Problem 1:

Problem 1:

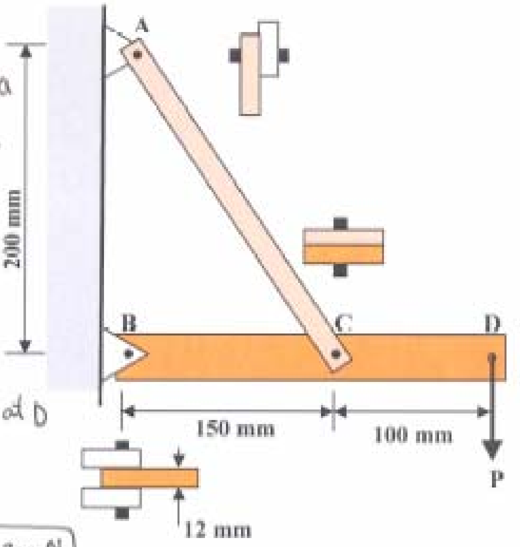
- Link AC (shown in the Figure below) has 125 MPa allowable normal stress with square cross section area of 12X12 mm. Each of the pins at A, C, and B has a diameter of 8 mm, and the rigid member BCD has a thickness of 12 mm. Determine the following:
  a) The largest load P which may be applied at D.
  b) The average shear stress in each of the pins at A and B.
  c) The bearing stress in the link at C.

Solution:

a) Draw F.B.D.

Using the allowable normal stress of 125 MPa

$\Rightarrow F_{AC} = 125 \times 10^6 \frac{N}{m^2} \times (0.012m)^2 = 18000\,N = 18\,KN.$



$+\circlearrowleft \Sigma M @ B = 0$

$P \cdot (250) - F_{AC} \cdot \left(\frac{200}{250}\right) \cdot 150 = 0$

$\Rightarrow P = \frac{12}{25} F_{AC} = \underline{8640\ N}.$

This is the largest load that may be applied at D

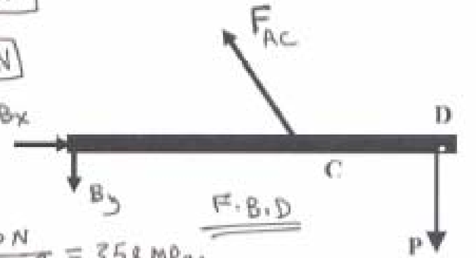
b) The reactions at B:

$\xrightarrow{+} \Sigma F_x = 0;\ +B_x - 18 \times 10^3 N \cdot \left(\frac{150}{250}\right) = 0$

$\Rightarrow \boxed{B_x = 10,800\ N}$

$+\uparrow \Sigma F_y = 0;\ -B_y + 18 \times 10^3 N \cdot \left(\frac{200}{250}\right) - 8640 = 0$

$\Rightarrow \boxed{B_y = 5760\ N}$



$\Rightarrow$ At B, Resultant Force, $F_B = \sqrt{B_x^2 + B_y^2}$

$\boxed{F_B = 12240\ N}$

$\therefore \tau_{Avg} @ A$:

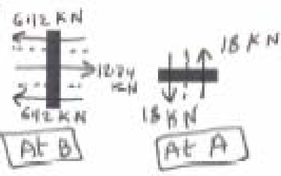
It is single shear $\Rightarrow (\tau_A)_{Avg} = \frac{18000\ N}{\frac{\pi}{4}(0.008)^2} = \underline{358\ MPa}.$

$\therefore \tau_{avg} @ B$:

It is double shear $\Rightarrow$ Shear force $= \frac{F_B}{2} = \frac{12.24\,KN}{2} = 6.12\,KN.$

$\Rightarrow (\tau_B)_{avg} = \frac{6.12 \times 10^3\ N}{\frac{\pi}{4}(0.008m)^2} = 121,757 \times 10^3\ N/m^2$

$= \underline{121.76\ MPa}.$

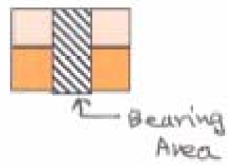


c) The bearing Stress @ C:

