King Fahd University of Petroleum & Minerals

DEPARTMENT OF CIVIL ENGINEERING

First Semester 1434-35 / 2013-14 (131)

CE 203 STRUCTURAL MECHANICS I

Major Exam I

Tuesday, October 29, 2012 7:00-9:00 P.M.

KEY SOLUTION

Note to Students

Even though the course is not "standard grading", ***being around the average does not indicate C performance, since there is a minimum amount of course comprehension needed to pass the course satisfactorily,*** **irrespective of the exam average and the performance of other students.**
Therefore, students who did poorly in this exam should do double effort in the remaining of the semester to avoid disappointing grade.

After reviewing the key solution and still having a concern about your mark, you may consult with the faculty members who prepared & Graded each problem.

The deadline for review is Tuesday November 12, 2013.

Problem	Solved & Graded by
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3	Dr. Ali Al-Gadhib
4	Dr. Shamshad Ahmad

Notes:

1. **A sheet that includes selected Basic Formulae and definitions is provided with this examination.**
2. **Write clearly and show all calculations, FBDs, and units.**

Problem 1: (25 points)

For the shown frame determine the following:

a. The required thickness of member BC.
b. The shear stress in the pin at B.
c. The required diameter of the pin at A.

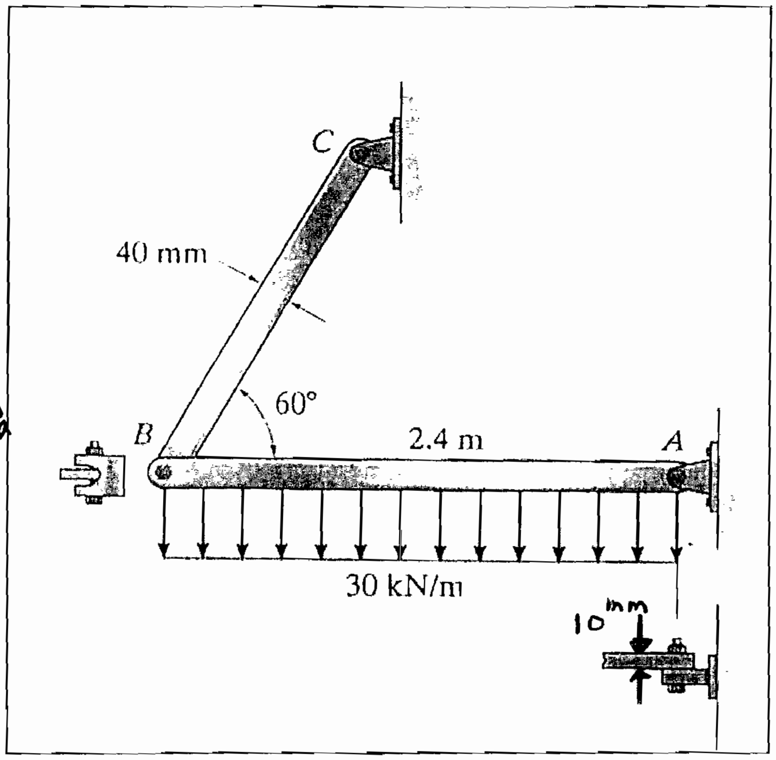
Given:

- *The diameter of the pin at B is 22 mm.*
- *The failure normal stress for member BC is (σ_{fial}) = 450 MPa.*
- *The failure shear stress for the pin at A is (τ_{fail}) = 160 MPa.*
- *The failure bearing stress (σ_b) = 430 MPa.*

Apply a factor of safety of F.S. = 2.

