King Fahd University of Petroleum & Minerals
DEPARTMENT OF CIVIL ENGINEERING
Secon Semester 1433-34 / 2012-13 (122)

CE 203 STRUCTURAL MECHANICS I

Major Exam I

Tuesday, March 12, 2013 - 6:30-8:45 P.M.

KEY SOLUTION

Problem	Solved & Graded by
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Notes:

1. **A sheet that includes selected Basic Formulae and definitions is provided with this examination.**
2. **Write clearly and show all calculations, FBDs, and units.**

Note to Students:

Even though the course is not "standard grading", *being around the average does not indicate C performance, since there is a minimum amount of course comprehension needed to pass the course satisfactorily,* irrespective of the exam average and the performance of other students. Therefore, students who did poorly in this exam should do double effort in the remaining of the semester to avoid disappointing grade.

After reviewing the key solution and still having a concern about your mark, you may consult with the faculty members who prepared each problem.

The deadline for review is Monday April 8, 2013.

Problem 1: (20 points)

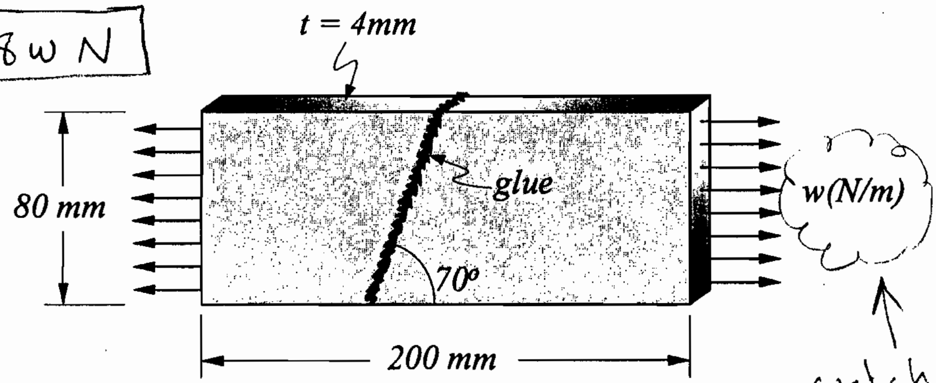
The given thin plate is made of two parts glued together as shown. The plate is subjected to an axial distributed load *w (N/m)*. Determine the largest value of *w* that can be applied.

For the plate material : ultimate normal stress = 60 *MPa*

For the glue : ultimate normal stress = 30 *MPa*, and ultimate shear stress = 15 *MPa*

For the whole problem, use safety factor *S.F.* = 3

Applied force $\boxed{F = .08w \text{ N}}$



watch units

a) check plate:

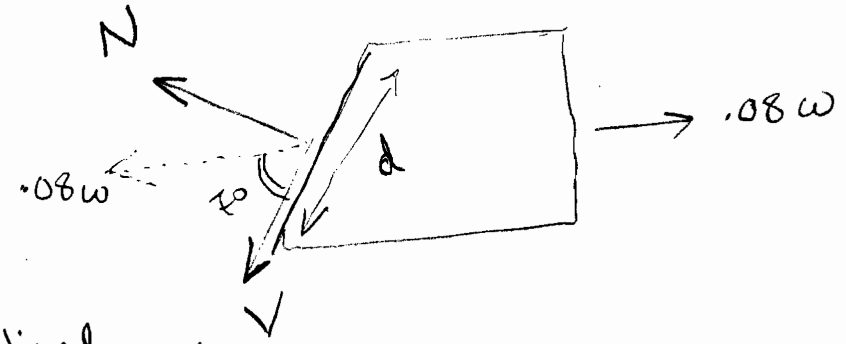
$$\sigma_{all} = \frac{60}{3} = 20 \text{ MPa}$$

$$\sigma_{all} = 20 = \frac{F}{A} = \frac{.08w}{(80)(4)} \rightarrow \boxed{w = 80{,}000 \text{ N/m}} \quad (1)$$

b) check glue

$N = (.08w) \sin 70$

$V = (.08w) \cos 70$



To find the inclined surface area

$$d = \frac{80}{\sin 70}$$

• check glue normal: $\sigma_{all} = \frac{30}{3} = 10 \text{ MPa} = \frac{\text{force}}{\text{area}} = \frac{.08w \sin 70}{(4)(80/\sin 70)}$

$$\boxed{w = 45{,}299 \text{ N/m}} \quad (2)$$

• check glue shear: $\tau_{all} = \frac{15}{3} = 5 \text{ MPa} = \frac{\text{force}}{\text{area}} = \frac{.08w \cos 70}{(4)(80/\sin 70)}$

