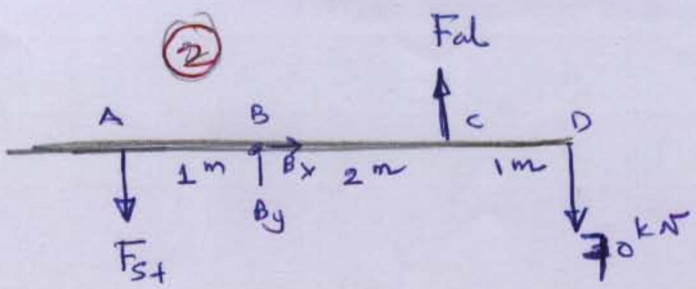
The rigid member *ABCD* is supported by a pin at B and two cables.

a) Calculate the stresses in the cables due to the application of the shown force.
b) Calculate the vertical displacement of point D.
c) In one **sentence**, explain what will happen to the stresses in the cables if the temperature of the steel cable *only* is increased.

Cable	Length	Area (mm^2)	Material	E (GPa)
AF	1.5 m	315	Steel	200
CE	1 m	600	Aluminum	70

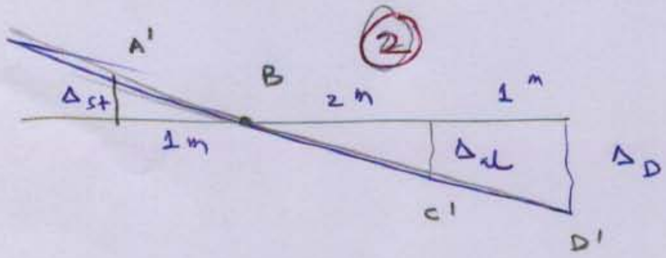


(2)



FBD ABCD

(2)



Geometry after deformation

Displacement of point D

using figure

$$\frac{\Delta_D}{3} = \frac{\Delta_{al}}{2} \quad (1)$$

$$\because \Delta_D = 1.5\, \Delta_{al} \quad (2)$$

$$= 1.5 \frac{F_{al} L_{al}}{A_{al} E_{al}}$$

$$= 1.5 \frac{(84)(1000)}{(600)(70)}$$

$$\Delta_D = 3 \text{ mm}$$

If steel temp is increased

σ_{steel} will decrease (2)

σ_{Alum} will increase

	L	A
steel	1.5	315
al	1	600

$$\circlearrowleft \Sigma M_B = 0$$

$$-F_{st}(1) - F_{al}(2) + 70(3) = 0$$

(2)

$$\boxed{F_{st} + 2F_{al} = 210} \quad -(1)$$

(statically indeterminate)

compatibility (2)

$$\frac{\Delta_{st}}{1} = \frac{\Delta_{al}}{2}$$

(2)

$$\boxed{\Delta_{al} = 2\,\Delta_{st}}$$

put interms of forces

$$\frac{F_{al} L_{al}}{A_{al} E_{al}} = 2 \frac{F_{st} L_{st}}{A_{st} E_{st}}$$

(2)

$$\boxed{F_{al} = 2F_{st}} \quad -(2)$$

Solving (1) & (2) we get

$F_{st} = 42^{kN}$; $F_{al} = 84^{kN}$

(2) (1)

$$\therefore \sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{42 \times 10^3}{315} = 133 \text{ MPa}$$

(1)

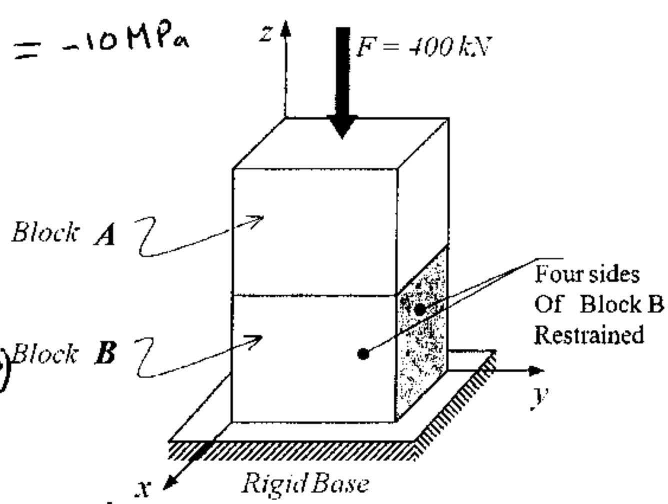
$$\sigma_{al} = \frac{84 \times 10^3}{600} = 140 \text{ MPa}$$

Block *A* rests on block *B* as shown. Each block is a cube with initial dimensions 200x200x200 *mm*. The 4 side-faces of block *A* are free to displace, while the 4 side-faces of block *B* are prevented from expanding (i.e. restrained in the *x* and *y* directions). Determine:

a) the vertical displacement of the force ***F***,
b) the stress σ_x for block ***A*** and for block ***B***,
c) the value of the Shear Modulus (***G***) for block ***A***.

Ignore self-weight and any friction.

E **= 10 *GPa*, and *ν* = 0.2**



a)

For block A $\sigma_z = \frac{-400,000}{(200)(200)} = -10\text{ MPa}$

sides are free, so can be considered an axial rod

$(\Delta h)_A = \frac{Nl}{EA} = \frac{(-400,000)(200)}{(10,000)(200)(200)}$

$(\Delta h)_A = -0.2\text{ mm}$

For block B, 4 sides are restrained,

$\sigma_z = -10\text{ MPa}$, $\sigma_x = ?$, $\sigma_y = ?$, $\epsilon_x = \epsilon_y = 0$, $\epsilon_z = ?$

using $\epsilon_x = 0 \rightarrow \sigma_x - .2\sigma_y - (.2)(-10) = 0$
using $\epsilon_y = 0 \rightarrow \sigma_y - .2\sigma_x - (.2)(-10) = 0$ } solve together

For block B $\sigma_x = \sigma_y = -2.5\text{ MPa}$, $\epsilon_z = \frac{1}{10,000}\left[-10 + (.2)(-5)\right]$

$\epsilon_z = -9\times10^{-4}$, → so $(\Delta h)_B = (\epsilon_z)(200) = -0.18\text{ mm}$

Displacement of Force $= (\Delta h)_A + (\Delta h)_B = -.2 - .18 = -0.38 =$ 0.38 mm ↓

b) For Block A: $\sigma_x = 0$ (free sides), Block B: $\sigma_x = 2.5\text{ MPa (c)}$ (from above)

c) $G = \frac{E}{2(1+\nu)} = \frac{10\text{ GPa}}{2(1+.2)} = 4.167\text{ GPa}$ Ans