The bearing area $= 0.008m \times 0.012m = 96 \times 10^{-6}\ m^2.$

$\Rightarrow$ The bearing Stress @ C $= \frac{18 \times 10^3\ N}{96 \times 10^{-6}\ m^2} = 187.5 \times 10^6\ Pa$

$= \underline{187.5\ MPa}.$



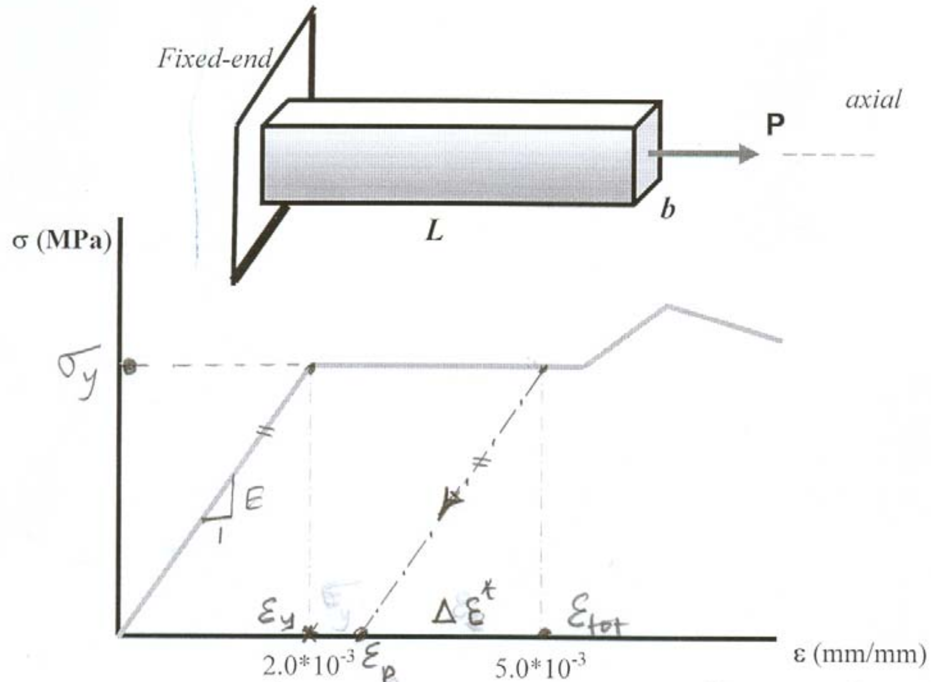
Problem 2:

A bar with square-cross section is made from a material with Young's modulus E = 80 GPa, Poisson's ratio $\nu = 0.30$ and has the $\sigma-\varepsilon$ diagram shown in the figure given below. Initial dimensions of the bar are $\boldsymbol{L} = 100$ cm, and $\boldsymbol{b} = 5$ cm. If the bar is fixed at the support and is subjected to an axial load **P** = 300 kN.

a) **Compute** the value of yield stress σ_y.

b) **Determine** the new dimension $\boldsymbol{b}$ under the *given* load.

c) If it is known to you that under a larger value of load **P** the axial strain $\varepsilon_a = 5.0*10^{-3}$ mm/mm, and the load is then removed completely, **determine** the final length $\boldsymbol{L}$ of the bar upon complete removal of load.



a) $\sigma_y/\varepsilon_y = E/1 \Rightarrow \sigma_y = \varepsilon_y E = 2\times10^{-3} \times 80\times10^{3}\ \text{MPa} = 160\ \text{MPa}.$

b) $\sigma_a = N/A_0 = P/A_0 = 300\ \text{kN}/(0.05)^2 = 120\ \text{MPa} < \sigma_y$

$\varepsilon_a = \sigma_a/E = 120\ \text{MPa}/80\times10^{3}\ \text{MPa} = 1.5\times10^{-3}\ \text{cm/cm}$

$\varepsilon_l = -\nu\,\varepsilon_a = -4.5\times10^{-4}\ \text{cm/cm}$

$b_f = b_0(1+\varepsilon_l) = 5(1-4.5\times10^{-4}) = 4.9978\ \text{cm}$

c) $\Delta\varepsilon^* = \varepsilon_y \Rightarrow \varepsilon_p = \varepsilon_{tot} - \Delta\varepsilon^* = 5\times10^{-3} - 2\times10^{-3}$

