بسم الله الرحمن الرحيم

King Fahd University of Petroleum & Minerals

DEPARTMENT OF CIVIL & ENVIRONMENTAL ENGINEERING

First Semester 1434-35 / 2013-14 (131)

CE 203 STRUCTURAL MECHANICS I

Major Exam II

Tuesday, November 26, 2013 7:00-9:30 P.M.

KEY SOLUTION

Note to Students

Even though the course is not "standard grading", ***being around the average does not indicate C performance, since there is a minimum amount of course comprehension needed to pass the course satisfactorily*****, irrespective of the exam average and the performance of other students.**
Therefore, students who did poorly in this exam should do double effort in the remaining of the semester to avoid disappointing grade.

After reviewing the key solution and still having a concern about your mark, you may consult with the faculty members who prepared, solved, and graded each problem.

The deadline for review is Tuesday December 10, 2013.

Problem	Solved & Graded by
1	Dr. Mesfer Al-Zahrani
2	Dr. Shamshad Ahmad
3	Dr. Hamdan Al-Ghamedy
4	Dr. Saeid Alghami
5	Dr. Mohammad Al-Suwaiyan

Notes:

1. **A sheet that includes selected Basic Formulae and definitions is provided with this examination.**
2. **Write clearly and show all calculations, FBDs, and units.**

<u>Problem 1</u>: (20 points)

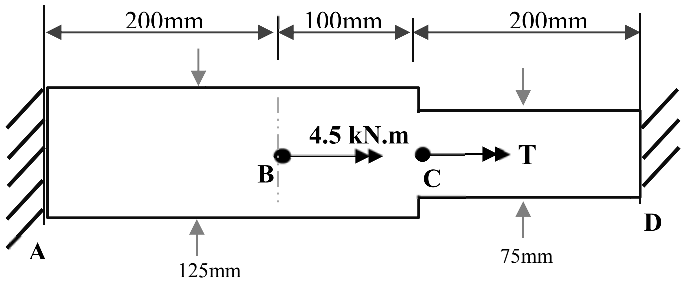
The solid cylinders AC and CD are made of steel and bonded together at C and attached to fixed ends at A and D. Two torques of **4.5 kN-m** and **T** are applied at B and C, respectively as shown in the Figure below.

Given:

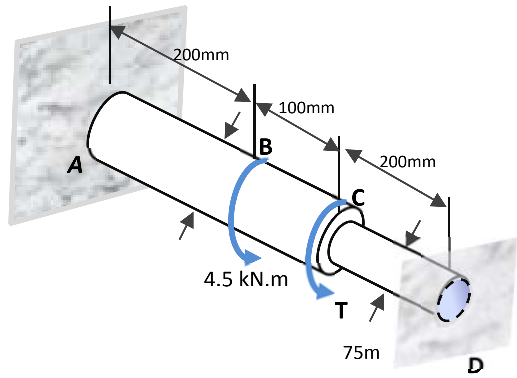
- The shear modulus, G_{st}=75GPa
- The allowable shear stress, τ_{st} = 25 MPa

Determine:

a) the maximum torque **T** that can be applied at C, then
b) determine the angle of twist $\mathbf{\Phi_{C/D}}$.



2D Figure



3D Figure

1- The solid cylinders AC and CD are made of steel and bonded together at C and attached to fixed ends at A and D. Two torques of **4.5 kN-m** and **T** are applied at B and C, respectively as shown in the Figure below.

