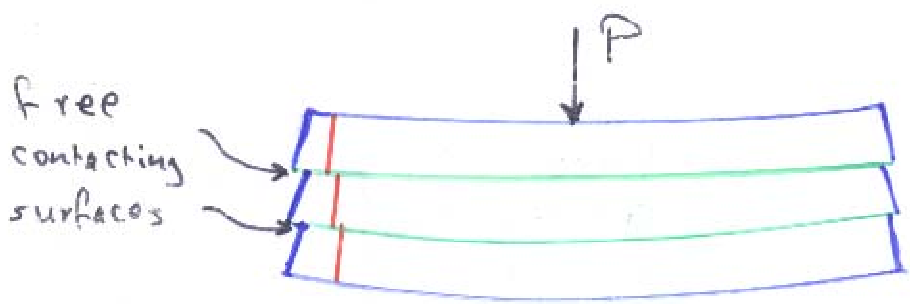
Shearing Stresses in Beams

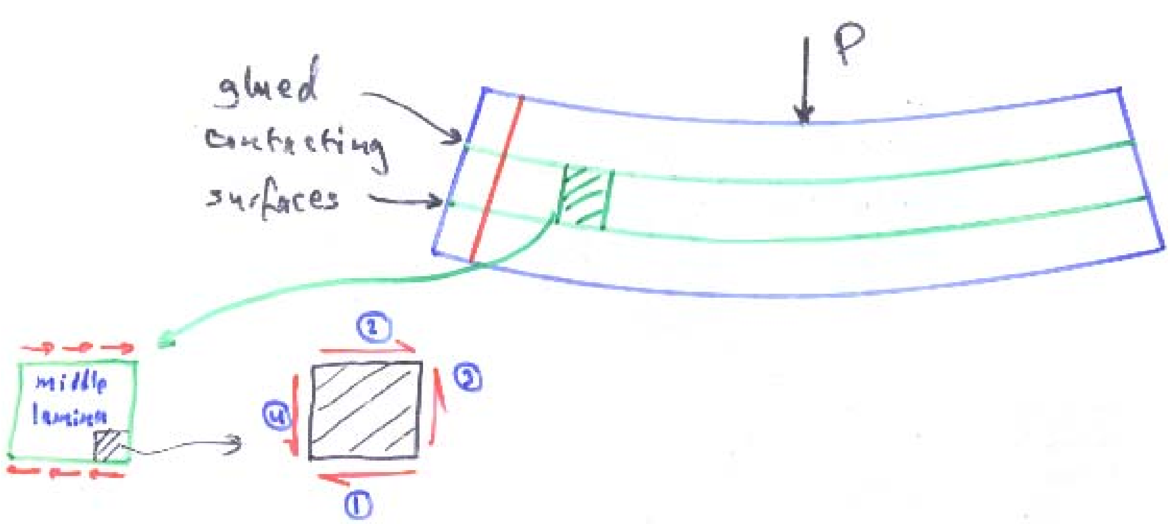
Previously, pure bending was assumed. ⇒ $V = 0$

In most beams, shear forces exist. ⇒ We need to take this into consideration.

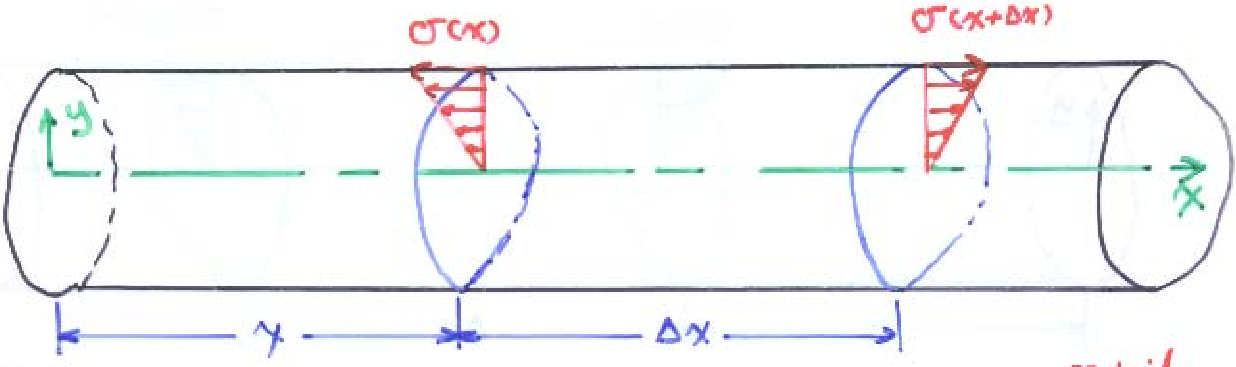
The cross section actually warps; however, the shearing strain which accompanies the warping has little effect on the normal strain; thus, the flexure formula remains approximately valid.

