King Fahd University of Petroleum & Minerals
DEPARTMENT OF CIVIL ENGINEERING
First Semester 1433-34 / 2012-13 (121)

CE 203 STRUCTURAL MECHANICS I
Major Exam II

Tuesday, December 4, 2012 7:00-9:30 P.M.

KEY SOLUTION

Problem	Graded by
1	Mohammad Al-Suwaiyan
2	Salah Al-Dulaijan
3	Hamdan AL-Ghamedy
4	Shamshad Ahmad
5	Saeid AL-Ghamdi

Notes:

1. **A sheet that includes selected Basic Formulae and definitions is provided with this examination.**
2. **Write clearly and show all calculations, FBDs, and units.**

Note to Students:

Even though the course is not "standard grading", being around the average does not indicate C performance, since there is a minimum amount of course comprehension needed to pass the course satisfactorily, irrespective of the exam average and the performance of other students.
Therefore, students who did poorly in this exam should do double effort in the remaining of the semester to avoid disappointing grade.

After reviewing the key solution and still having a concern about your mark, you may consult with the faculty members who prepared each problem.

The deadline for review is Monday December 17, 2012.

Problem 1: (20 points)

The shown shaft which is made from a circular steel tube AB and a solid circular brass shaft BC, is subjected to torques at B and C as shown.

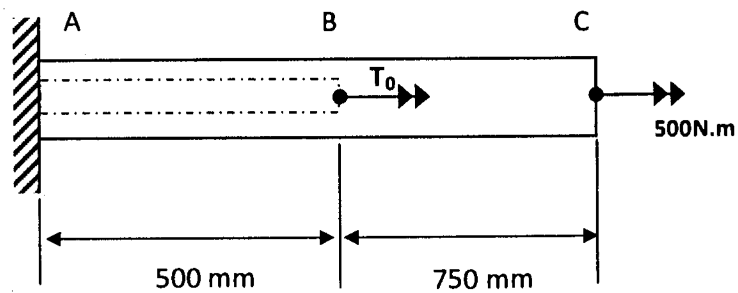
a) Determine the maximum value for T_0 if the allowable shear stress for both materials is 150MPa and the allowable angle of twist at B is 4°.
b) Using the results obtained above, determine the rotation of end C.
c) Calculate the maximum shear stress in brass.
d) Calculate $\Phi_{B/C}$

For steel (AB)
d_{out} = 30 mm
d_{in} = 20 mm
G_{steel} = 80 GPa

For brass (BC)
d = 30 mm
G_{brass} = 40 GPa

$\Sigma M = 0$ $\quad T_{BC} = 500$ N.m

$\Sigma M = 0$ $\quad T_{AB} = T_o + 500$



$J_{AB} = \frac{\pi}{2}(15^4 - 10^4) = 6.38 \times 10^4 \ mm^4$

$J_{BC} = \frac{\pi}{2}(15)^4 = 7.95 \times 10^4 \ mm^4$

* If τ controls

(a) $\tau_{max_{AB}} = \frac{T_{AB}\, C}{J_{AB}} \le \tau_{allowable}$

$\frac{10^3 (500 + T_0)(15)}{\frac{\pi}{2}(15^4 - 10^4)} = 150$ (4)

$T_o = 138$ N.m

Note that the stress in the brass will not depend on To and is less that τ allowable.

* If φ_B controls

$\varphi_B = \frac{T_{AB} L_{AB}}{J_{AB} G_{AB}} \le \varphi_{allowable}$ (4)

$\frac{(T_o + 500)(500)}{\frac{\pi}{2}(15^2 - 10^2)(80)} = 4 \times \frac{\pi}{180}$

$T_o = 213$ N.m

∴ shearstress controls & max value of $T_o = 138$ N.m ⇐ Ans

(b) $\varphi_c = \varphi_{A/C} = \frac{T_{AB} L_{AB}}{J_{AB} G_{AB}} + \frac{T_{BC} L_{BC}}{J_{BC} G_{BC}}$ (4)

$= \frac{638(500)}{6.38 \times 10^4 (80)} + \frac{500(750)}{7.95 \times 10^4 (40)}$

$\varphi_c = 0.0625 + 0.118 = 0.18$ rad $= 10.3°$ Ans

(c) $\tau_{max_{BC}} = \frac{T_{BC}\, C}{J_{BC}}$ (4)