Normal stress :

$$\sigma_{all} = \frac{\sigma_{fail}}{F.S} = \frac{450}{2} = 225\ MPa$$



Shear stress :

2

$$\tau_{all} = \frac{\tau_{fail}}{F.S} = \frac{160}{2} = 80\ MPa$$

Bearing stress :

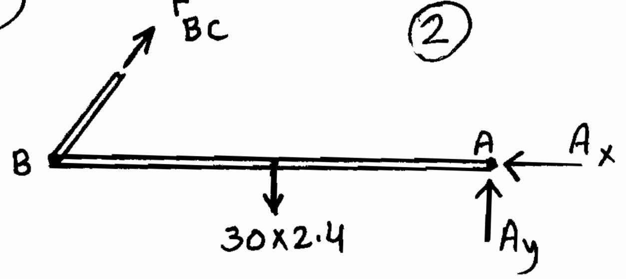
$$(\sigma_b)_{all} = \frac{(\sigma_b)_{fail}}{F.S} = \frac{430}{2} = 215\ MPa$$

a) The required thickness of member BC, t_{BC}.

$\circlearrowleft + \sum M_A = 0$;

$$30 \times 2.4 \times 1.2 - F_B \cdot \sin 60^\circ \times 2.4 = 0$$

$$\Rightarrow F_{BC} = \frac{30 \times 2.4 \cdot 1.2}{\sin 60^\circ \times 2.4} = 4.57\ KN.$$ 3



2

F.B.D. (1)

For Member BC :

$$\sigma_{BC} = \sigma_{all} \Rightarrow 225 \times 10^6\ Pa = \frac{F_{BC}}{A_{BC}}$$

$$A_{BC} = 0.04m \times t_{BC} = \frac{F_{BC}}{225 \times 10^6\ Pa}$$

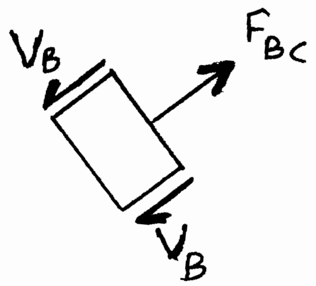
$$\Rightarrow t_{BC} = \frac{41.57 \times 10^3\ N}{(225 \times 10^6 \frac{N}{m^2}) \times (0.04m)}$$

$$= 0.004619\ m$$ 3

$$= \underline{\underline{4.619\ mm.}}$$

b) The shear stress in the pin at B.

$d_B = 22\ mm$



5 Double Shear $\Rightarrow 2V_B = F_{BC}$

$$\Rightarrow V_B = \frac{41.57}{2} = 20.785\ KN$$

$$\Rightarrow \tau_B = \frac{V_B}{A_B} = \frac{20.785 \times 10^3\ N}{\pi \times \left(\frac{0.022m}{2}\right)^2} = 54.678 \times 10^6\ Pa = 54.678\ MPa$$

c) The required diameter of the pin at A:

The resultant force at A (see F.B.D. 1)

$$\overset{+}{\rightarrow} \Sigma F_x = 0;\ -\cancel{A_x} + F_{BC} \times \cos 60^\circ = 0 \Rightarrow A_x = 41.57 \cos 60^\circ = 20.785\ KN$$

$$+\uparrow \Sigma F_y = 0;\ A_y - 72\ KN + F_{BC} \sin 60^\circ = 0$$

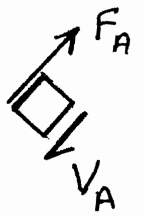
2

$$\Rightarrow A_y = 36\ KN$$

$$\Rightarrow F_A = \sqrt{(A_x)^2 + (A_y)^2} = \sqrt{(20.785)^2 + (36)^2} = \underline{\underline{4.57\ KN.}}$$

- Considering allowable shear stress:

It is single shear at pin A $\Rightarrow V_A = F_A$



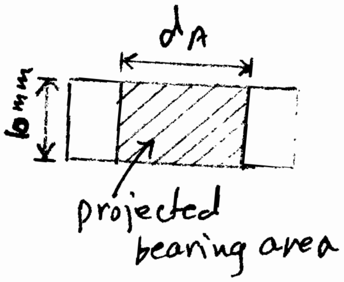
3 $\Rightarrow \tau_{all} = \dfrac{F_A}{A_A} \Rightarrow 80\ MPa = \dfrac{41.57 \times 10^3\ N}{\pi (r_A)^2}$

$$\Rightarrow r_A = 0.01286\ mm \Rightarrow d_A = 0.02572\ m = \underline{\underline{25.72\ mm.}}$$

