General States of Stress and Strain

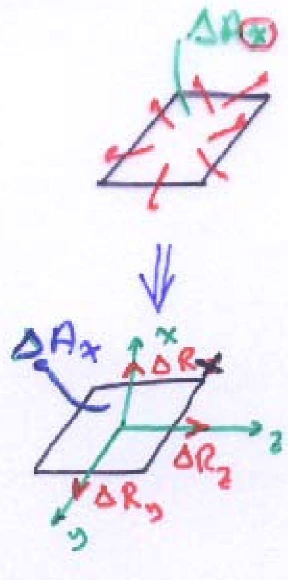
* Stresses

The forces per unit area (stresses) are:

$$\frac{\Delta R_x}{\Delta A_x}$$

$$\frac{\Delta R_y}{\Delta A_x}$$

$$\frac{\Delta R_z}{\Delta A_x}$$



Let's now take the limit as

$\Delta A_x \longrightarrow 0 \Rightarrow$

$$\lim_{\Delta A_x \to 0} \frac{\Delta R_x}{\Delta A_x} = \frac{dR_x}{dA_x} = \sigma_{x\,x}$$

x (second) ↳ means direction in x-axis

x (first) ↳ area $\perp$ to x-axis

" Since x is repeated in σ_{xx}, we can drop one $x \Rightarrow \boxed{\sigma_x}$ ← normal

$$\lim_{\Delta A_x \to 0} \frac{\Delta R_y}{\Delta A_x} = \frac{dR_y}{dA_x} = \sigma_{xy} \text{ or } \boxed{\tau_{xy}}$$ ← shear

$$\lim_{\Delta A_x \to 0} \frac{\Delta R_z}{\Delta A_x} = \frac{dR_z}{dA_x} = \sigma_{xz} \text{ or } \boxed{\tau_{xz}}$$

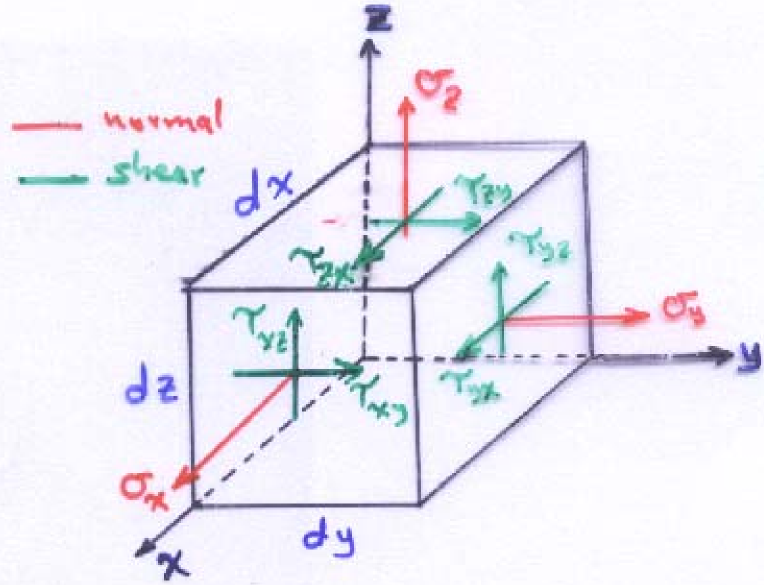
Similarly for $\Delta A_y \Rightarrow \underline{\underline{\tau_{yx}}}$, $\underline{\underline{\sigma_{y}}}$, $\underline{\underline{\tau_{yz}}}$

Similarly for $\Delta A_z \Rightarrow \underline{\underline{\tau_{zx}}}$, $\underline{\underline{\tau_{zy}}}$; $\underline{\underline{\sigma_{z}}}$

Now, let's consider 3-D differential volume element:

There are 9 stress components on the positive faces

((Note that the stresses on the negative faces are equal & opposite.))