$\varepsilon_p = 3\times10^{-3}\ \text{cm/cm} \Rightarrow L_f = L_0(1+\varepsilon_p) = 100.3\ \text{cm}.$

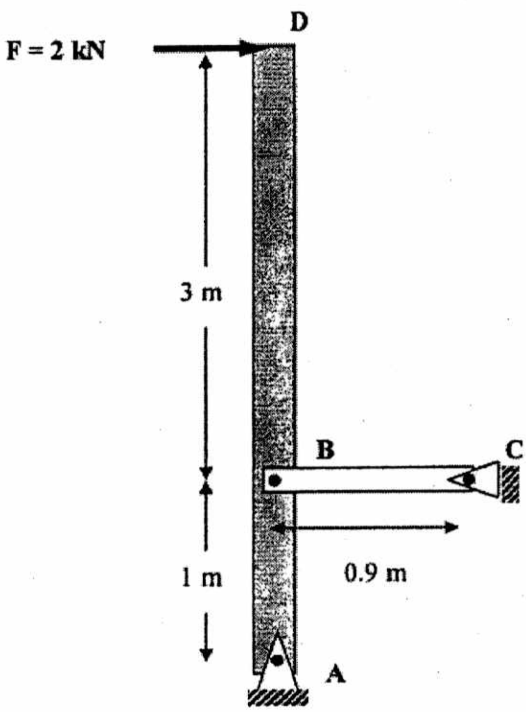
Problem 3:

Rigid member ABD (shown in the figure given below) is supported by steel link BC and pin A. Before application of the load member ABD was vertical. If the steel temperature is raised by $\Delta T = 35^{\circ}C$ and a force ($F = 2$ kN) is applied as shown:

a) Calculate the stress in the steel link.
b) Calculate the horizontal displacement of point D.

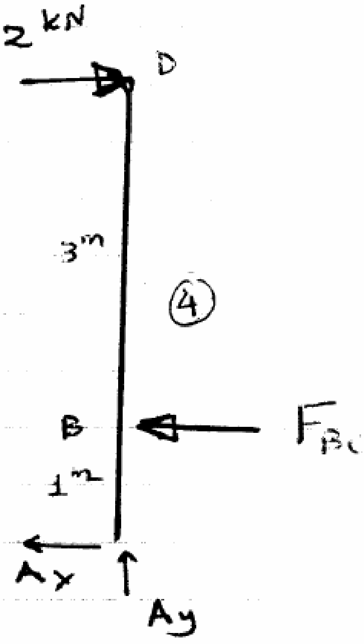
Given:

$E_{steel} = 200$ GPa; $\alpha_{steel} = 12 \times 10^{-6}/^{\circ}C$; diameter of steel link (BC) = 10 mm



Problem 3:

Determine force in link
using FBD for rigid member



$\curvearrowleft \Sigma M_A = 0$

$$2 \times (3+1) - F_{BC}(1) = 0$$

$$F_{BC} = 8^{kN} \quad 2$$

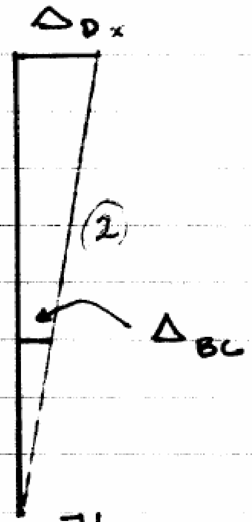
$$\because \quad \sigma_{steel} = \frac{F_{BC}}{A} \; 2 = \frac{8 \times 10^3}{\frac{\pi}{4}(10)^2}$$

$$\sigma_{steel} = 102 \text{ MPa} \quad 2$$

From shape after load & T application

$$\frac{\Delta_{Dx}}{4} \overset{1}{=} \frac{\Delta_{BC}}{1}$$

steel due to load + Temp.



$$\because \quad \Delta_{Bx} = 4 \left| \Delta_{steel} \right|$$

$$= 4 \left| \left[\overset{2}{\frac{F_{BC} L_{BC}}{EA}} + \overset{2}{\alpha_{st} L_{BC} \Delta T} \right] \right|$$

$$= 4 \left| \left[\frac{-8 \times 900}{\frac{\pi}{4}(10)^2 * 200} + 12 \times 10^{-6} \times 900 * 35 \right] \right|$$

$$= 4 \left| \left[-0.458 + 0.378 \right] \right| = 4(.08) = 0.32^{mm} \rightarrow$$

