بسم الله الرحمن الرحيم

King Fahd University of Petroleum & Minerals

DEPARTMENT OF CIVIL & ENVIRONMENTAL ENGINEERING

CE 203 STRUCTURAL MECHANICS I

Semester (132)

Major Exam II

Tuesday, April 22, 2014 8:00-10:30 P.M.

KEY SOLUTION

Note to Students

Even though the course is not "standard grading", *being around the average does not indicate C performance, since there is a minimum amount of course comprehension needed to pass the course satisfactorily*, irrespective of the exam average and the performance of other students.
Therefore, students who did poorly in this exam should do double effort in the remaining of the semester to avoid disappointing grade.

After reviewing the key solution and still having a concern about your mark, you may consult with the faculty members who prepared, solved, and graded each problem.

The deadline for review is Thursday May 8, 2014.

Problem	Solved & Graded by
1	Dr. Mohammad Al-Osta
2	Dr. Abdulrahman Khathlan
3	Dr. Hamdan Al-Ghamedy
4	Dr. Alfarabi Sharif
5	Dr. Mohammad Al-Suwaiyan

Notes:

1. **A sheet that includes selected Basic Formulae and definitions is provided with this examination.**
2. **Write clearly and show all calculations, FBDs, and units.**

Problem 1: (20 points)

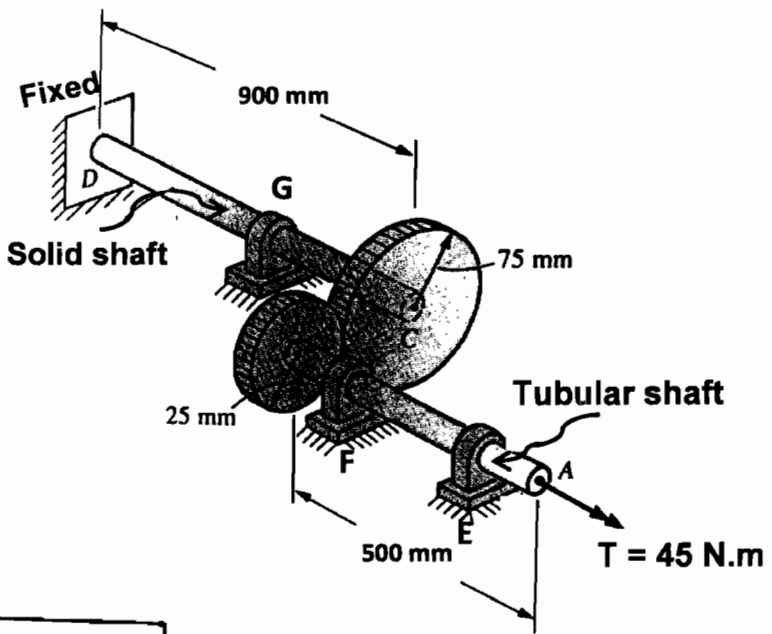
Two shafts are connected by the gears at B and C as shown in the figure. The bearings E, F, and G allow free rotation of the shafts and shaft DC is fixed at D.

Determine the minimum required diameter of the solid shaft DC to the nearest mm if:
The allowable shear stress is τ_{allow} = 50 MPa, and
The allowable angle of twist of A is 0.2 (0.2 rad) (Should be in radian)

Take:

G = 80 GPa.

Tubular shaft AB has an outer diameter of 15 mm and an inner diameter of 10 mm.



$J_{DC} = \frac{\pi}{2} c^4$

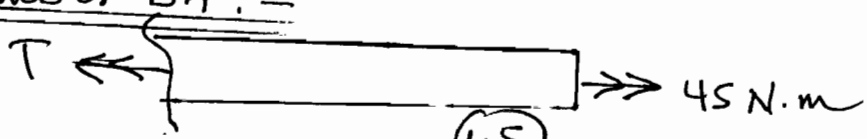
$c = \frac{d}{2}$

$J_{BA} = \frac{\pi}{2}(c_o^4 - c_i^4)$

(0.5) $= \frac{\pi}{2}(7.5^4 - 5^4)$

$= 3988.4\ mm^4$

Member BA :-



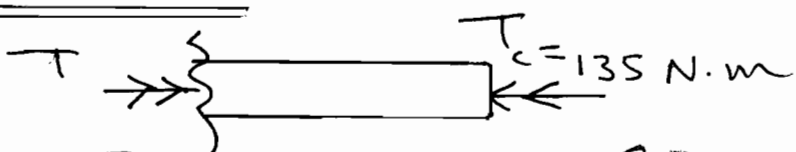
(1.5)

$T_{BA} = 45\ N.m$

∴ Torque at B = 45 N.m

Torque at $C = -45 * \frac{75}{25} = -135\ N.m$ (2)

Member CD :



$T_{DC} = -135\ N.m$ (1.5)

Find d from τ_{all}.