- Considering allowable bearing stress:

$\sigma_b = 215\ MPa \Rightarrow \sigma_b = \dfrac{F_A}{\text{Bearing Area}}$

3 $= \dfrac{F_A}{(0.01\ m) \cdot d_A}$



$$\Rightarrow d_A = 0.01933\ m = \underline{\underline{19.33\ mm.}}$$

$\Rightarrow$ Therefore, the required diameter of the pin at A = $\boxed{25.72\ mm}$

2

Problem 2: (25 points)

Two bars, A and B are made from the same material and have the same diameter, d=10 mm. Bar A has an initial length of 400 mm and bar B had an initial length of 250 mm.

When bar A is subjected to a tensile force of 4 kN its length increased by 0.37 mm and its diameter decreased by 0.0023 mm.

When bar B is subjected to a tensile force of 10 kN its length increased by 3 mm.

Calculate:

a) The elastic modulus of the material,
b) Poisson's ratio of the material,
c) The final length of each bar when the load is removed.

Given: the yield stress = 80 MPa

$$\sigma_A = \frac{4\times10^3}{\frac{\pi}{4}(10)^2} = 51.0\,\text{MPa} \quad (2)$$

$$\varepsilon_{A\,(\text{long})} = \frac{0.37}{400} = 0.000925 \quad (2)$$

$$\sigma_B = \frac{10\times10^3}{\frac{\pi}{4}(10)^2} = 127.3\,\text{MPa} \quad (2)$$

$$\varepsilon_B = \frac{3}{250} = 0.012 \quad (2)$$

$$\varepsilon_{\text{lateral}\,A} = -\frac{.0023}{10} = -0.00023 \quad (2)$$

(a) $$E = \frac{\sigma}{\varepsilon} = \frac{51\times10^{-3}}{0.000925} = \underline{55.1}\ \text{GPa} \quad (4)$$

in linear portion

(b) $$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} = -\frac{(-.00023)}{.000925} = \underline{0.25} \quad (4)$$

(c) Bar A will be $\underline{400\text{ mm}}$ since it is within Elastic range (2)

Bar B recovered strain $= \frac{\sigma_B}{E} = \frac{127.3\times10^{-3}}{55.1} = 0.0023$ (2)

∴ Permenat strain $= 0.0097$

∴ " deformation $= 2.422$ mm

Final length $= \underline{252.422}$ mm (3)

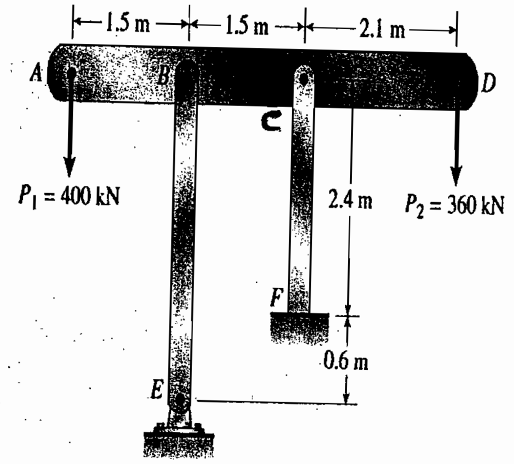
Problem 3: (25 points)

The horizontal rigid beam ABCD is supported by vertical Bars BE and CF and is loaded by vertical forces P_1 and P_2 as shown. Bars BE and CF are made of steel for which E = 200 GPa and have cross-sectional areas $A_{BE} = 11{,}100\ mm^2$ and $A_{CF} = 9{,}280\ mm^2$.

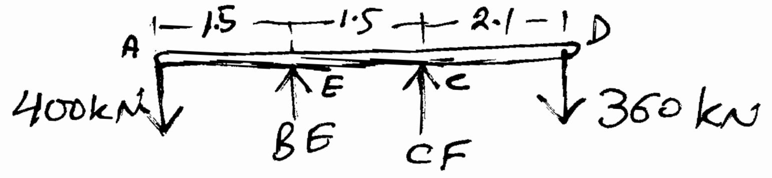
a) Determine the vertical displacement at A, δ_A.

b) What is the change in temperature of bar CE so that the rigid beam ABCD will remain horizontal?