$= \frac{500 \times 10^3 \times 15}{\frac{\pi}{2}(15)^4} = 94.3$ MPa Ans

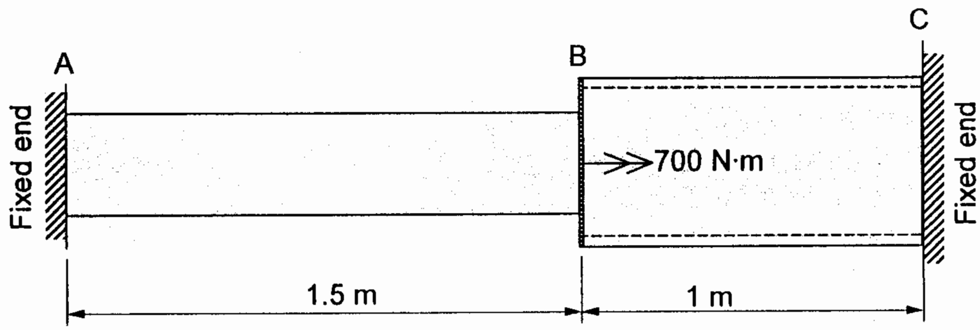
(d) $\varphi_{BC} = \frac{T_{BC} L_{BC}}{J_{BC} G_{BC}} = 0.118$ rad $= 6.8°$ (4) Ans

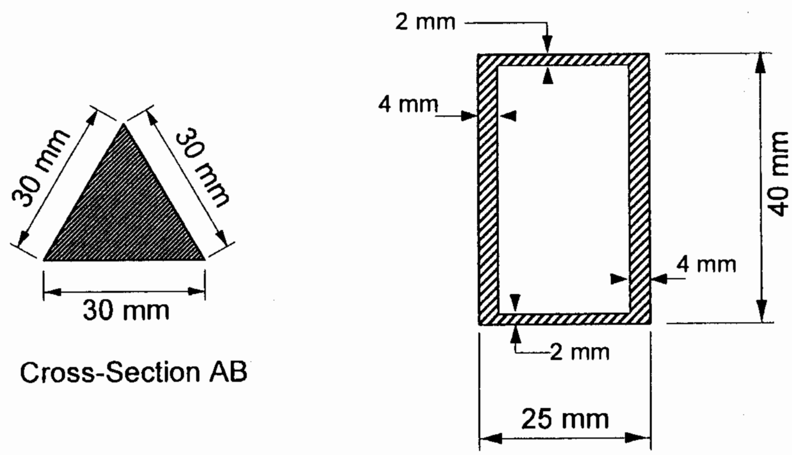
Problem 2: (20 points)

The steel shaft is made from two segments: AB is a solid triangular section, and BC is a thin tube.

a) Determine the reactions at A and C.
b) Determine the maximum shear stress in the whole shaft and indicate its location.
c) Determine the angle of twist of B.

$G_{steel} = 75$ GPa





Cross-Section AB

Cross-Section BC

(a) Reactions at A & C

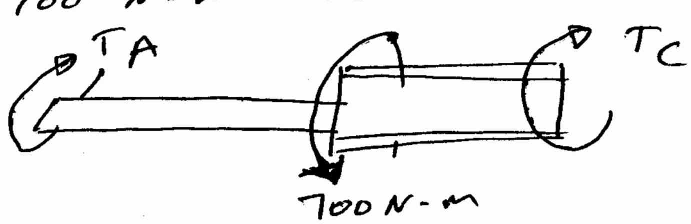
This problem is statically indeterminate;

[1] From equilibrium

$A_m = 7.98 \times 10^{-4}$ m [1]

$T_A + T_C = 700$ N·m —[1]

[2]



[2] From compatibility

$\Sigma\phi = 0$ [2]

$\phi_{A/B} + \phi_{B/C} = 0$ [1] [1] [1] [1]

$$\frac{46\,T_A\,(1.5)}{(0.03)^4(75\times10^9)} + \frac{-T_C\,(1)}{4\left[0.021\times0.038\right]^2\left[75\times10^9\right]}\left[2\,\frac{21}{2} + 2\,\frac{38}{4}\right.$$

$1.13580 \times 10^{-3}\,T_A - 2.093789 \times 10^{-4}\,T_C = 0$ —(2)

(b) Max. shear stress

in shaft AB

[2]

$$\tau_{AB} = \frac{20\,(109)}{(0.03)^3} = 80.74 \text{ MPa}$$

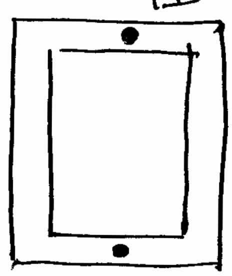
Location of shear max in AB.