$$\boxed{w = 62{,}229 \text{ N/m}} \quad (3)$$

$$\boxed{w_{max} = 45{,}299 \text{ N/m}}$$ Answer

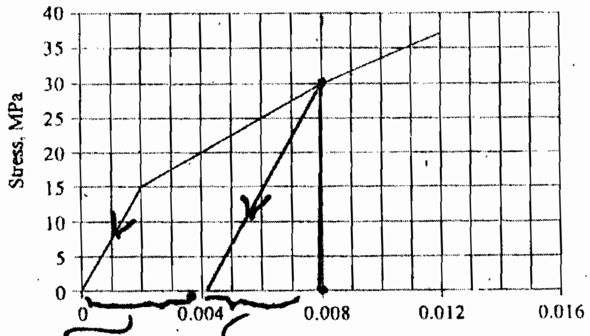
Compare (1), (2), (3)
the smallest controls

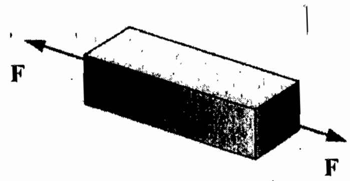
Problem 2: (20 points)

A bar with the stress-strain diagram shown was originally 1 m long with a square cross-sectional area of 100 mm x 100 mm.
When an axial tension load F is applied, the square cross-section became 99.95 mm x 99.95 mm.
Determine the following:

(6) a) The magnitude of the applied force F.
(3) b) The final length of the bar when the load F is applied.
(2) c) The final length of the bar when the load F is released.
(5) d) The final length of the bar when the applied load is 300 kN.
(4) e) The final length of the bar when the 300 kN load is released.

Poisson's ratio, ν = 0.25





Permentant Strain

recovered strain

Solution

(a) $\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} \Rightarrow \varepsilon_{long} = \frac{-\varepsilon_{lat}}{\nu}$

$\varepsilon_{lat} = \frac{99.95-100}{100} = -0.0005 \frac{mm}{mm}$ (2)

$\varepsilon_{long} = \frac{-(-0.0005)}{0.25} = 0.002 \frac{mm}{mm}$ (1)

From σ-ε diagram when $\varepsilon_{long} = 0.002 \Rightarrow \boxed{\sigma = 15\ MPa}$ (1)

$\sigma = \frac{P}{A_o} \Rightarrow P = \sigma A_o = 15 \times 10000 = \boxed{150000\ N = 150\ kN}$ (2)

(b) $\varepsilon_{long} = \frac{L_f - L_o}{L_o} \Rightarrow L_f = (\varepsilon_{long} \times l_o) + l_o = \boxed{1.002\ m = l_f}$ (3)

(c) when the load F is released will go back ov original Length $\sigma = \sigma_y$,
$\therefore L_f = 1\ m.$ (2)

(d) $\sigma = \frac{300000}{10000} = 30\ MPa$ (2), in the plastic range.
at $\sigma = 30\ MPa$, $\varepsilon_{long} = 0.008\ mm/mm$ (2)
$L_f = (0.008)(1) + 1 = \boxed{1.008\ m}$ (1)

(e) $E = \frac{\sigma}{\varepsilon_{long}} = \frac{15}{0.002} = 7500\ MPa$

(3)

recovered strain $= \frac{30}{7500} = 0.004 \frac{mm}{mm}$, or directly form the graph

permentant strain $= 0.008 - 0.004 = .004\ mm/mm$

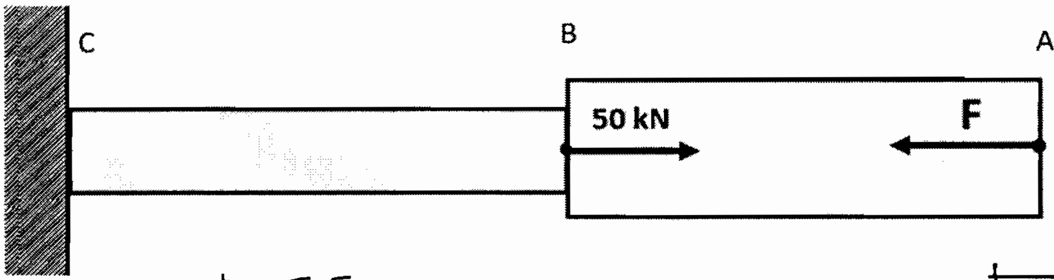
$L_f = (1 \times .004) + 1 = \boxed{1.004\ m}$ (1)

Problem 3: (20 points)

The rods AB and BC are subjected to the *loads and temperature changes* shown in the figure and table below. Determine the **maximum allowable force F** that can be applied (in the shown direction) if

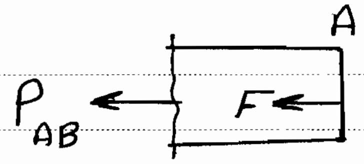
- the maximum allowable normal stress in AB is 150 MPa (tension or compression), and
- the maximum allowable normal stress in BC is 100 MPa (tension or compression), and
- the maximum allowable displacement of point A is 5 $(10)^{-4}$ m.