2

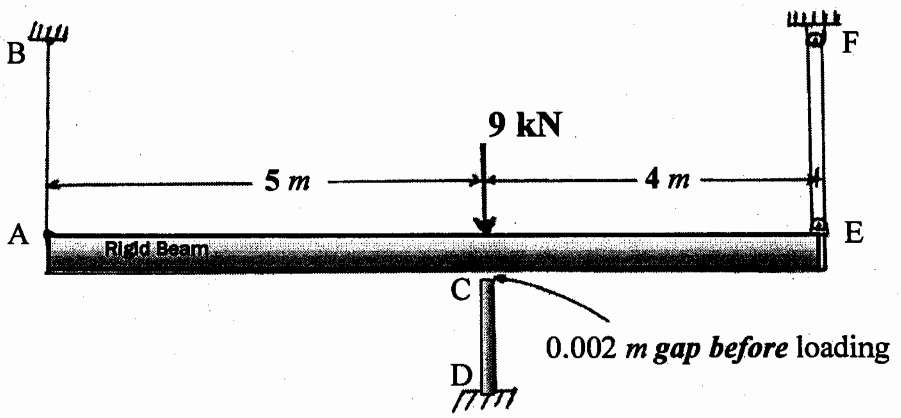
Problem 4: (20 pts.)

Determine the stress in member CD shown.

Detailed Solution

Not expected from students!

	L(m)	A (m^2)	E (GPa)
Cable **AB**	2	$4(10)^{-6}$	250
Post **CD**	1.5	$6(10)^{-6}$	100
Rod **EF**	1.8	$9(10)^{-6}$	200



First, check if the gap closes.

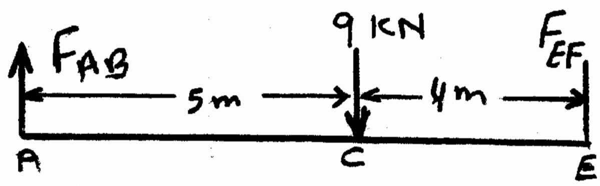
If it closes, then the problem is statically indeterminate (SI) and we need to use the geometric compatibility. If not, the problem is statically determinate and there is no need to use geom. comp. as "Statics" is enough to solve the prob.

Assume the gap does not close. Thus, the load is carried by AB and EF only, and CD carries no load ($\sigma = 0$).

(pts.)

①

From the FBD



$\circlearrowleft + \sum M_E = 0 \Rightarrow$

$$9(4) - F_{AB}(9) = 0 \Rightarrow F_{AB} = 4 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0 \Rightarrow 4 - 9 + F_{EF} = 0 \Rightarrow F_{EF} = 5 \text{ kN}$$

Now, check the elongations of AB and EF

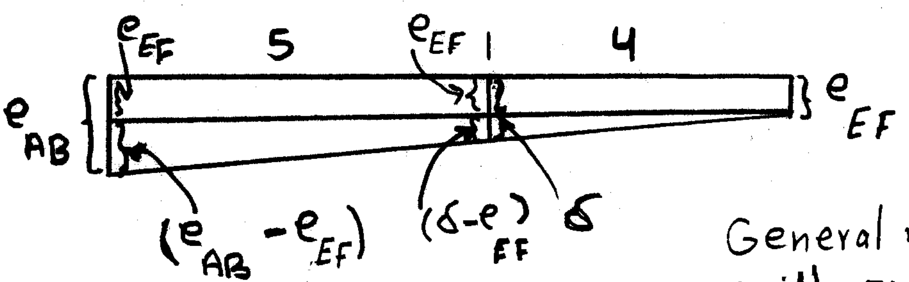
$$e = \frac{PL}{AE} \Rightarrow e_{AB} = \frac{4(10)^3(2)}{4(10)^{-6}\; 250(10)^9} = 0.008 \text{ m}$$

$$(8 \times 10^{-3}) \text{ m}$$

$$e_{EF} = \frac{5(10)^3(1.8)}{9(10)^{-6}\; 200(10)^9} = 0.005 \text{ m}$$

Now, we draw the geometry and check the gap.