* 3 normal stresses:

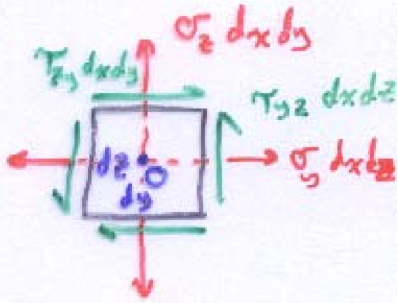
$$\sigma_x \, , \, \sigma_y \, , \, \sigma_z$$

* 6 shearing stresses:

$$\tau_{xy} \, , \, \tau_{xz} \, , \, \tau_{yx} \, , \, \tau_{yz} \, , \, \tau_{zx} \, , \, \tau_{zy}$$

Now, we need to find some relations among shearing stresses (if there are any).

Consider the differential area shown ⇒



$\overset{+}{\circlearrowleft} \Sigma M_o = 0 \Rightarrow$

$$2\left[\tau_{yz}\, dx dz \left(\frac{dy}{2}\right)\right] - 2\left[\tau_{zy}\, dx dy \left(\frac{dz}{2}\right)\right] = 0$$

$$\Rightarrow \quad \tau_{yz} = \tau_{zy}$$

Similarly,

$$\tau_{xy} = \tau_{yx}$$

$$\tau_{xz} = \tau_{zx}$$

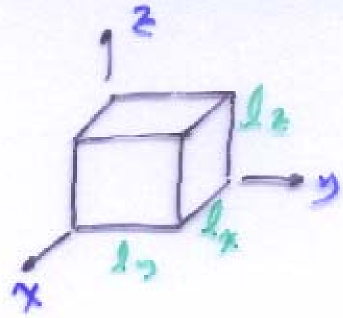
⇒ There are only <u>6</u> <u>independent</u> stress components at any point:

3 normal : $\sigma_x \, , \, \sigma_y \, , \, \sigma_z$

3 shear : $\tau_{xy} \, , \, \tau_{xz} \, , \, \tau_{yz}$

Strains:

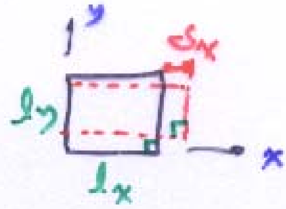
General strain state:



3-D

Consider normal strains:

Take x as an example



2-D

$$\varepsilon_x = \frac{\delta_x}{l_x}$$

Similarly,

$$\varepsilon_y = \frac{\delta_y}{l_y}$$

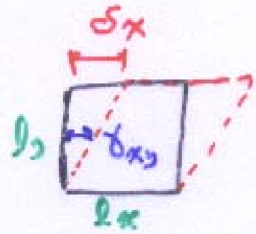
$$\varepsilon_z = \frac{\delta_z}{l_z}$$

Normal strains are a measure of volumetric alteratio[n]

Note that the 90° angles remain 90°

Now, consider shearing strains:

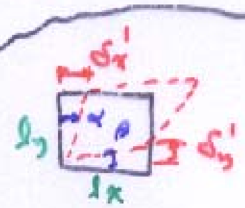
$$\gamma_{xy} = \frac{\delta_x}{l_y}$$



Similarly,

$$\gamma_{xz} = \frac{\delta_x}{l_z}$$

$$\gamma_{yz} = \frac{\delta_y}{l_z}$$



optional $\Leftarrow$ $\gamma_{xy} = \underbrace{\frac{\delta_x'}{l_y}}_{\alpha} + \underbrace{\frac{\delta_y'}{l_x}}_{\beta}$; $\gamma_{xz} = \frac{\delta_x'}{l_z} + \frac{\delta_z'}{l_x}$; $\gamma_{yz} = \frac{\delta_y'}{l_z} + \frac{\delta_z'}{l_y}$

Summary
- 3 normal strains: ε_x, ε_y, ε_z
- 3 Shearing strains: γ_{xy}, γ_{xz}, γ_{yz}

* Normal strains change lengths: $+\Delta l$ gives $\oplus$ $\Rightarrow$ T (C $\Rightarrow$ $\ominus$)

* Shearing strains change the 90° angles: $<90° = \oplus$; $>90° = \ominus$