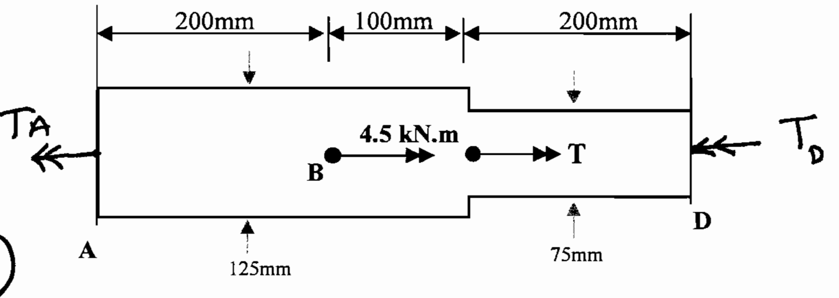
Given:

- The shear modulus, G_{st} = 75 GPa
- The allowable shear stress, τ_{st} = 25 MPa

Determine the following:

a. The maximum torque **T** that can be applied at C

b. The angle of twist $\Phi_{C/D}$



Solution :

a) Equil. Equ :

3

$\sum M_x = 0$;

$4.5 \times 10^3 \text{ N.m} + T = T_A + T_D$ — ①

F.B.D

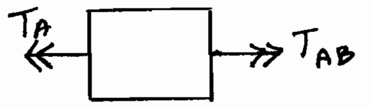
$J_{AB} = J_{BC} = \frac{\pi}{2}\left(\frac{0.125m}{2}\right)^4$
$= 2.397 \times 10^{-5} m^4$

$J_{CD} = \frac{\pi}{2}\left(\frac{0.075m}{2}\right)^4$
$= 3.106 \times 10^{-6} m^4$

Compatibility :

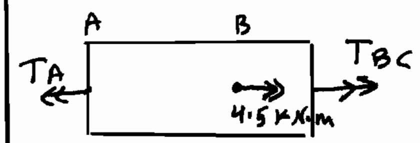
1. $\phi_{A/D} = 0 = \phi_{A/B} + \phi_{B/C} + \phi_{C/D}$ — ②

3 Segment AB :



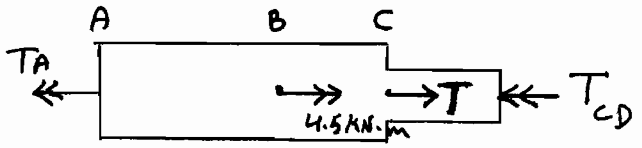
$T_{AB} = T_A$

Segment BC :



$T_{BC} = T_A - 4.5 \text{ KN.m}$

Segment CD :



$T_{CD} = 4.5 \text{ KN.m} + T - T_A$

$\phi_{A/D} = 0 = \phi_{A/B} + \phi_{BC} + \phi_{CD}$

4

$$\frac{T_A \cdot (0.2m)}{J_{AB} \cdot \cancel{G_{st}}} + \frac{(T_A - 4.5 \times 10^3 \text{ N.m}) \cdot (0.1m)}{J_{BC} \cdot \cancel{G_{st}}} - \frac{(4.5 \times 10^3 \text{ N.m} + T - T_A) \cdot (0.2m)}{J_{CD} \cdot G_{st}} = 0$$

$0.2\, T_A + 0.1\,(T_A - 4.5 \times 10^3 \text{ N.m}) - 0.2 * 7.717 * (4.5 \times 10^3 \text{ N.m} + T - T_A) = 0$

$1.843\, T_A - 1.543\, T - 7395.3 = 0$ — ②'

$\Rightarrow \boxed{T_A = 4012.64 + 0.837\, \mathbf{T}}$

- In segment AB :

$$T_{AB} = T_A \quad , \quad \tau = \frac{T \cdot c}{J} \quad , \quad (\tau_{st})_{allowable} = 25 \times 10^6 \text{ Pa}$$

$$\Rightarrow 25 \times 10^6 \text{ Pa} = \frac{T_{AB} \cdot c_{AB}}{J_{AB}} = \frac{(4012.64 + 0.837T) \cdot \left(\frac{0.125m}{2}\right)}{\frac{\pi}{2} \cdot \left(\frac{0.125m}{2}\right)^4}$$