not to scale



General method with any value for the gap

Using similar triangles,

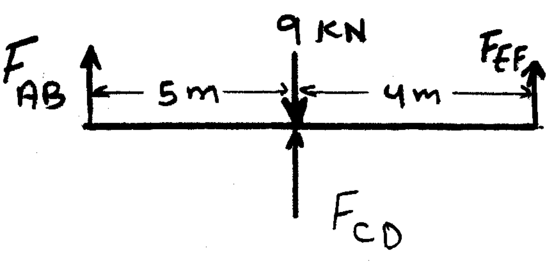
$$\frac{\delta - e_{EF}}{4} = \frac{e_{AB} - e_{EF}}{(4+5)} \Rightarrow$$

4 $$\frac{\delta - 0.005}{4} = \frac{0.008 - 0.005}{9} \Rightarrow \delta = 0.006333 \text{ m}$$

$$> \text{gap} = 0.002 \text{ m}$$

$\Rightarrow$ The gap closes, and the prob. is SI. $\Rightarrow$

⊗ Equilibrium:



2 From the FBD

$$+\uparrow \Sigma F_y = 0 \Rightarrow$$

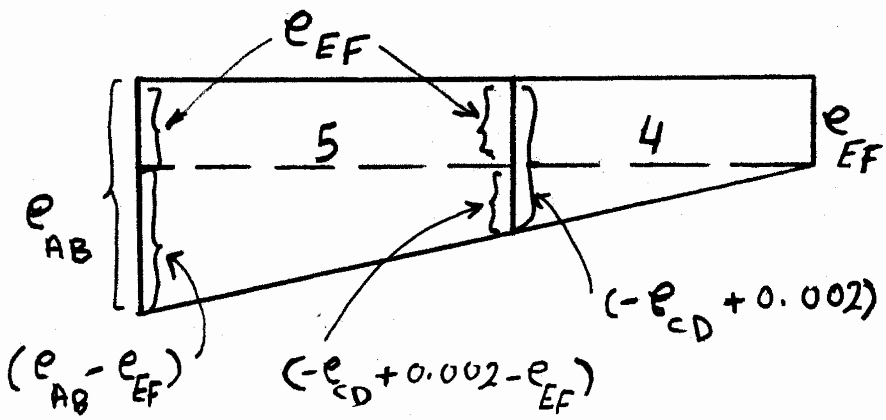
1 $$F_{AB} + F_{CD} + F_{EF} - 9(10)^3 = 0 \quad (1)$$

Note that F_{CD} is assumed "C" since we know it will be so. However, it may be assumed "T" as you will see in the other method.

* Geometric Compatibility:

not to scale

3



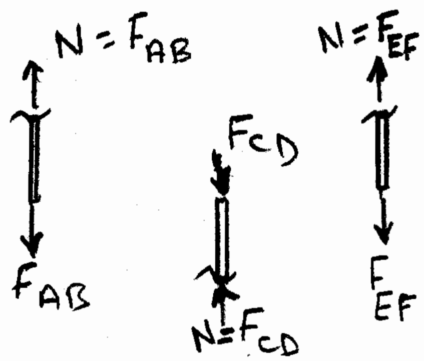
Using similar triangles

3 $$\frac{(-e_{CD}+0.002-e_{EF})}{4}=\frac{e_{AB}-e_{EF}}{9} \quad (2)$$

Note that minus (-) sign is used in e_{CD} as CD shrunk (shortened) (compared with AB and EF).