$\therefore \tau_{DC} = \frac{T_{DC} \rho_{DC}}{J_{DC}} = \frac{T_C}{J_{DC}} = \frac{135 * 10^3 (d/2)}{\frac{\pi}{2}(d/2)^4}$

$$= \frac{687549.4\,\cancel{d}}{\cancel{d^4}\,d^3}$$

$$\text{So} = \frac{687549.4}{d^3} \Rightarrow d = \sqrt[3]{13750.9} = 23.96\text{mm} \quad (5)$$

$$\simeq 24\text{mm}$$

Find d from $(\phi)_{all} = 0.2$ rad

(2) $\phi_{C/D} = \dfrac{(-135\times10^3)\times900}{80000\,J_{DC}} = \dfrac{-135\times10^3\times900}{80000\times\frac{\pi}{2}(d/2)^4}$

(2.5) $\therefore \phi_B = -\frac{75}{25}\phi_{CD} = \frac{75}{25}\left(\dfrac{135\times10^3\times900}{80000\times\frac{\pi}{2}(d/2)^4}\right) = \dfrac{46409.6}{d^4}$

(2) $\Rightarrow \phi_{A/B} = \dfrac{45\times1000\times500}{80000\left(\frac{\pi}{2}(7.5^4-5^4)\right)} = 0.0705$ rad

(3) $\phi_{A/D} = \phi_{A/B} + \phi_B$

$$= 0.0705 + \frac{46409.58}{d^4}$$

$\because (\phi_A)_{all} = 0.2$ rad

$$\Rightarrow 0.2 = 0.0705 + \frac{46409.58}{d^4}$$

$$\Rightarrow d = 24.46\text{ mm} \simeq 25\text{ mm}$$

The minimum req. diameter of the shaft is the largest.

$\Rightarrow$ Use $d = 25$ mm

Any student used 0.2° instead of 0.2 radian did not lose any point.

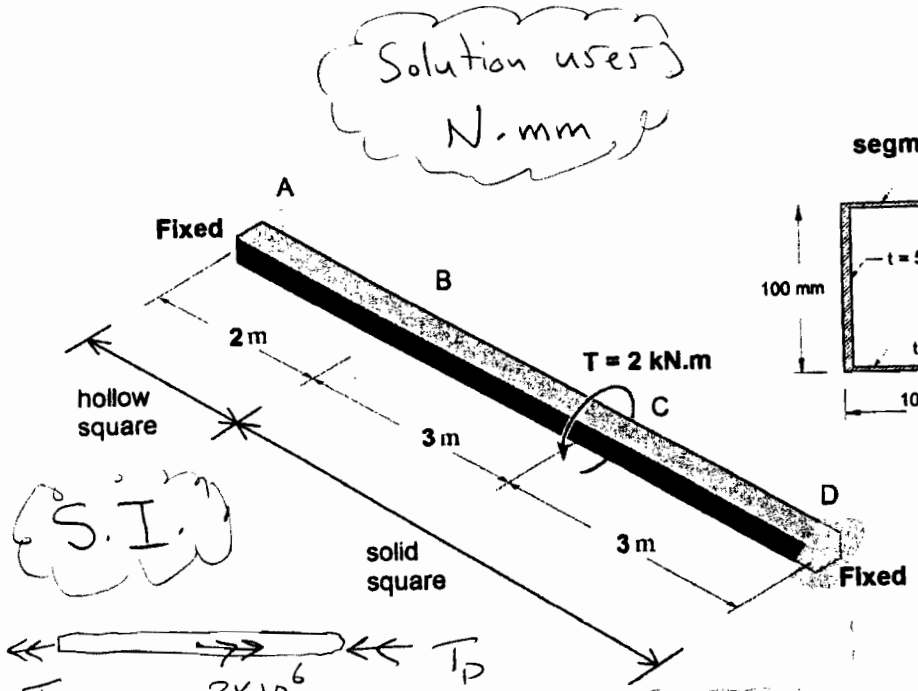
Problem 2: (20 points)

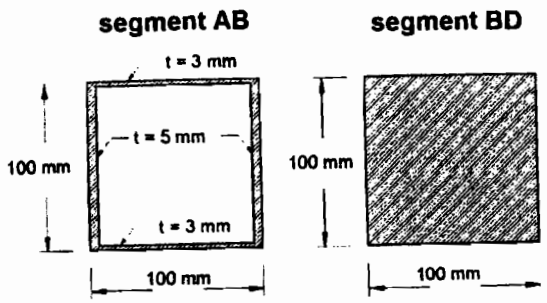
For the given shaft, segment AB has a hollow square cross section, while segment BD has a solid square cross section as shown. **Use *G = 100 GPa.***

a) Determine the magnitude and **exact location** of the maximum shear stress in the whole shaft.

b) Determine the magnitude and direction of the angle of twist of point ***C*** with respect to ***B***.