in shaft BC

$$\tau_{BC} = \frac{591}{2\,[(.021)(.038)]\,(0.002)} = 185.15 \text{ MPa}$$

[2]

[1]



location for τ_{max} in BC

(C) Angle of twist of B

$$\frac{46\,(109)\,(1.5)}{(0.03)^4\,(75 \times 109)} = 0.124 \text{ rad}$$

[4]

or

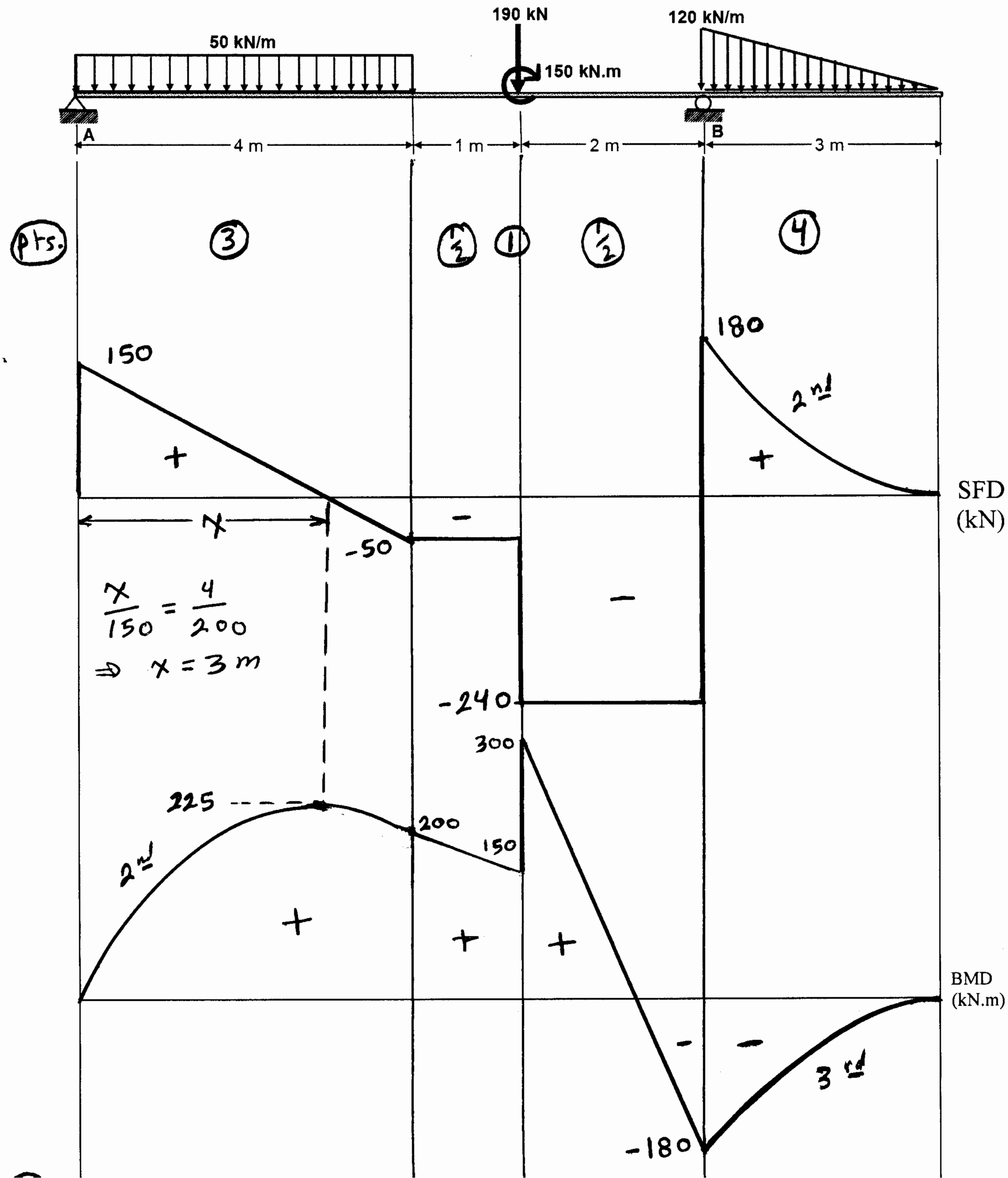
$$\frac{(591)(1)}{4\,[(0.021)(0.038)]^2\,(75 \times 109)} \left[2\,\frac{21}{2} + 2\,\frac{38}{4}\right]$$

$$= 0.124 \text{ rad}$$

Problem 3 (20 pts.)