* Material Behavior:

$$\delta = e = \frac{PL}{AE} \quad (3)$$



From eq. (3) into (2)

$$(-e_{CD}+0.002-e_{EF})=\frac{4}{9}(e_{AB}-e_{EF})$$

$$-\frac{-F_{CD}(1.5)}{6(10)^{-6}100(10)^9}+0.002-\frac{F_{EF}(1.8)}{9(10)^{-6}200(10)^9}=\frac{4}{9}\left[\frac{F_{AB}(2)}{4(10)^{-6}250(10)^9}-\frac{F_{EF}(1.8)}{9(10)^{-6}200(10)^9}\right]$$

$\Rightarrow\ 2.5(10)^{-6} F_{CD} + 0.002 - 8.88889(10)^{-7} F_{AB} - 5.55556(10)^{-7} F_{EF} = 0$

2 or $25 F_{CD} - 8.88889 F_{AB} - 5.55556 F_{EF} + 20{,}000 = 0$ (4)

Now we have two eqs. (# 1 and 4) and three unknowns (F_{AB}, F_{CD}, and F_{EF}). The moment equation will be utlized. $\Rightarrow$

$\overset{+}{\curvearrowleft} \Sigma M_c = 0 \Rightarrow$

1 $4 F_{EF} - 5 F_{AB} = 0 \Rightarrow F_{EF} = 1.25 F_{AB}$ (5)

From eq. 5 into 1, $F_{AB} + F_{CD} + 1.25 F_{AB} - 9(10)^3 = 0$

$\Rightarrow\ F_{AB} = 4000 - 0.44444 F_{CD}$ (6)

From 6 into 5,

$F_{EF} = 5000 - 0.55556 F_{CD}$ (7)

From eqs. 6 & 7 into 4,

$25 F_{CD} - 35{,}556 + 3.9506 F_{CD} - 27{,}778 + 3.0864 F_{CD} + 20{,}000 = 0$

2 $\Rightarrow\ F_{CD} = \dfrac{43{,}334}{32.037} \Rightarrow F_{CD} = 1{,}353$ N $= 1.353$ kN as shown (CD)

($F_{AB} \simeq 3.4$ kN ; $F_{EF} \simeq 4.25$) ← not required

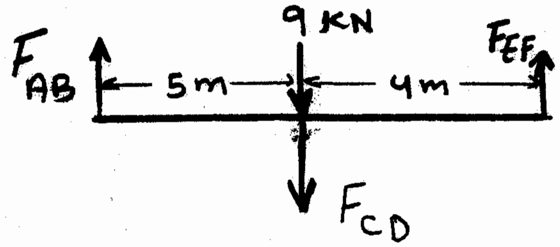
1 $\sigma_{CD} = \dfrac{F_{CD}}{A} = \dfrac{-1{,}353}{6(10)^{-6}} \Rightarrow \boxed{\sigma_{CD} = 225.4 \text{ MPa "C"}}$

Are the values of the forces reasonable ?!

If there is no gap, how will this affect F_{CD} and other forces?!

Another Method: Assume F_{CD} is "T"

Equilibrium:

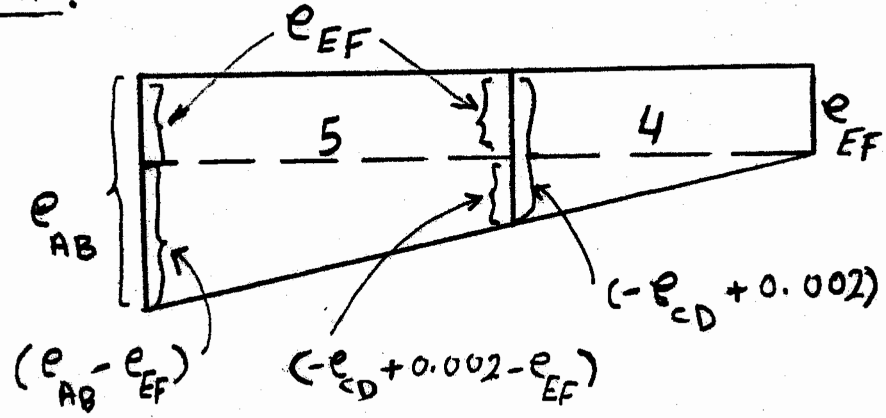


From the FBD

$$+\uparrow \sum F_y = 0 \Rightarrow$$

$$F_{AB} + F_{CD} + F_{EF} - 9(10)^3 = 0 \quad (1)$$

Geom. Compat.:



drawing not to scale

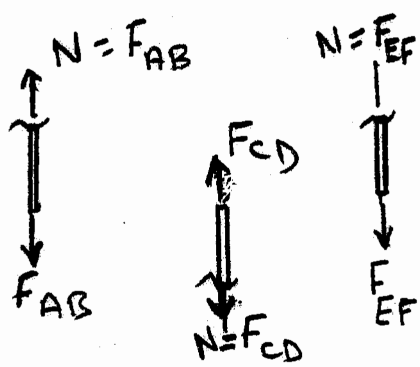
Using similar triangles

$$\frac{(-e_{CD} + 0.002 - e_{EF})}{4} = \frac{e_{AB} - e_{EF}}{9} \quad (2)$$

Note that minus (-) sign is used in e_{CD} as CD shrunk (got shorter) (compared with AB and EF).