2

$$\Rightarrow \boxed{T = 6{,}660.37 \text{ N}\cdot\text{m}}$$

- In segment BC :

$$T_{BC} = T_A - 4.5 \times 10^3 \text{ N}\cdot\text{m}$$

2 From equ 2' $\Rightarrow T_{BC} = 4012.64 + 0.837T - 4.5 \times 10^3 \text{ N}\cdot\text{m}$

$$= 0.837T - 487.36$$

$$\Rightarrow 25 \times 10^6 \text{ Pa} = \frac{(0.837T - 487.36) \cdot \left(\frac{0.125m}{2}\right)}{\frac{\pi}{2}\left(\frac{0.125m}{2}\right)^4} \Rightarrow \boxed{T = 12{,}036.88 \text{ N}\cdot\text{m}}$$

- In segment CD :

$$T_{CD} = 4.5 \times 10^3 \text{ N}\cdot\text{m} + T - T_A = 4.5 \times 10^3 \text{ N}\cdot\text{m} + T - 4012.64 - 0.837T$$

2

$$\Rightarrow T_{CD} = 487.36 + 0.163T$$

$$\Rightarrow 25 \times 10^6 \text{ Pa} = -\frac{(487.36 + 0.163T) \cdot \left(\frac{0.075m}{2}\right)}{\frac{\pi}{2}\left(\frac{0.075m}{2}\right)^4} = -5883507.4 - 1967.72T$$

1

$$\Rightarrow \boxed{T = 15{,}694.67 \text{ N}\cdot\text{m}}$$

$$\therefore \boxed{T_{max} = 6{,}660.37 \text{ N}\cdot\text{m}}$$

b) $\phi_{C/D} = \dfrac{-T_{CD} \cdot L_{CD}}{J_{CD} \cdot G_{st}}$, $T_{CD} = 4500 \text{ N}\cdot\text{m} + 6660.37 \text{ N}\cdot\text{m} - 4012.64 - 5574.73 \text{ N}\cdot\text{m}$

3

$$T_{CD} = 1573 \text{ N}\cdot\text{m}$$

$$\therefore \phi_{C/D} = \frac{-(1573 \text{ N}\cdot\text{m}) \cdot (0.2m)}{\frac{\pi}{2} \cdot \left(\frac{0.075}{2}\right)^4 \cdot (75 \times 10^9 \text{ Pa})}$$

$$\phi_{C/D} = -1.350 \times 10^{-3} \text{ rad}.$$

$$\phi_{C/D} = -0.0774^\circ$$

Problem 2: (20 points)

A **thinned-wall square shaft** fixed at C and subjected to torques at A and B is shown below. Considering τ_{allow} = 50 MPa, magnitude of ϕ_{allow} at end A = 5°, and G = 75 GPa, **determine** smallest permissible outer dimension $\boldsymbol{h}$.

Internal torques

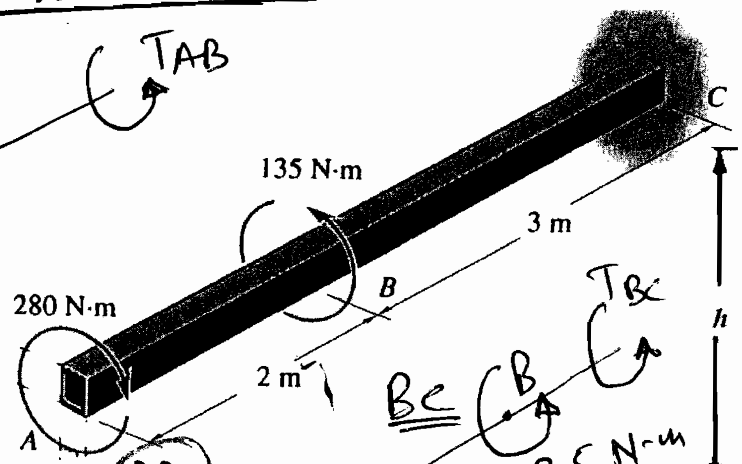
AB

280 N-m, T_{AB}

$\Sigma T = 0$

$\Rightarrow -280 + T_{AB} = 0$

$\Rightarrow T_{AB} = 280$ N-m (02)