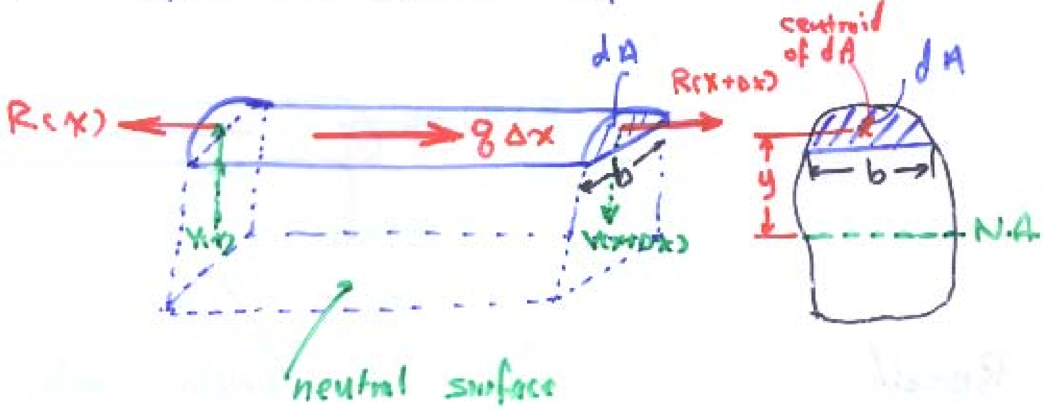
The existance of horizontal shear in beams:





Shear Flow & Shearing Stress





Recall

$$\sigma(x,y) = -\frac{M(x)\,y}{I}$$

$$\sigma(x+\Delta x,y) = -\frac{M(x+\Delta x)\,y}{I}$$

$$R(x) = \int_A \sigma(x,y)\,dA = \int_A -\frac{M(x)\,y}{I}\,dA = -\frac{M(x)}{I}\int_A y\,dA$$

$$R(x+\Delta x) = \int_A \sigma(x+\Delta x,y)\,dA = -\frac{M(x+\Delta x)}{I}\int_A y\,dA$$

Let $\int y\,dA = Q$ = first moment of the area wrt the C.A.

$$\Rightarrow R(x) = -\frac{M(x)}{I}\,Q$$

$$R(x+\Delta x) = -\frac{M(x+\Delta x)}{I}\,Q$$

Recall that $\dfrac{dR}{dx} = \lim\limits_{\Delta x\to 0}\dfrac{R(x+\Delta x)-R(x)}{\Delta x} = \lim\limits_{\Delta x\to 0} -\left(\dfrac{M(x+\Delta x)-M(x)}{\Delta x}\right)\dfrac{Q}{I}$

$$\Rightarrow \frac{dR}{dx} = -\frac{dM}{dx}\,\frac{Q}{I} \Rightarrow \frac{dR}{dx} = -\frac{VQ}{I} \quad (1)$$

From Equilibrium of the FBD above,

$$\Sigma F_x = 0 \Rightarrow R(x+\Delta x) - R(x) + q\,\Delta x = 0$$

Dividing by Δx $\Rightarrow \dfrac{R(x+\Delta x) - R(x)}{\Delta x} + q = 0 \Rightarrow \lim_{\Delta x \to 0} \dfrac{dR}{dx} + q = 0$

$$\Rightarrow \frac{dR}{dx} = -q \qquad (2)$$

$\Rightarrow$ From eqs. (1), (2) above:

$$\boxed{q = \frac{VQ}{I}}$$

This is the well-known shear flow (q) formula $\Rightarrow$

τ (shearing stress) has a complicated distribution over the width of the cutting plane (b); however, it is assumed constant over such width.

$$\Rightarrow \boxed{\tau = \frac{q}{b} = \frac{VQ}{Ib}} \Leftarrow \text{Shearing Stress formula}$$

* Very Very Important (!!!):

How to calculate Q ??!!

Something is "strange" in this derivation! What is it?!

Only equilibrium consideration is utilized in this derivation. Kinematic (Geom. Compat.) assumptions are not used!

This is one of the very few formulas in structural mechanics in which compatibility is not utilized in deriving the formula!