Properties / *Member*	L (m)	A (m^2)	E (GPa)	ΔT (°C)	α (/°C)
AB	0.5	4 $(10)^{-4}$	200	+40	20 $(10)^{-6}$
BC	0.6	3 $(10)^{-4}$	100	-60	15 $(10)^{-6}$



Pb

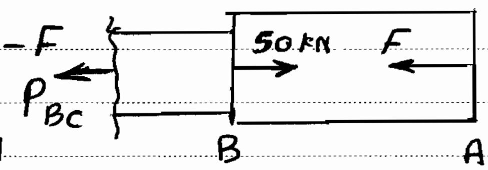
(1) FBD : AB $\quad \xrightarrow{+} \sum F_x = 0 \Rightarrow$



(1) $-F - P_{AB} = 0 \Rightarrow P_{AB} = -F$ "C"

(1) FBD : BC $\quad \xrightarrow{+} \sum F_x = 0 \Rightarrow$

(1) $-F + 50(10)^3 - P_{BC} = 0 \Rightarrow P_{BC} = 50(10)^3 - F$



$\sigma^{AB}_{max\,allow} = P_{AB}/A_{AB} = \pm 150(10)^6 \Rightarrow$

(2) $-F/4(10)^{-4} = -150(10)^6 \Rightarrow F^{(1)}_{max} = 60$ kN

$\sigma^{BC}_{max\,allow} = P_{BC}/A_{BC} = \pm 100(10)^6 \Rightarrow$

(2) $[50(10)^3 - F]/3(10)^{-4} = -100(10)^6 \Rightarrow F^{(2)}_{max} = 80$ kN

displ. of A $= \sum \delta = (\delta_{mech} + \delta_{therm})_{AB} + (\delta_{mech} + \delta_{therm})_{BC}$

(2) $\delta^{AB}_{mech} = e_{load} = \frac{PL}{AE} = \frac{-F(0.5)}{4(10)^{-4}\,200(10)^9} = -6.25(10)^{-9}F \quad (\leftarrow)$

(1) $\delta^{AB}_{therm} = e_{\Delta T} = \alpha \Delta T L = 20(10)^{-6}(+40)(0.5) = +4(10)^{-4}$ m $(\rightarrow)$

(2) $\delta^{BC}_{mech} = e_{load} = \frac{PL}{AE} = \frac{[50(10)^3 - F](0.6)}{3(10)^{-4}\,100(10)^9} = 1(10)^{-3} - 2(10)^{-8}F$

(2) $\delta^{BC}_{therm} = e_{\Delta T} = \alpha \Delta T L = 15(10)^{-6}(-60)(0.6) = -5.4(10)^{-4}$ m $(\leftarrow)$

displ. of A $= -6.25(10)^{-9}F + 4(10)^{-4} + 1(10)^{-3} - 2(10)^{-8}F - 5.4(10)^{-4}$

(1) $= 8.6(10)^{-4} - 2.625(10)^{-8}F \Rightarrow$

$8.6(10)^{-4} - 2.625(10)^{-8}F = -5(10)^{-4}$ [Note the minus sign! Why ?!]

(2) $\Rightarrow F^{(3)}_{max} = 1.36(10)^{-3}/2.625(10)^{-8}$
$= 51.81$ kN

(2) $F_{max} = \min(F^{(1)}_{max}, F^{(2)}_{max}, F^{(3)}_{max}) \Rightarrow \boxed{F_{max} = 51.81 \text{ kN}}$