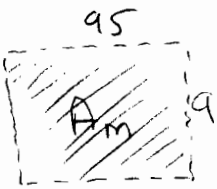
Solution uses N.mm





S.I.

$T_A + T_D = 2\times10^6$ (1)



$A_m = (95)(97) = 9215\ mm^2$

$\sum \frac{S}{t} = 2\left[\frac{95}{3} + \frac{97}{5}\right] = 102.13$

S.I., Comp. Eqn $\quad \phi_{A/D} = 0$

$\phi_{A/B} + \phi_{B/C} + \phi_{C/D} = 0$ (2)

Hollow Square

$\phi_{A/B} = \frac{T_A(2000)}{4(A_m^2)(100{,}000)} \sum \frac{S}{t}$

$\phi_{A/B} = 6.0138\times10^{-9}\, T_A$

$T_{AB} = +T_A$, $T_{BC} = +T_A$, $T_{CD} = T_A - 2\times10^6$

solid Sq. $\quad \phi_{BC} = \frac{7.1\, T_A(3000)}{(100)^4(100{,}000)} = 2.13\times10^{-9}\, T_A$

Solid Sq $\quad \phi_{CD} = \frac{7.1(T_A - 2\times10^6)(3000)}{(100)^4(100{,}000)} = 2.13\times10^{-9}(T_A - 2\times10^6)$

Sub in (2) above $\quad 6.0138\times10^{-9} T_A + 2.13\times10^{-9} T_A + 2.13\times10^{-9}(T_A - 2\times10^6) = 0$

$10.2738\, T_A = 4.26\times10^6$

$T_A = +.414\times10^6$ N.mm, $\quad T_D = +1.585\times10^6$ N.mm

$(\tau_{max})_{CD} = \frac{4.81\, T_D}{(100)^3} = 7.626$ MPa

$(\tau_{max})_{BC} = \frac{4.81\, T_A}{(100)^3} = 1.99$ MPa

$(\tau_{max})_{AB} = \frac{T_A}{(2)(3)\,A_m} = 7.499$ MPa

τ_{max} for whole shaft is 7.626 MPa in CD

$\phi_{C/B} = \phi_{B/C}$ (from above)