Given: $\alpha_{steel} = 12 \times 10^{-6}\ (\frac{1}{°C})$



Finding forces in bars CF and BE.



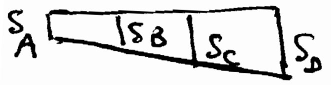
$\Sigma M_{@A} = 0 \; +\circlearrowleft \qquad BE(1.5) + CF(3.0) - 360\text{ kN}(5.1) = 0 \quad \text{— (1)} \quad \boxed{6}$

$\Sigma F_y = 0 \; +\uparrow \qquad BE + CF - 400 - 360 = 0 \quad \text{— (2)} \quad \boxed{2}$

Solving (1) & (2) $\Rightarrow$ $BE = 296\text{ kN}$ $\boxed{1}$

$CF = 464\text{ kN}$ $\boxed{1}$

Finding δ_{BE}, δ_{CF}, and δ_A



(a)

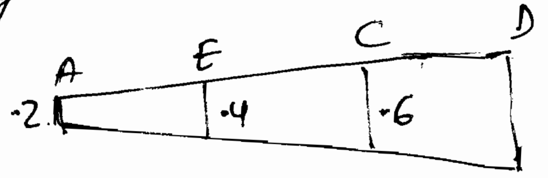
$$\delta_{CF} = \left(\frac{N_{CF} L}{EA}\right)_{CF} = \frac{(464\times10^3)(2.4\times10^3)}{(200\times10^3)\,9280} = 0.6\text{ mm} \quad \boxed{3}$$

$$\delta_{BE} = \left(\frac{N_{BE} L}{EA}\right)_{BE} = \frac{((296)\times10^3)(3.0\times10^3)}{(200\times10^3)(1100)} = 0.4\text{ mm} \quad \boxed{3}$$

Since AECD is rigid and from fig (a). $\boxed{4}$

$$\frac{\delta_B - \delta_A}{1.5} = \frac{\delta_C - \delta_A}{3.0} \Rightarrow \delta_A = 2\delta_B - \delta_C \Rightarrow \delta_A = -0.2\text{ mm}$$

For the rigid beam ABCD to remain horizontal.



bar CF should shrink by 0.2 mm due to temperature.

so

$$-0.2 = \alpha \,\Delta T\, L$$

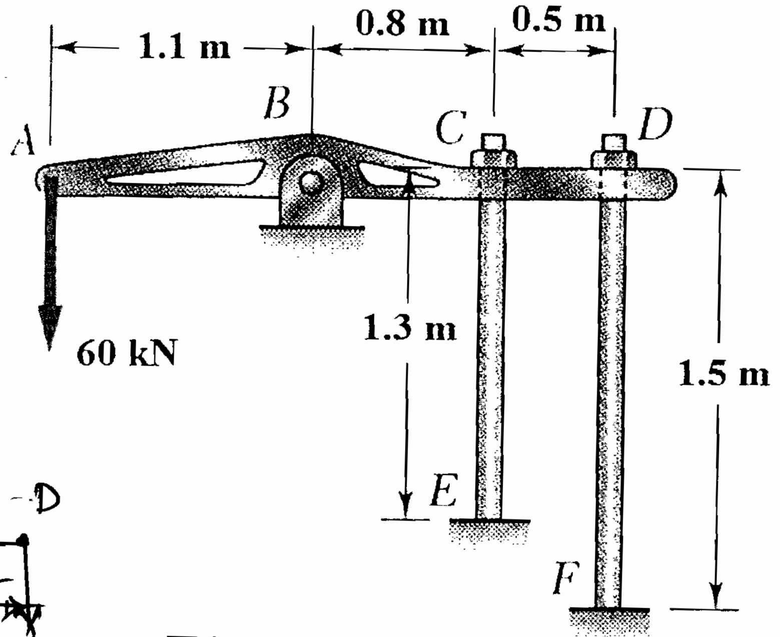
$$-0.2 = (12\times10^{-6})\,\Delta T\,(2.4\times10^3)\text{ mm} \quad \boxed{5}$$

$$\therefore \Delta T = -6.94\,C^\circ$$

Problem 4: (25 points)