("Why ?!") {Note that σ_{BC} is still "ok"}

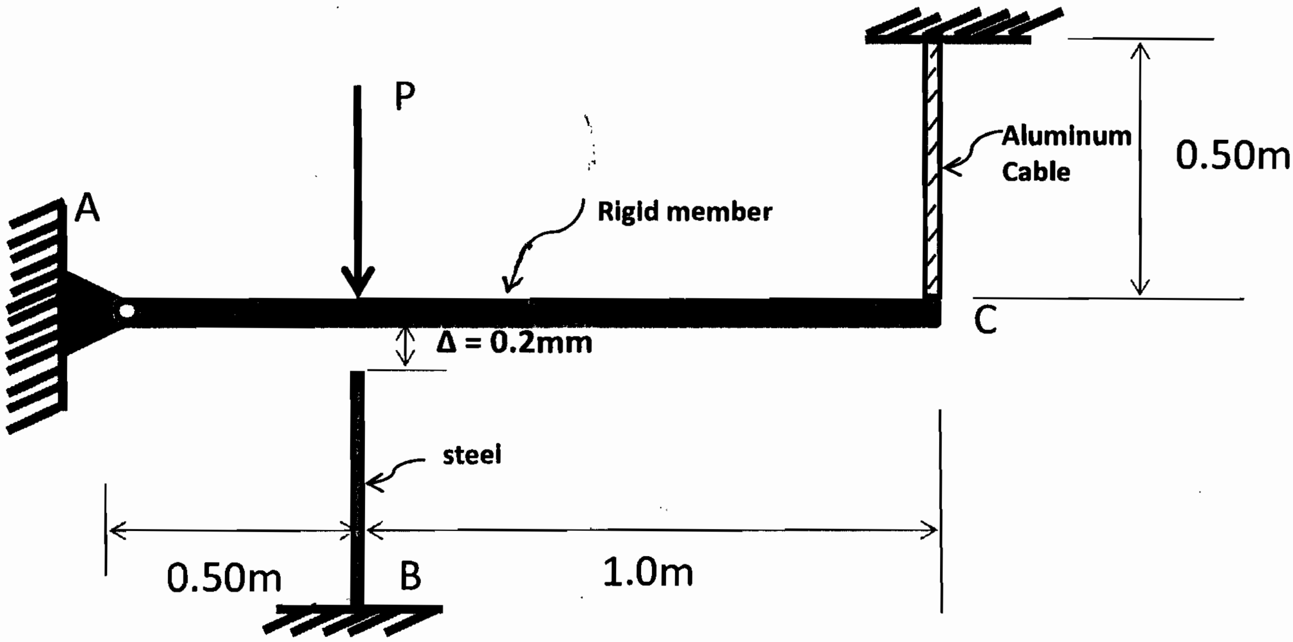
Problem 4: (20 points)

Rigid member AC is hinged at A and is supported by an aluminum cable at C. Before applying the load, AC was horizontal and a gap, $\Delta = 0.2$ mm separated it from a steel rod as shown.

If P = 24 kN, determine the following:

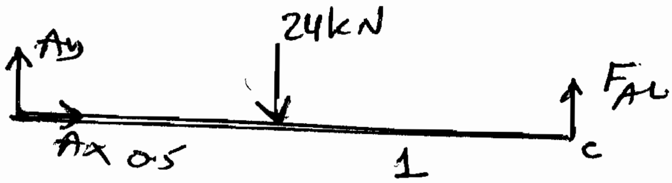
a) the stress in the aluminum cable.
b) the displacement of point C.

$E_{aluminum} = 70$ GPa, $E_{steel} = 200$ GPa, $L_{steel} = 0.5$ m
$A_{aluminum} = A_{steel} = 50\ mm^2$



(I) First need to check if the gap closes or not and the problem need to be treated as statically determinate :

FBD :



(1) $\sum M @ A = 0 \ +\circlearrowleft \quad 24 \times 10^3 (0.5) = 1.5\ (F_A) \implies F_A = 8000\ N$

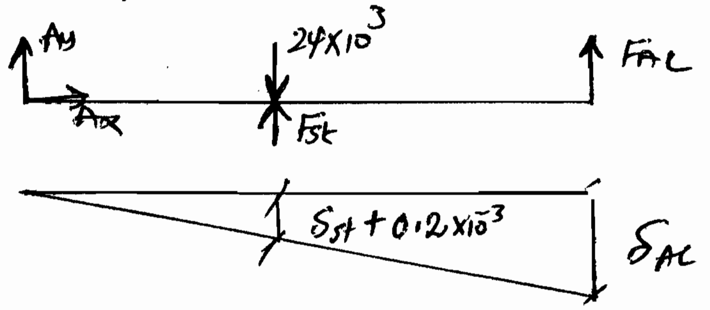
(1) $\delta_c = \dfrac{F_{AL}\, \ell}{E_{AL} A_{AL}} = \dfrac{(8000)(0.5)}{(70 \times 10)^9 (50 \times 10^{-6})} = 1.142$ mm

(1) $\delta_B = \delta_c \left(\dfrac{.5}{1.5}\right) = 0.38$ mm > 0.2 mm

∴ The problem is statically indeterminate as the gap closes

(II) Now treat the problem as statically indeterminate problem

(i) FBD



(δ – in m)

Equilibrium Eq.