$\phi_{C/B} = 2.13\times10^{-9}\, T_A$

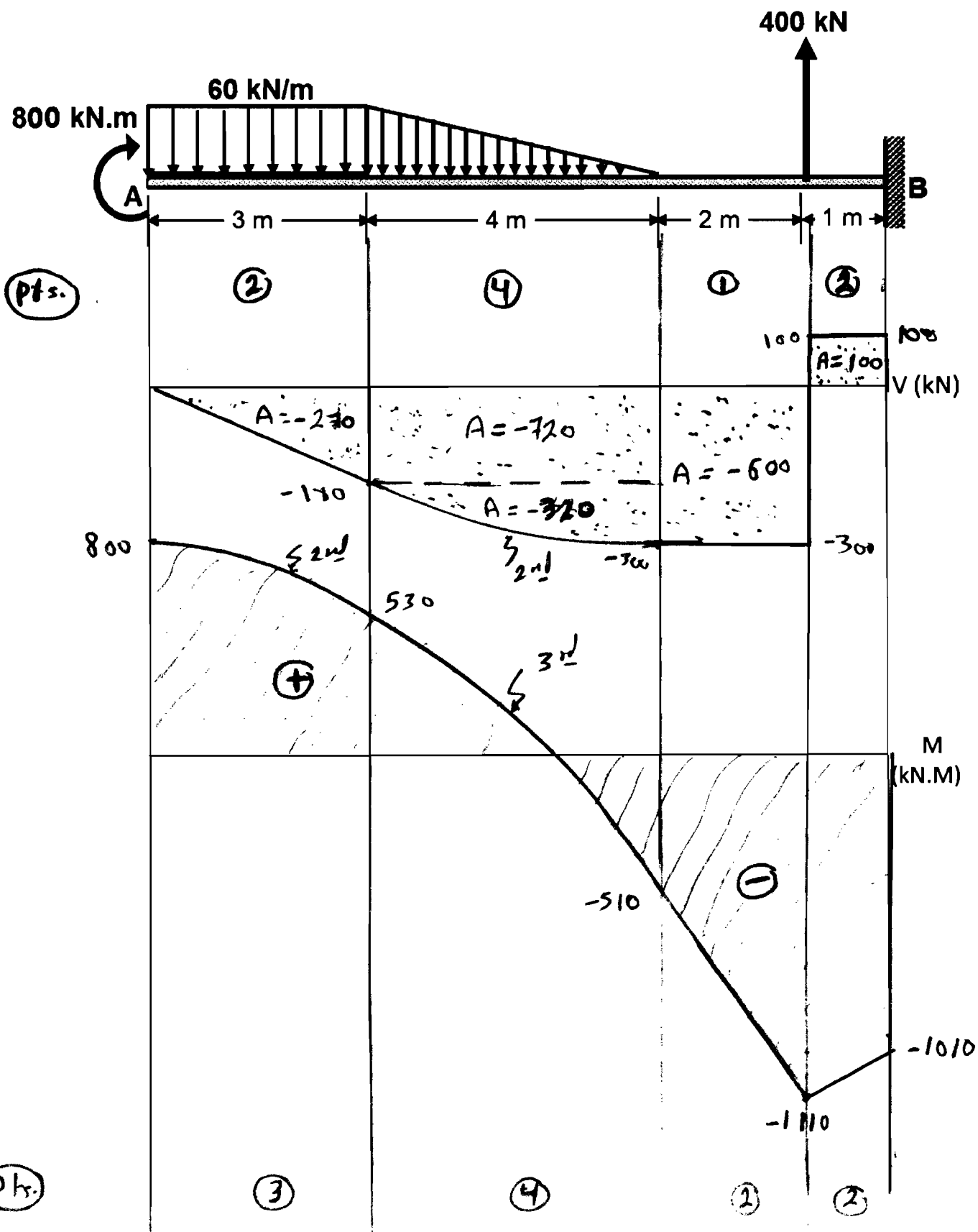
$= +.883\times10^{-3}$ rad

+ means ↺

Problem 3 (20 pts.)

Draw the **shear force and bending moment diagrams** for the beam shown below using the ***summation (graphical) method***. *Write the degree (2, 3, etc.) of the curve on each one.*
Put all values on the diagrams, but you do NOT need to show the calculations.
Use appropriate scale. ***No credit will be given if another method is used.***

The reactions are: $B_y = 100$ kN↓ ; $M_B = 1010$ kN.m ↻

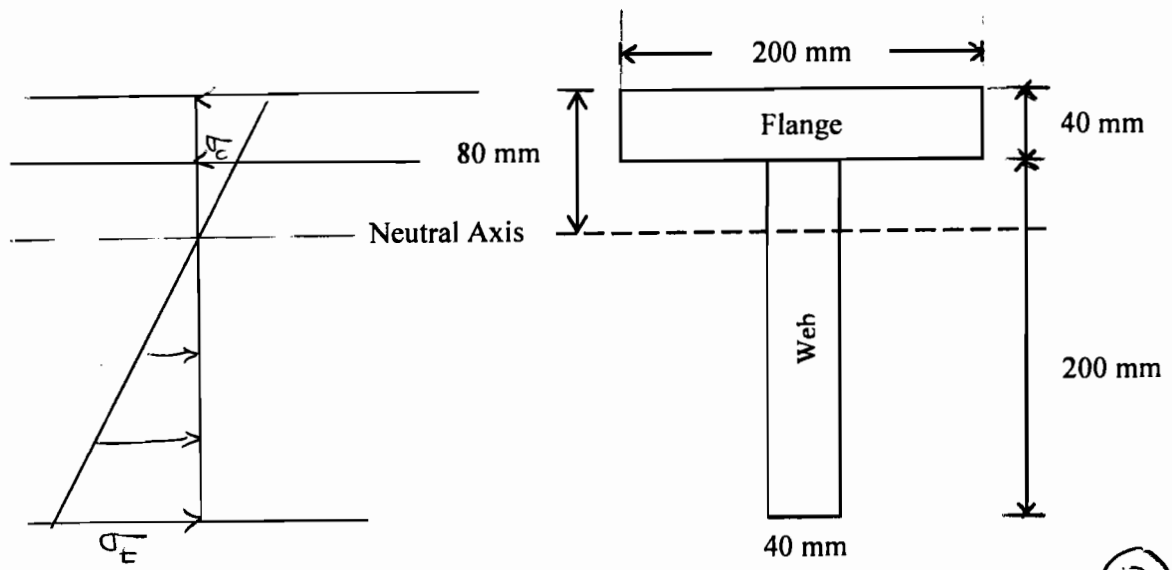


Problem 4: (20 points)

