King Fahd University of Petroleum & Minerals
DEPARTMENT OF CIVIL ENGINEERING

# CE 201 STATICS

## First Major Exam

Tue. 3/Nov./09
8:00 p.m. → 10:00 p.m.

Student Name : Solved by the instructors who put the problems (Coordinated course) .........

Student I.D. # : ........................................

SECTION:

| Question | Grade | Score |
|---|---|---|
| 1 | 25 | |
| 2 | 25 | |
| 3 | 25 | |
| 4 | 25 | |
| **TOTAL** | **100** | |

*Good luck !*

**Question # 1 (25%)**

For the figure shown below:

8% a) Express forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in Cartesian vector forms.

8% b) Use the dot product to determine the angle between ($\mathbf{F}_1$) and ($\mathbf{F}_2$).

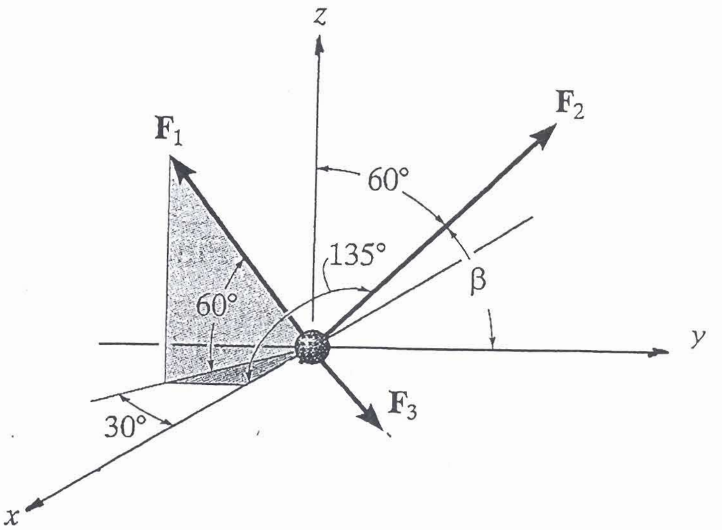
9% c) Use the dot product to determine the projection of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_3$.

Use the following information for this problem:

$|\tilde{F}_1| = 200$ N,

$|\tilde{F}_2| = 100$ N,

$\mathbf{F}_3 = \{80\tilde{\mathbf{i}} + 60\tilde{\mathbf{j}} - 40\tilde{\mathbf{k}}\}$ N




1) For F1

$F_1'$ (on x-y plane)

$= 200 \cos 60 = 100$ N

$\Rightarrow F_{1x} = 100 \cos 30$ (+ve x)

$= 86.60$ N

$F_{1y} = 100 \sin 30$ (−ve y-axis)

$= 50$ N (−ve y-axis)

$F_{1z} = 200 \sin 60$

$= 173.21$ N

4 $\Rightarrow \bar{F}_1 = \{86.60\,\hat{i} - 50\,\hat{j} + 173.21\,\hat{k}\}$ N

For $F_2$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \Rightarrow \cos^2\beta = 1 - \cos^2\alpha - \cos^2\gamma$

$\cos^2\beta = 1 - \cos^2 135 - \cos^2 60 \Rightarrow \beta = 60°$

$\bar{F}_2 = 100\,N\,(\cos 135\,\hat{i} + \cos 60\,\hat{j} + \cos 60\,\hat{k})$

4 $= \{-70.71\,\hat{i} + 50\,\hat{j} + 50\,\hat{k}\}$ N

b) $$\cos\theta = \frac{\bar{F}_1 \circ \bar{F}_2}{F_1 * F_2}$$

$$= \frac{86.60 * -70.71 - 50 * 50 + 173.21 * 50}{200 * 100}$$

$$\cos\theta = \frac{37.014}{20000} \Rightarrow \theta = 89.89^\circ$$

c) Projection of $\bar{F}_1$ along the Line of Action of $\bar{F}_3$

$|F_3| = 107.70$

4 $$F_{1\,\text{along}\,\bar{F}_3} = \bar{F}_1 \cdot \bar{u}_{F3}$$

4 $$\bar{u}_{F3} = \frac{80\hat{i} + 60\hat{j} - 40\hat{k}}{\sqrt{(80)^2 + (60)^2 + (-40)^2}} = \{0.742\hat{i} + 0.558\hat{j} - 0.371\hat{k}\}\ m$$

$$F_{1\,\text{along}\,\bar{F}_3} = \{86.6 * 0.742 - 50 * 0.558 - 173.21 * 0.371\}\ N$$