(5) $\sum M_{@A}=0 \;+\!\uparrow \qquad 1.5\,F_{AL}+F_{st}(0.5)=24\times10^{3}(0.5)$ —— (A)

Compatibility Equation

(6) $\dfrac{\delta_{st}+0.2\times10^{-3}}{0.5}=\dfrac{\delta_{AL}}{1.5} \Rightarrow 3\delta_{st}+0.6\times10^{-3}=\delta_{AL}$

$$3\,\frac{F_{st}(0.5)}{(200\times10^{9})(50\times10^{-6})}+0.6\times10^{-3}=\frac{F_{AL}(0.5)}{(70\times10^{9})(50\times10^{-6})}$$

$$1.5\,F_{st}+.6\times10^{4}=1.429\,F_{AL} \quad \text{— B}$$

Solving (A) & B $\Rightarrow$

(1) $F_{st}=2747\ N$

$F_{AL}=7084\ N$

(2) (a) stress in Aluminium $=\dfrac{F_{AL}}{A_{AL}}=\dfrac{7084}{50\times10^{-6}}=142\ MPa$

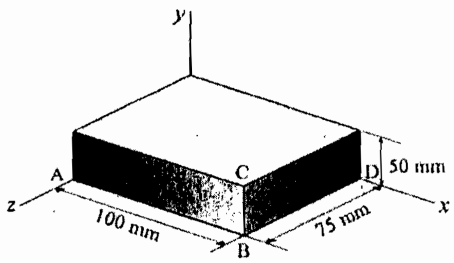
(2) (b) Displacement of point C $=\dfrac{F_{AL}\,l_{AL}}{E_{AL}\,A_{AL}}=\dfrac{(7084)(0.5)}{(70\times10^{9})(50\times10^{-6})}$

$$\delta_c=1.012\times10^{-3}\ m$$
$$=1.012\ mm$$

Problem 5: (20 points)

The steel block shown is subjected to a uniform pressure p on all the faces. Knowing that the change in length of edge AB is -30×10^{-3} mm and using E = 200 GPa, and G = 75 GPa, determine the followings:

(8) a) The magnitude of the applied pressure, p.
(3) b) The strains in the x, y, and z directions.
(6) c) The new length of AB, CB, and BD after the application of the uniform pressure p.
(3) d) The change in volume, using any approach.



Initial Dimensions

Solution

(a)

$$\varepsilon_x = \frac{(\Delta L)_{AB}}{L_{AB}} = \frac{-30\times10^{-3}}{100} = -3\times10^{-4}\ mm/mm \quad (2)$$

$$\varepsilon_x = -3\times10^{-4} = \frac{1}{200\times10^9}\left[-p - 0.333(-p-p)\right]$$

$$\boxed{p = 179.64\ MPa} \quad (4)\ \text{compression}$$

(b)

$$\boxed{\varepsilon_x = -3\times10^{-4}} \quad (1)$$

$$G = \frac{E}{2(1+\nu)}, \quad 75\times10^9 = \frac{200\times10^9}{2(1+\nu)} \Rightarrow \nu = 0.333 \quad [2]$$

$$\varepsilon_y = \frac{1}{200\times10^9}\left[-179.64\times10^6 - 0.333(-2 * 179.64\times10^6)\right]$$

$$\boxed{\varepsilon_y = -3\times10^{-4}\ mm/mm} \quad (1)$$

similarly $\Rightarrow$ $\boxed{\varepsilon_z = -3\times10^{-4}\ mm/mm}$ [2]

(c)

$$(L_{AB})_{new} = (-30\times10^{-3}) + 100 = \boxed{99.97\ mm} \quad [2]$$

$$(L_{CB})_{new} = (50 * -3\times10^{-4}) + 50 = \boxed{49.985\ mm} \quad [2]$$

$$(L_{BD})_{new} = (75 * -3\times10^{-4}) + 75 = \boxed{74.9775\ mm} \quad [2]$$

(d) change in Volume $= \Delta V =$

$$(99.97)(49.985)(74.9775) - (100)(50)(75) =$$

$$\boxed{\Delta V = -337.40\ mm^3} \quad [3]$$

$-337.4\ mm^3$

or

$$e = \frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\frac{\Delta V}{375000} = 3(-3\times10^{-4})$$

$$\Delta V = -337.5\ mm^3$$