BC

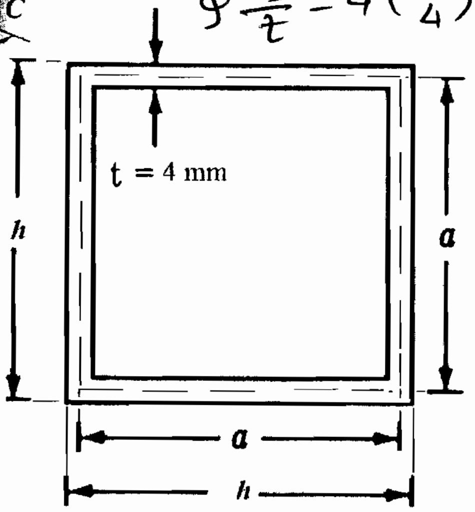
T_{BC}, 135 N-m, 280 N-m

$\Sigma T = 0 \Rightarrow -280 + 135 + T_{BC} = 0 \Rightarrow T_{BC} = 145$ N-m (02)

A_m & $\oint \frac{ds}{t}$

$A_m = a^2$ (02)

$\oint \frac{ds}{t} = 4\left(\frac{a}{4}\right) = a$ (02)



(i) Considering τ_{allow}

$$\tau_{av,max} = \frac{T_{max}}{2tA_m} = \tau_{allow} \Rightarrow \frac{280\times10^3}{2\times4\times a^2} = 50$$

$$\Rightarrow a = \sqrt{\frac{280\times10^3}{2\times4\times50}} = 26.45 \text{ mm} \quad (04)$$

(ii) Considering ϕ_{allow}

$$\phi_A = \phi_{AB} + \phi_{BC} = \phi_{allow} = 5° = \frac{5\times\pi}{180}$$

$$\Rightarrow \frac{280\times10^3\times2\times10^3}{4(a^2)^2\times75\times10^3}\times a + \frac{145\times10^3\times3\times10^3}{4(a^2)^2\times75\times10^3}\times a = \frac{5\pi}{180}$$

$$\Rightarrow \frac{3316.67}{a^3} = \frac{\pi}{36} \Rightarrow a = \left[\frac{3316.67\times36}{\pi}\right]^{1/3}$$

$$\Rightarrow a = 33.62 \text{ mm} \quad (04)$$

Therefore, required $a = 33.62$ mm (greater of the two values) (02)