Material Behavior:

$$\delta = e = \frac{PL}{AE} \quad (3)$$



From eq. (3) into (2)

$$(-e_{CD} + 0.002 - e_{EF}) = \frac{4}{9}(e_{AB} - e_{EF})$$

$$-\frac{F_{CD}(1.5)}{6(10)^{-6}\,100(10)^9} + 0.002 - \frac{F_{EF}(1.8)}{9(10)^{-6}\,200(10)^9} = \frac{4}{9}\left[\frac{F_{AB}(2)}{4(10)^{-6}\,250(10)^9} - \frac{F_{EF}(1.8)}{9(10)^{-6}\,200(10)^9}\right]$$

$$\Rightarrow -25(10)^{-6} F_{CD} + 0.002 - 8.88889(10)^{-7} F_{AB} - 5.55556(10)^{-7} F_{EF} = 0$$

or $-25\, F_{CD} - 8.88889\, F_{AB} - 5.55556\, F_{EF} + 20{,}000 = 0$ ④

Now we have two eqs. (# ① and ④) and three unknowns (F_{AB}, F_{CD}, and F_{EF}). The moment equation will be utlized. ⇒

$$+\circlearrowleft \Sigma M_c = 0 \Rightarrow$$

$$4F_{EF} - 5F_{AB} = 0 \Rightarrow F_{EF} = 1.25\, F_{AB} \quad ⑤$$

From eq. ⑤ into ①, $F_{AB} - F_{CD} + 1.25 F_{AB} - 9(10)^3 = 0$

$$\Rightarrow F_{AB} = 4000 + 0.44444\, F_{CD} \quad ⑥$$

From ⑥ into ⑤,

$$F_{EF} = 5000 + 0.55556\, F_{CD} \quad ⑦$$

From eqs. ⑥ & ⑦ into ④,

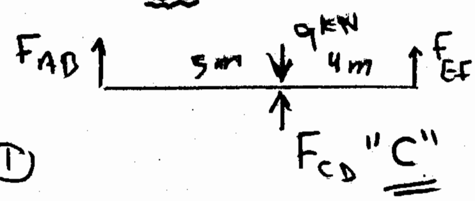
$$-25\, F_{CD} - 35{,}556 + 3.9506\, F_{CD} - 27{,}778 - 3.0864 F_{CD} + 20{,}000 = 0$$

$$\Rightarrow F_{CD} = \frac{-43{,}334}{32.037} \Rightarrow F_{CD} = -1{,}353 \text{ N} \quad \text{"opp. dir."}$$
$$= 1.353 \text{ kN "C"}$$

($F_{AB} \simeq 3.4$ kN ; $F_{EF} \simeq 4.25$) ⟵ not required

$$\sigma_{CD} = \frac{F_{CD}}{A} = \frac{-1{,}353}{6(10)^{-6}} \Rightarrow \boxed{\sigma_{CD} = 225.4 \text{ MPa}} \text{ "C"}$$

Common Mistake : e_{CD} taken as ⊕ : extension



Equil : $\Sigma F_y = 0$

$$F_{AB} + F_{CD} + F_{EF} - 9{,}000 = 0 \quad (1)$$

Geom. Comp. :

$$-25\, F_{CD} - 8.8889\, F_{AB} - 5.5556\, F_{EF} + 20{,}000 = 0 \quad (2)$$

Equil : $\Sigma M_c = 0$

$$F_{EF} = 1.25\, F_{AB} \quad (3)$$

From (3) into (1),

$$F_{AB} = 4000 - 0.44444 F_{CD} \quad (4)$$

From (4) into (3)