Draw the **shear force and bending moment diagrams** for the beam shown below using the ***summation (graphical) method***. *Write the degree (2, 3, etc.) of the curve on each one.* Put the values on the diagrams, but you do NOT need to show the calculations. *Use appropriate scale. **No credit will be given if another method is used.***

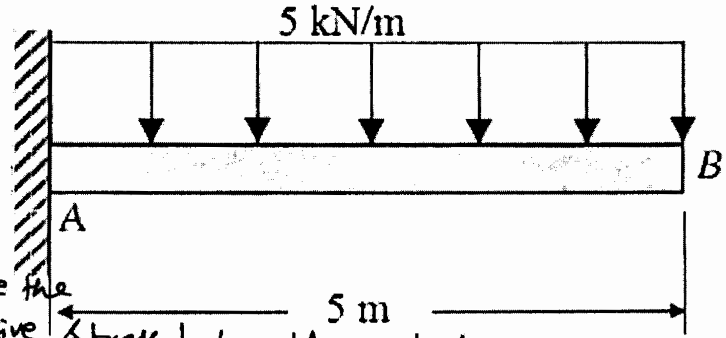
The reactions are: A_y = 150 kN ↑ ; B_y = 420 kN ↑



Problem 4: (20 points)

A cantilever beam along with its cross-section is shown in the figure below. Determine the following at point *A* of the beam:

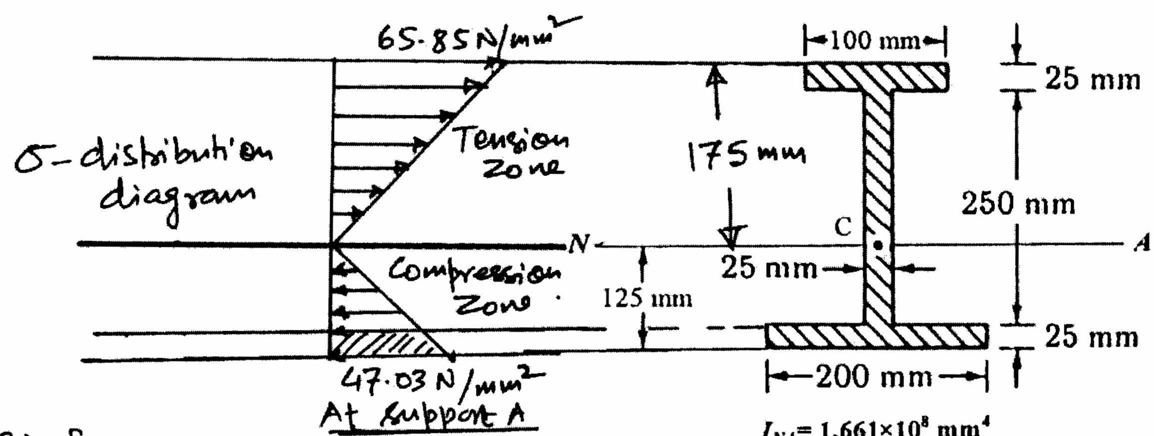
a) Maximum tensile stress and its location
b) Maximum compressive stress and its location
c) Normal stress distribution along the depth of the cross-section
d) Magnitude of the normal force on the lower flange.



M at support 'A':

$M_A = 5 \times 5 \times 2.5$
$= 62.5$ kN-m (↻)

The moment will cause tensile stress above the neutral axis and compressive stress below the neutral axis.



(a) $[\sigma_{max,t}]_A = \dfrac{M_A C_{top}}{I_{NA}} = \dfrac{62.5 \times 10^6 \times 175}{1.661 \times 10^8} = 65.85 \text{ N/mm}^2 \text{ (T)}$

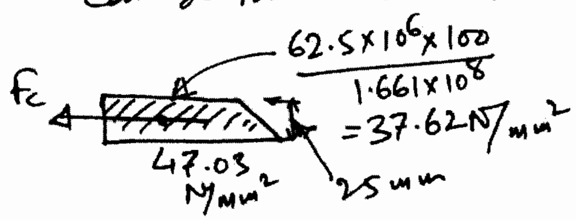
(Location: top most point of the section) Ans

(b) $[\sigma_{max,c}]_A = \dfrac{M_A C_{bottom}}{I_{NA}}$

$= \dfrac{62.5 \times 10^6 \times 125}{1.661 \times 10^8} = 47.03 \text{ N/mm}^2$ (C) Ans (Location: bottom most point of the section).