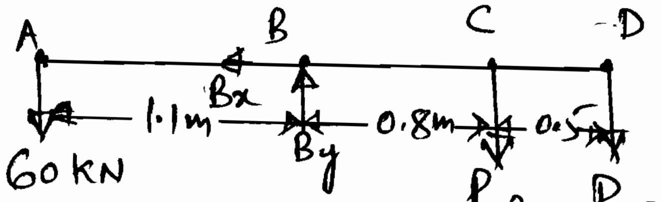
The 12 mm diameter rod *CE* and the 18 mm diameter rod *DF* are attached to the rigid bar *ABCD* as shown below. Taking $E = 75$ GPa for both rods (*CE* and *DF*), determine:

a) the stress in each rod,
b) deflection of point *A*.



Let P_{CE} and P_{DF} are the internal forces in rods CE and DF, respectively.

Equilibrium Condition



Free-body diagram

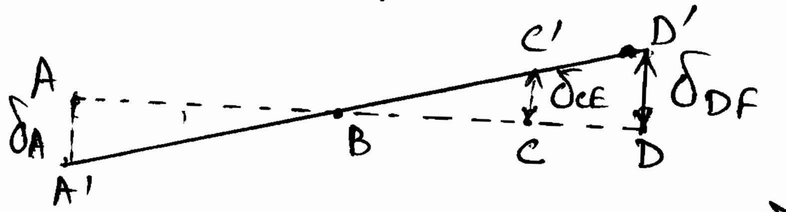
ΣM about $B = 0$

$\Rightarrow 60 \times 1.1 - 0.8 P_{CE} - 1.3 P_{DF} = 0$

$\Rightarrow \boxed{P_{CE} + 1.625 P_{DF} = 82.5}$ — Eq (1)

(05)

Compatibility Condition



$$\frac{\delta_{DF}}{1.3} = \frac{\delta_{CE}}{0.8} = \frac{\delta_A}{1.1}$$

$\Rightarrow 0.8\,\delta_{DF} = 1.3\,\delta_{CE}$

$\Rightarrow \delta_{DF} = 1.625\,\delta_{CE} \Rightarrow \dfrac{P_{DF} L_{DF}}{A_{DF} E_{DF}} = 1.625 \dfrac{P_{CE} L_{CE}}{A_{CE} E_{CE}}$

$$\Rightarrow \frac{1.5 \times 10^3 P_{DF}}{\frac{\pi}{4}(18)^2 \times 75 \times 10^3} = \frac{1.625 \times 1.3 \times 10^3 P_{CE}}{\frac{\pi}{4}(12)^2 \times 75 \times 10^3}$$

$\Rightarrow \boxed{P_{DF} = 3.169 P_{CE}}$ --- Eq. (2) (07)

Solving Eqs. (1) & (2), $P_{CE} = 13.42$ kN and $P_{DF} = 42.51$ kN

(03)

a) $\sigma_{CE} = \dfrac{13.42 \times 10^3}{\frac{\pi}{4}(12)^2} = 118.66$ N/mm² Ans

(03)

$\sigma_{DF} = \dfrac{42.51 \times 10^3}{\frac{\pi}{4}(18)^2} = 167.05$ N/mm² Ans

b) $\delta_A = \dfrac{1.1}{0.8}\delta_{CE} = 1.375 \dfrac{P_{CE} L_{CE}}{A_{CE} E_{CE}}$

$= \dfrac{1.375 \times 13.42 \times 10^3 \times 1.3 \times 10^3}{\frac{\pi}{4}(12)^2 \times 75 \times 10^3}$

$= 2.828$ mm Ans (07)