$$F_{EF} = 5000 - 0.55556\, F_{CD} \quad (5)$$

From (4) and (5) into (2),

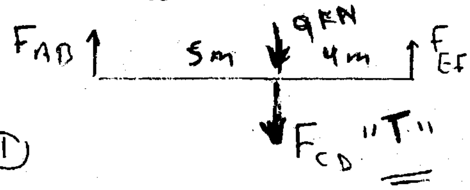
$$-25\, F_{CD} - 35{,}556 + 3.9506\, F_{CD} - 27{,}778 + 3.0864\, F_{CD} + 20{,}000 = 0$$

$$\Rightarrow \quad F_{CD} = -\frac{43334}{17.963} = -2.412 \text{ kN}$$

It means in the opposite direction ⇒ X

$\Rightarrow \sigma = -(-402) = 402$ MPa "T" X

Common Mistake: e_{CD} taken as (+): extension



Equil: $\Sigma F_y = 0$

$$F_{AB} - F_{CD} + F_{EF} - 9{,}000 = 0 \quad (1)$$

Geom. Comp.:

$$25\,F_{CD} - 8.8889\,F_{AB} - 5.5556\,F_{EF} + 20{,}000 = 0 \quad (2)$$

Equil.: $\Sigma M_c = 0$

$$F_{EF} = 1.25\,F_{AB} \quad (3)$$

From (3) into (1),

$$F_{AB} = 4000 + 0.44444 F_{CD} \quad (4)$$

From (4) into (3)

$$F_{EF} = 5000 + 0.55556\,F_{CD} \quad (5)$$

From (4) and (5) into (2),

$$25\,F_{CD} - 35{,}556 - 3.9506\,F_{CD} - 27{,}778 - 3.0864\,F_{CD} + 20{,}000 = 0$$

$$\Rightarrow \quad F_{CD} = \frac{43334}{17.963} = 2.412 \text{ kN}$$

It means it is as shown (T) $\Rightarrow$ X

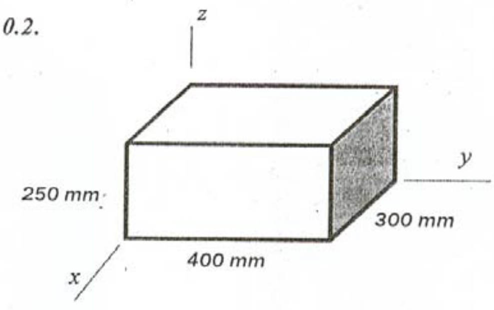
$\Rightarrow \sigma = 402$ MPa (T) X

Problem 5:

A solid block has the initial dimensions shown in the figure (given below). If the block is subjected to **tensile stresses** in the x and y directions equal to 40 MPa, and to **compressive stress** in the z direction equal to 20 MPa:

a) determine the final length of the block in the z direction;
b) calculate the change in volume due to the applied stresses (*use either the exact or approximate method*).

Given: $E = 10\ GPa$, **and** $\nu = 0.2$.



$\sigma_x = +40$ MPa

$\sigma_y = +40$ MPa

$\sigma_z = -20$ MPa

a) $\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] = \frac{1}{10 \times 10^3}\left[-20 - (.2)(40+40)\right]$

$\epsilon_z = -3.6 \times 10^{-3}$

$\epsilon_z = \frac{(\Delta l)_z}{l_z}$, $(\Delta l)_z = (-3.6 \times 10^{-3})(250) = -0.9$ mm

$\therefore$ final length in z-dir $= 250 - .9 = \boxed{249.1 \text{ mm}}$

b) $\epsilon_x = \frac{1}{10 \times 10^3}\left[40 - (.2)(40-20)\right] = +3.6 \times 10^{-3}$

$\epsilon_y = \frac{1}{10 \times 10^3}\left[40 - (.2)(40-20)\right] = +3.6 \times 10^{-3}$

$e \simeq \epsilon_x + \epsilon_y + \epsilon_z = +3.6 \times 10^{-3} = \frac{\Delta V}{V}$

$\therefore \Delta V = (e)(\text{Volume})$

$= (3.6 \times 10^{-3})\left[400 \times 300 \times 250\right] = \boxed{+108000 \text{ mm}^3}$

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OR: you can calculate final volume and subtract from it the initial volume. (more accurate but longer)