$$= -27.80\ N$$

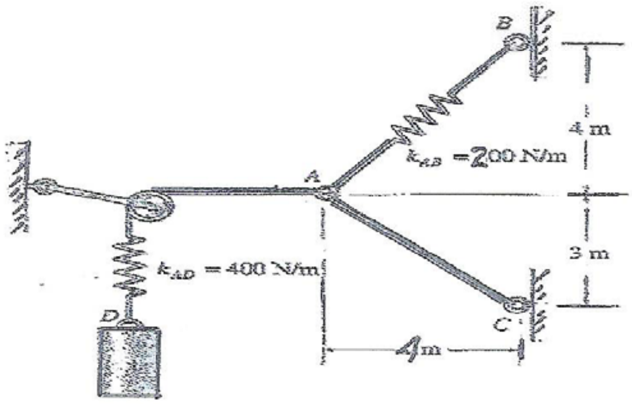
1 or $20.69\hat{i} - 15.52\hat{j} + 10.35\hat{k}$

### Problem-2 (25 points)

Equilibrium has been reached in the cables, pulley, and spring system as shown in the figure below. If the mass $m_D$, attached to $D$ is $20$ kg, determine;

(a) The extension in spring $AD$.
(b) The force in cable $AC$, $F_{AC}$.
(c) The distance between $A$ and $B$ before mass, $m_D$ is added to the system.

Note: Draw appropriate *free-body-diagrams* (**FBD**) to illustrate and justify your answers.

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***Solution:***

(a) Extension in Spring AD;

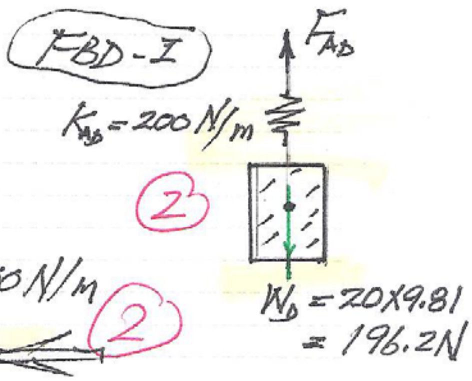
$\sum F_y + \uparrow = 0;$

$F_{AD} - W_D = 0; \quad F_{AD} = 196.2N$

But $F_{AD} = K_{AD}\Delta_{AD}; \quad \Delta_{AD} = F_{AD}/K_{AD}$

$= 196.2N / 400 N/m$

$= 0.4905\ m$ 2



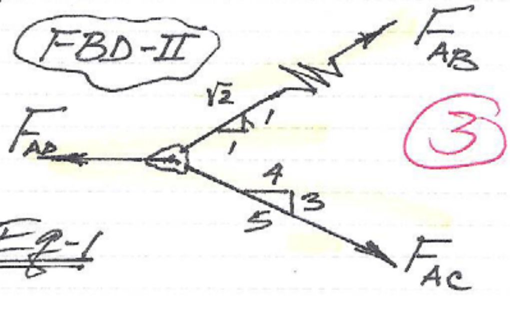

(b) Force in Cable AC, $F_{AC}$;

From FBD-II;

$\overset{+}{\rightarrow}\sum F_x = 0;$

3 $-F_{AD} + \frac{1}{\sqrt{2}}F_{AB} + \frac{4}{5}F_{AC} = 0,$

$-196.2 + \frac{1}{\sqrt{2}}F_{AB} + \frac{4}{5}F_{AC} = 0;$ Eq-I




$\sum F_y + \uparrow = 0;$

$\frac{1}{\sqrt{2}}F_{AB} - \frac{3}{5}F_{AC} = 0,$

$F_{AB} = \frac{3\sqrt{2}}{5}F_{AC}$ Eq-II

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**Solution:** Substituting for $F_{AB}$ in Eq-I;

$$-196.2 + \frac{1}{\sqrt{2}}\left(\frac{3\sqrt{2}}{5}\right)F_{AC} + \frac{4}{5}F_{AC} = 0$$

3

$$\therefore \quad F_{AC} = \underline{140.143\,N}$$

3, 4

Ⓒ The distance between A and B before $m_D$ is added to the system;