A beam has a cross section consisting of a top flange and a web as shown. The moment of intertia (I) about the neutral axis is $6.93(10)^{-4}$ m^4.

a) If the beam is subjected to a positive moment M = 40 kN-m, determine the resultant force the bending stress produces on the web.

b) If σ_{all} for compression = 60 MPa, and σ_{all} for tension = 100 MPa, determine the maximum positive moment that can be applied.



a) $\sigma_c = \dfrac{M(\bar{Y} - .04)}{I} = \dfrac{40(.08 - .04)}{6.93(10)^{-4}} = 2.31$ MPa (3)

$\sigma_t = \dfrac{40(.24 - .08)}{6.93(10)^{-4}} = 9.24$ MPa (3)

$R = R_t - R_c = \frac{1}{2}(9.24)(.04)(.24 - .08) - \frac{1}{2}(2.31)(.04)(.08 - .04) = 27.7$ kN (2) (2) (2)

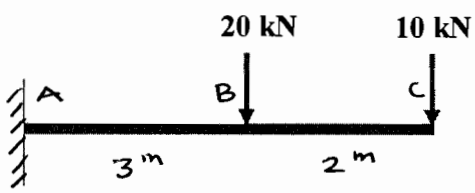
b) $\sigma_{all\,t} = 100 = \dfrac{M(.24 - .08)}{6.93(10)^{-4}} \Rightarrow M = 433.1$ kN-m (3)

$\sigma_{all\,c} = 60 = \dfrac{M(.08)}{6.93(10)^{-4}} \Rightarrow M = 519.8$ kN-m (3)

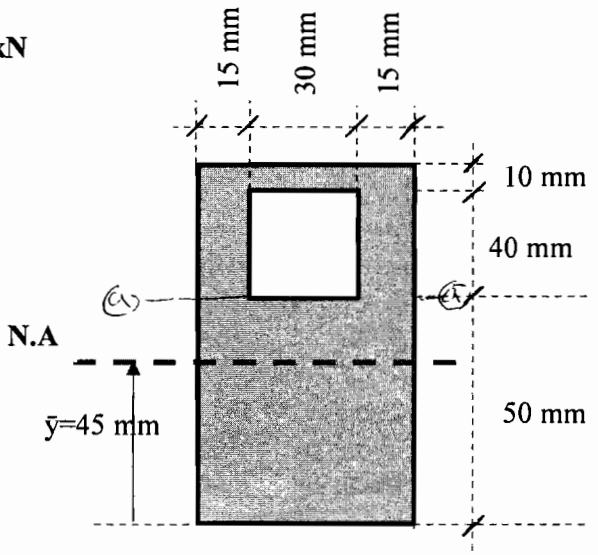
$\Rightarrow M_{max} = 433.1$ kN-m (2)

Problem 5: (20 points)

Beam ABC, with the shown cross section is loaded as shown below.
Determine the maximum shear stress in the beam and indicate where it acts,

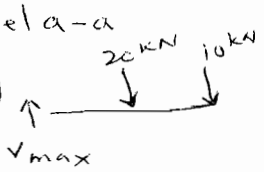


Fixed



$\tau = \frac{VQ}{It}$ (2)

the max τ will be at max V between A & B and at N/A or level a-a



$V_{max} = 30\ kN$ (2)

Find $I_{N/A} = I_{big\text{-}rect} - I_{small\ rec}$ (2)

$= \frac{1}{12}(60)(100)^3 + (60 \times 100 \times 5^2) - \left[\frac{1}{12}(30)(40)^3 + 30 \times 40 \times 25^2\right]$ (2)

$= 5\times10^6 + 0.15\times10^6 - (0.16\times10^6 + 0.75\times10^6)$

$I_{N/A} = 4.24\times10^6\ mm^4$ (2)

$Q_{N/A} = 60 \times 45 \times 45/2 = 60750\ mm^3 \Rightarrow t_{N/A} = 60\ mm$

$Q_{a-a} = 60 \times 50 \times 20 = 60{,}000\ mm^3 \Rightarrow t_{a-a} = 30\ mm$

$\tau_{N/A} = \frac{30\times10^3 \times 60750}{4.24\times10^6 \times 60} = 7.2\ MPa$ (2) (1) (1)

$\tau_{a-a} = \frac{30\times10^3 \times 60000}{4.24\times10^6 \times 30} = 14.2\ MPa$ (2) (2)

τ_{max} <= Ans

at level a-a between A & B (1)