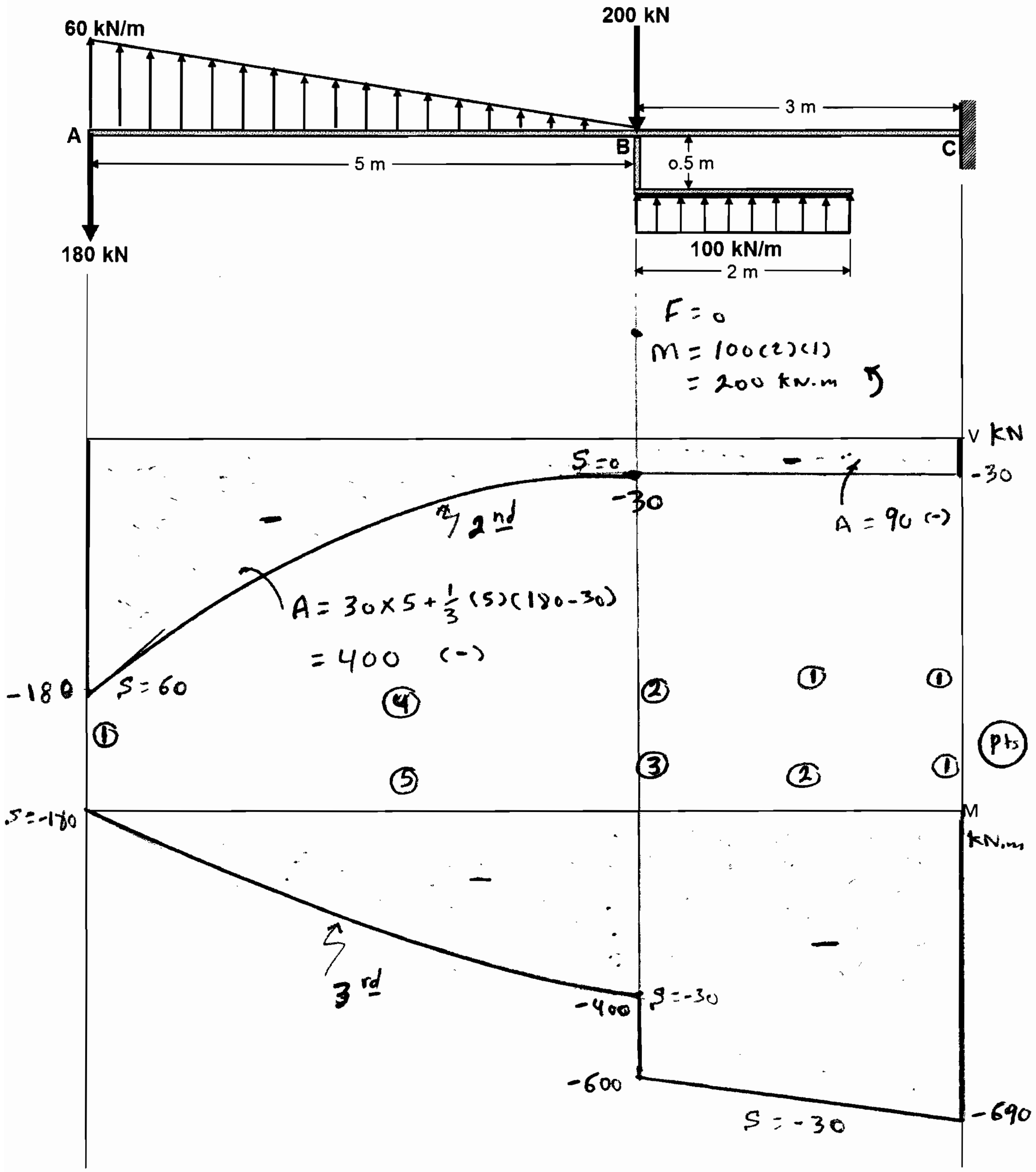
$\therefore$ Smallest permissible 'h' $= 33.62 + \frac{4}{2} + \frac{4}{2} = 37.62$ mm $= 37.62$ mm Ans (02)

Problem 3 (20 pts.)

Draw the **shear force and bending moment diagrams** for the beam **ABC** shown below using the ***summation (graphical) method***. *Write the degree (2, 3, etc.) of the curve on each one.* Put all values on the diagrams, but you do NOT need to show the calculations.

Use appropriate scale. ***No credit will be given if another method is used.***

The reactions are: C_y **= 30 kN ↑ ;** M_C **= 690 kN.m**



Problem 4: (20 points) key - solution

The beam **ABCD** has an U-inverted cross-section with dimensions given including common thickness t = 12 mm, and is loaded with two concentrated loads as shown.