We have; $F_{AB} = \frac{3\sqrt{2}}{5}F_{AC} = \frac{3\sqrt{2}}{5}(140.143)$ 3

$= \underline{118.915\,N}$ $\sqrt{2}$, 1, 1

But $F_{AB} = K_{AB}\,\Delta_{AB}$

$$\therefore \quad \Delta_{AB} = \frac{F_{AB}}{K_{AB}} = 118.915\,N / 200\,N/m$$

$= \underline{0.595\,m}$ 3

and $\Delta_{AB} = L_{AB} - L'_{AB}$ 3

where $L_{AB} = \sqrt{4^2+4^2} = \underline{5.657\,m}$

and $L'_{AB}$ = distance between A and B before $m_D$ is added to the system.

$$\therefore \quad L'_{AB} = L_{AB} - \Delta_{AB}$$

$= 5.657 - 0.595$

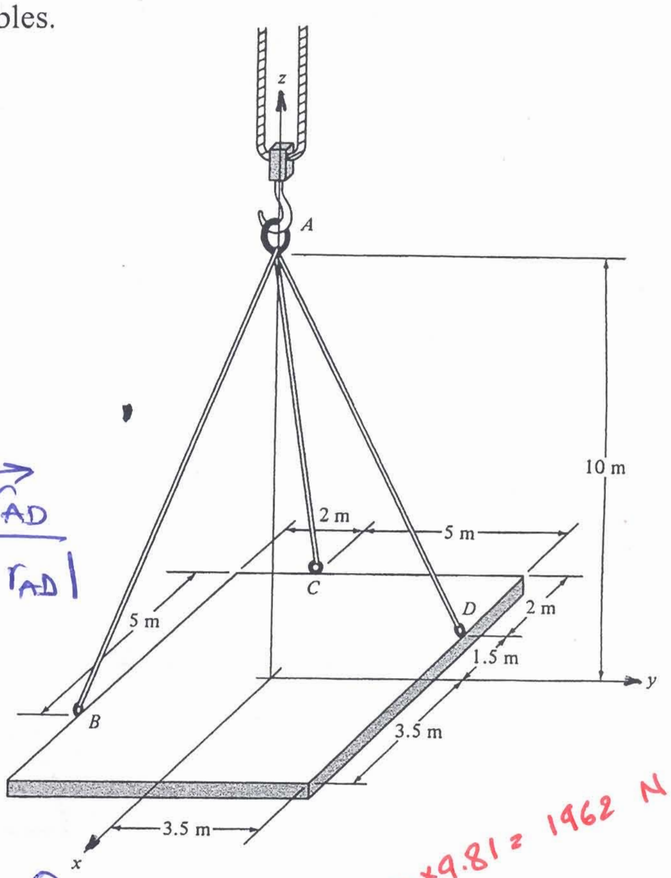
$= \underline{5.062\,m}$ 3

**Question # 3 (25%)**

A 200 kg plate is held by three cables (AB, AC & AD) as shown below:

21% A) Set the governing equations for the determination of tensile forces in the three cables.

4% B) Find the tension in these three cables.