(c) Normal stress distribution diagram shown above.

(d) From σ-distribution diagram, the stress over lower flange can be shown below:



$F_c = \left[\dfrac{47.03 + 37.62}{2}\right] \times 25 \times 200$

$= 211625$ N

$= 211.62$ kN Ans

Problem 5: (20 points)

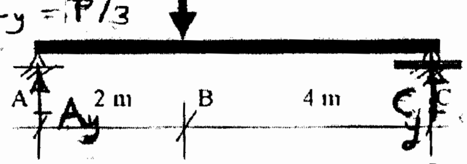
The beam ABC shown with details of the cross-section given below is made from a material whose ultimate shear strength is 450 MPa.

(5%) a) Show that the centroidal values for the cross section are:
$\bar{y} =$ 0.4233 m , and $I_{CA} =$ 1.2818 x 10^{-2} m^4.

(10%) b) Using a factor of safety (FS) value of 1.5 and the centroidal values given in part a, determine the allowable value for load P.

(5%) c) Sketch (*qualitatively*) the shear-stress distribution τ(y) along the beam-depth at the location of maximum shear force. On the diagram clearly indicate the maximum and minimum values.

From statics:
$A_y = 2P/3$
$C_y = P/3$

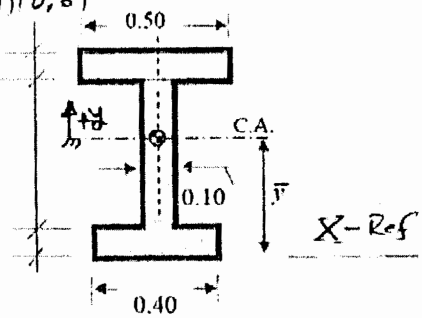


a) $\bar{y} = \dfrac{0.1\times0.4\times0.05 + 0.1\times0.6\times0.4 + 0.1\times0.5\times0.75}{0.040 + 0.060 + 0.050}$

$\therefore \bar{y} = 0.4233$ m (3%)

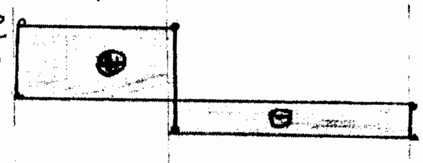
$I_{CA} = \Sigma \bar{I}_e + \Sigma A_e d_{ey}^2$

$\therefore I_{CA} = \frac{1}{12}(0.5)(0.1)^3 + \frac{1}{12}(0.40)(0.1)^3 + \frac{1}{12}(0.1)(0.6)^3$
$+ 0.05(0.75 - 0.4233)^2$
$+ 0.06(0.4 - 0.4233)^2$
$+ 0.04(0.4233 - 0.05)^2$
$= 1.2818 \times 10^{-2}$ m^4 (2%)



Cross-section (units: in *m*)

b) $\tau_{all} = 450/FS = 300$ MPa



$\Rightarrow V_{max} = 2P/3$ (3%)

$\therefore \tau_{max} = \dfrac{V_{max} Q}{I_{CA} B}$ where $Q = Q_{CA}$ (1%)

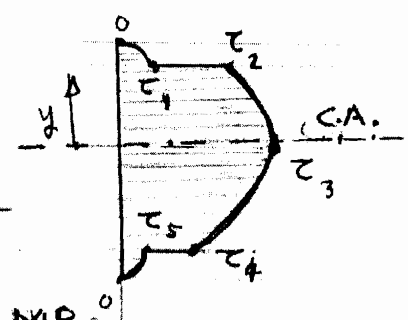
$Q_{CA} = 0.5\times0.1\times0.3267 + 0.2767\times0.1\times\frac{0.2767}{2} = 0.02016$ m^3 (3%)

Setting $\tau_{max} = \tau_{all} \Rightarrow 2P/3 = \dfrac{\tau_{all} \times I_{CA} B}{Q_{CA}} = 19.1$ MN

$\therefore P_{all} = 3/2 \times 19.10 = 28.6$ MN (2%)

c) Stress distribution τ(y):

$\tau_{min} = 0$; $\tau_{max} = \tau_3$ (2%)



$\tau_{max} = \dfrac{V_{max} Q_{max}}{I_{CA} B} = \dfrac{2/3 P_{all} \times 0.02016}{1.2818\times10^{-2}\times0.10}$ (3%)

$= \dfrac{19.07 \times 0.02016}{1.2818\times10^{-2}\times0.10} = 300$ MPa