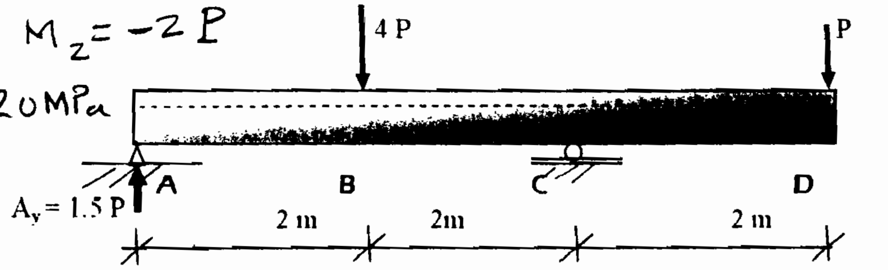
Given: Ultimate tensile material strength is 180 MPa and E = 80 GPa.

a) Compute an allowable value of load P based on FS value = 1.5.
b) Draw the **strain**-diagram at the cross section at C.

Scores:

Assume that the material can carry very high compressive stress.

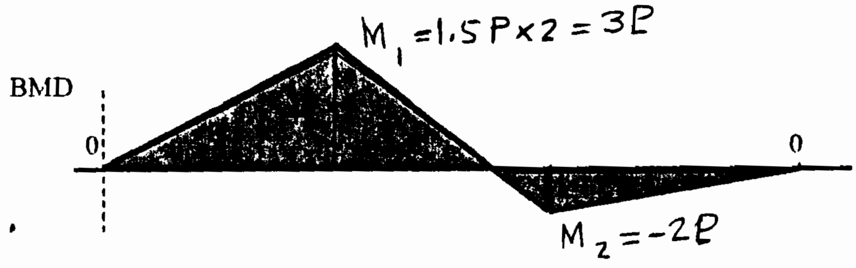
$M_1 = 3P$ & $M_2 = -2P$



1% $\sigma_{all} = \dfrac{180}{1.5} = 120\,MPa$

Max. tensile stresses

$\sigma^* \,\&\, \sigma^{**}$

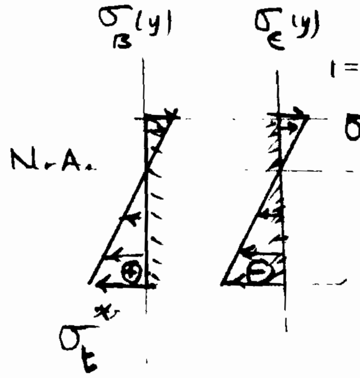


$M_1 \rightarrow \sigma^*$

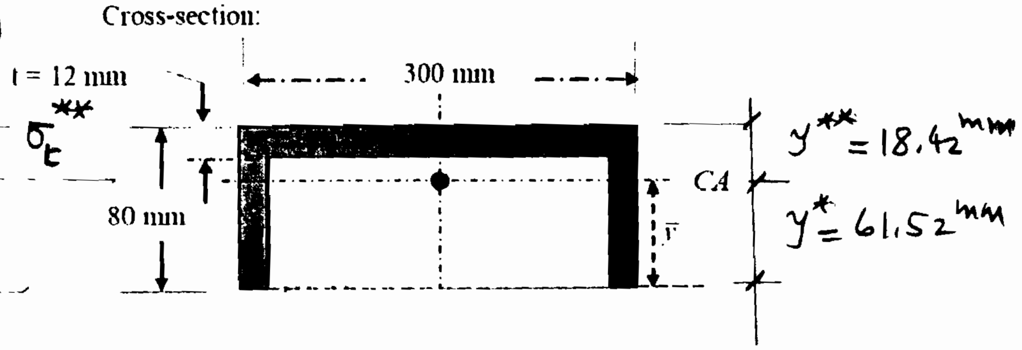
$M_2 \rightarrow \sigma^{**}$

$\varepsilon(y) = \sigma(y)/E$

$\sigma(y)$ at c/s :



Cross-section:



$\sigma_t^* \,\&\, \sigma_t^{**} \not> \sigma_{all}$

Given: $\bar{y}$ = 61.52 mm

$I_{CA} = 2.469 \times 10^6\ mm^4$

a) $\sigma(y) = -\dfrac{M(x)\,y}{I_{CA}} \Rightarrow 120 \times 10^3 = -\dfrac{3P^*(-61.52 \times 10^{-3})}{2.469 \times 10^{-6}} = 74.8\,P^*$