PART A $\boxed{21}$

$$\vec{T}_{AB} = T_{AB}(\vec{u}_{AB})$$

$$\vec{T}_{AC} = T_{AC}(\vec{u}_{AC})$$

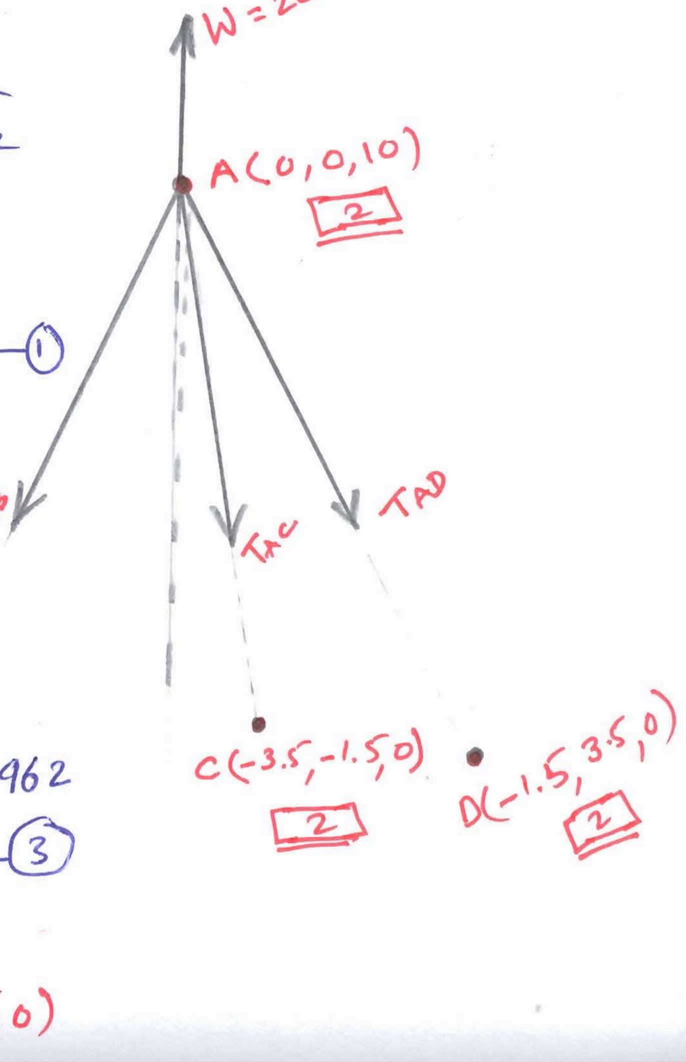
$$\vec{T}_{AD} = T_{AD}(\vec{u}_{AD})$$

$$\vec{u}_{AB} = \frac{\vec{r}_{AB}}{|r_{AB}|} \; ; \; \vec{u}_{AC} = \frac{\vec{r}_{AC}}{|r_{AC}|} \; ; \; \vec{u}_{AD} = \frac{\vec{r}_{AD}}{|r_{AD}|}$$

$$\vec{u}_{AB} = 0.14\hat{i} - 0.327\hat{j} - 0.9345\hat{k}$$ $\boxed{2}$

$$\vec{u}_{AC} = -0.327\hat{i} - 0.14\hat{j} - 0.9345\hat{k}$$ $\boxed{2}$

$$\vec{u}_{AD} = -0.14\hat{i} + 0.327\hat{j} - 0.9345\hat{k}$$ $\boxed{2}$




$\Sigma F_x = 0$

$$0.14\,T_{AB} - 0.327\,T_{AC} - 0.14\,T_{AD} = 0 \quad \text{—} (1)$$ $\boxed{2}$

$\Sigma F_y = 0$

$$-0.327\,T_{AB} - 0.14\,T_{AC} + 0.327\,T_{AD} = 0 \quad \text{—} (2)$$ $\boxed{2}$

$\Sigma F_z = 0$

$$+0.9345\,T_{AB} + 0.9345\,T_{AC} + 0.9345\,T_{AD} = 1962 \quad \text{—} (3)$$ $\boxed{3}$

PART B $\boxed{4}$

$$0.14\,T_{AB} - 0.327\,T_{AC} - 0.14\,T_{AD} = 0 \quad \text{— (1)}$$

$$-0.327\,T_{AB} - 0.14\,T_{AC} + 0.327\,T_{AD} = 0 \quad \text{— (2)}$$

Multiply (1) by 2.336 $\boxed{1}$

$$0.327\,T_{AB} - 0.764\,T_{AC} - 0.327\,T_{AD} = 0 \quad \text{— (1)}$$

Add (1) and (2) to get

$$-0.904\,T_{AC} = 0$$

$$\therefore \boxed{T_{AC} = 0} \quad \boxed{1}$$

from (1)

$$T_{AB} = T_{AD}$$

Substitute in (3) to get

$$\boxed{T_{AB} = T_{AD} = 1049.7 \simeq 1050\ N}$$

$\boxed{2}$

**Problem # 4** (25 points)

A force **P** of magnitude 500 N is acting along a line $AB$ as shown in Fig. below :

i) Express the force **P** in Cartesian Vector Form [10 Points]

ii) Determine the moment of the force **P** about point $Q$ [15 Points]