8% $\therefore P^* = 1.605$ kN

Also at C: $120 \times 10^3 = -\dfrac{-2P^{**}(18.42 \times 10^{-3})}{2.469 \times 10^{-6}} = 14.92\,P^{**}$

6% $\therefore P^{**} = 8.042$ kN

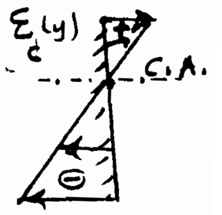
$\therefore P_{all} = \min\,[P^* \,\&\, P^{**}] = 1.605$ kN

b) Using $P_{all} = 1.605$ kN $\Rightarrow M_c = -2P = -3.21$ kN.m

2½% $\sigma_{t;c} = -\dfrac{-3.21 \times (18.42 \times 10^{-3})}{I_{CA}} = 23.94\,MPa \Rightarrow \varepsilon_{t,c} = 2.99 \times 10^{-4}$ mm/mm

2½% $\sigma_{c;c} = -\dfrac{-3.21 \times (-61.52 \times 10^{-3})}{I_{CA}} = -79.98\,MPa$

$\therefore \varepsilon_{t;c} = 2.99 \times 10^{-4}$ mm/mm ; $\varepsilon_{c;c} = -1.0 \times 10^{-3}$ mm/mm

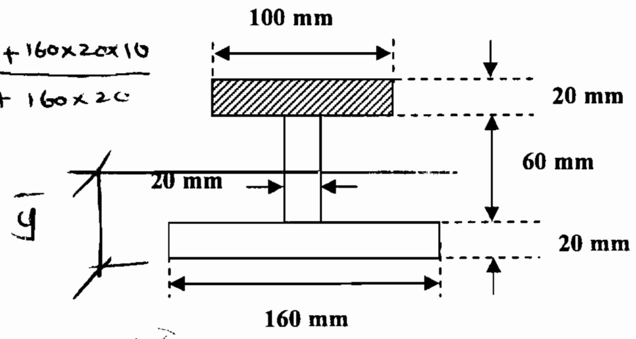


Problem 5: (20 points)

A beam having the shown cross section is subjected to a moment of +20 kN.m, determine:

a) The location of the neutral axis.
b) The moment of inertia about the neutral axis.
c) The resultant force acting on the top flange (shaded in the figure).

(a) Divide x-section into 3 rectangles



$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{100\times20\times90 + 60\times20\times50 + 160\times20\times10}{100\times20 + 60\times20 + 160\times20} \quad 2$$

$$\bar{y} = 42.5 \text{ mm} \quad 3 \quad \Leftarrow \text{Ans}$$

(b) $I_{NA} = I^{(1)} + I^{(2)} + I^{(3)}$ 1

$$= \frac{1}{12}(100)(20)^3 + 100\times20(\overset{47.5}{100-42.5-10})^2 \qquad 4579\times10^3$$

$$+ \frac{1}{12}(20\times60^3) \; 2 \; + 60\times20(\overset{(7.5)}{50-42.5})^2 \qquad 427.5\times10^3$$

$$+ \frac{1}{12}(160\times20^3) + 160\times20(42.5-10)^2 \qquad 3487\times10^3$$

$$= 8.493\times10^6 \text{ mm}^4 \quad 2$$

(c) $F_{flange} = \bar{\sigma}A$ 2

$$\bar{\sigma} = -\frac{M\bar{y}}{I} = -\frac{(20\times10^6)(\overset{47.5}{90-42.5})}{8.493\times10^6} = -111.9 \text{ MPa} \quad 5$$

$$F_{flange} = \bar{\sigma}(100\times20) = -\underline{\underline{224 \text{ kN}}} \quad 3$$