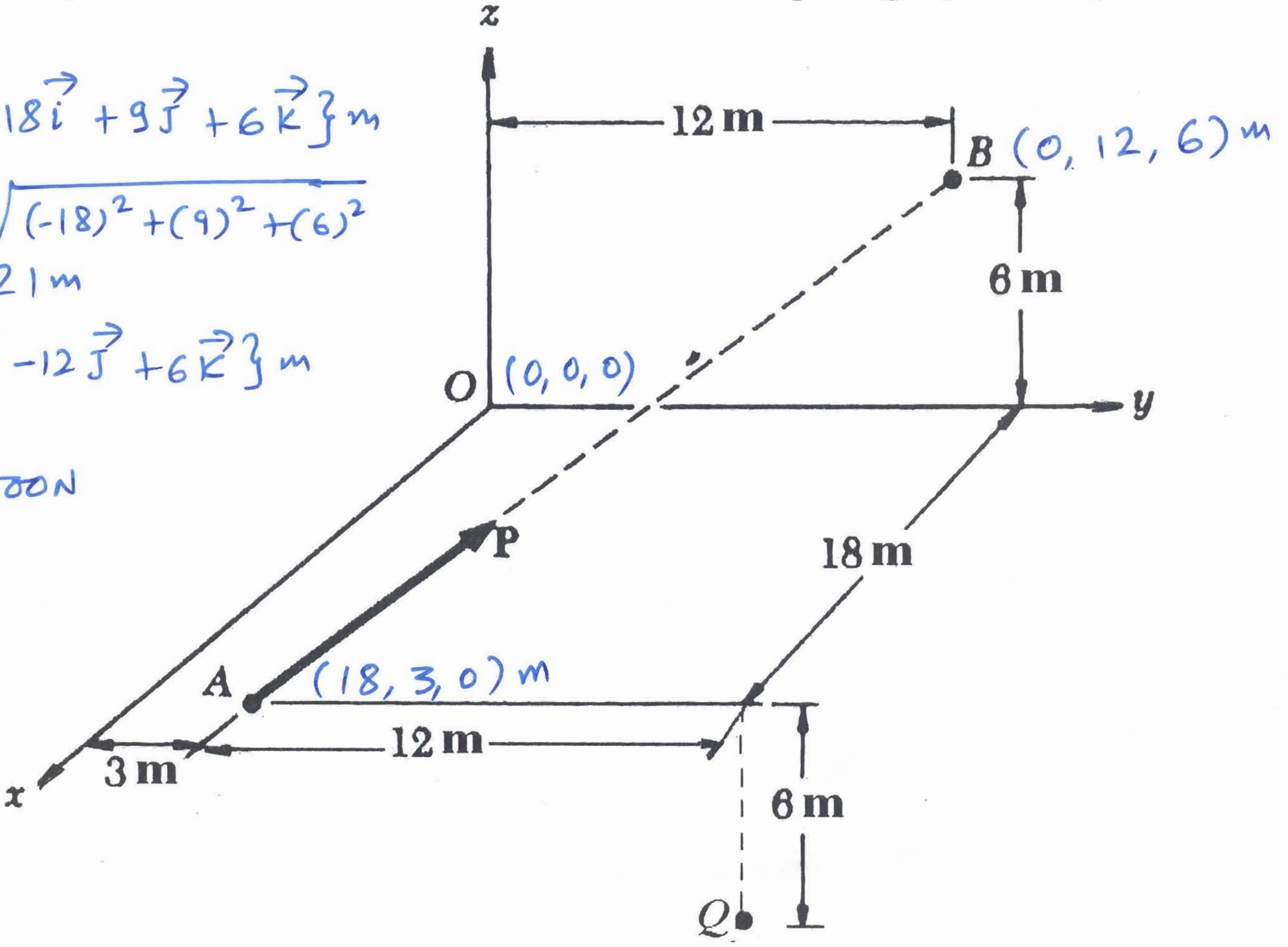
$\vec{r}_{AB} = \{-18\vec{i} + 9\vec{j} + 6\vec{k}\}$ m

$|\vec{r}_{AB}| = \sqrt{(-18)^2 + (9)^2 + (6)^2}$

$= 21$ m

$\vec{r}_{QA} = \{-12\vec{j} + 6\vec{k}\}$ m

$|\vec{P}| = 500$ N




(i) $\vec{P} = |\vec{P}| \dfrac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$

$= \dfrac{500}{21}\{-18\vec{i} + 9\vec{j} + 6\vec{k}\} = \{-428.57\vec{i} + 214.28\vec{j} + 142.85\vec{k}\}$ N

Ans

(ii) $\vec{M}_Q = \vec{r}_{QA} \times \vec{P} = \{-12\vec{j} + 6\vec{k}\} \times \{-428.57\vec{i} + 214.28\vec{j} + 142.85\vec{k}\}$

$= (-12)(-428.57)(-\vec{k}) + (-12)(142.85)(\vec{i})$

$+ (6)(-428.57)(\vec{j}) + (6)(214.28)(-\vec{i})$

$= \{-3000\vec{i} - 2571\vec{j} - 5143\vec{k}\}$ N-m

$|\vec{M}_Q| = \sqrt{(-3000)^2 + (-2571)^2 + (-5143)^2} = 6485$ N-m

Ans