

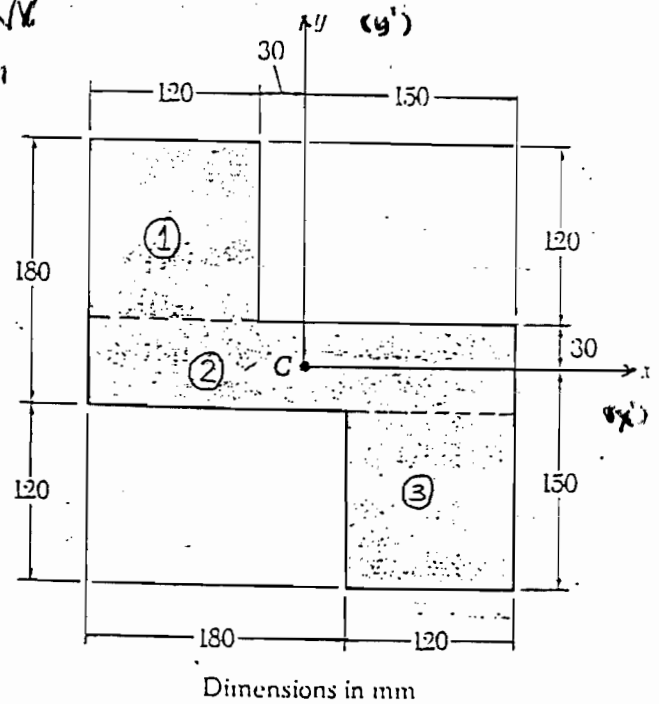
Last H.W.
Solution

PROBLEM # 1

Given: The section shown

Required: I_x and I_y

Solution:



Part	$\bar{I}_{x'}$	$\bar{I}_{y'}$	Area A	d_y	d_x	$\bar{I}_{x'} + Ad_y^2$	$\bar{I}_{y'} + Ad_x^2$
1	$\frac{120(120)^3}{12}$ $= 17.28 \times 10^6$	$\frac{120(120)^3}{12}$ $= 17.28 \times 10^6$	$(120)^2$ $= 14400$	$30+60$ $= 90$	$30+60$ $= 90$	133.9×10^6	133.9×10^6
2	$\frac{300(60)^3}{12}$ $= 5.4 \times 10^6$	$\frac{60(300)^3}{12}$ $= 135 \times 10^6$	60×300 $= 18000$	0	0	5.4×10^6	135×10^6
3	$\frac{120(120)^3}{12}$ $= 17.28 \times 10^6$	$\frac{120(120)^3}{12}$ $= 17.28 \times 10^6$	$(120)^2$ $= 14400$	$30+60$ $= 90$	$30+60$ $= 90$	133.9×10^6	133.9×10^6
Σ						273.2×10^6	402.8×10^6

$$I_x = \Sigma(\bar{I}_{x'} + Ad_y^2) \Rightarrow I_x = 273 \times 10^6 \text{ mm}^4 \quad \underline{\underline{\text{Ans}}}$$

$$I_y = \Sigma(\bar{I}_{y'} + Ad_x^2) \Rightarrow I_y = 403 \times 10^6 \text{ mm}^4 \quad \underline{\underline{\text{Ans}}}$$

$$\begin{aligned} \bar{I}_{x'} &= I_x & \text{as } x \text{ is } x' \\ \bar{I}_{y'} &= I_y & \text{as } y \text{ is } y' \end{aligned}$$

PROBLEM # 2

Given: The section shown

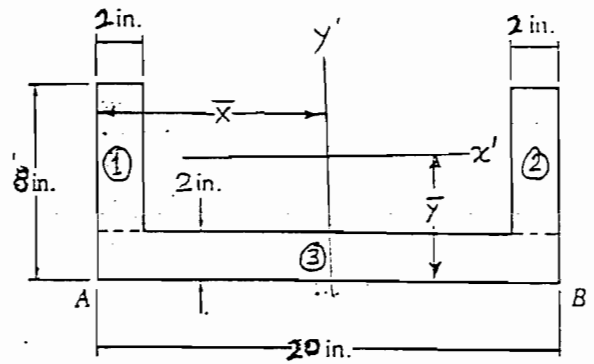
Required: \bar{I}_x and \bar{I}_y

Solution:

∵ Section is symmetrical w.r.t vertical axis

$$\therefore \bar{x} = \frac{20}{2} = 10 \text{ in}$$

$$\bar{y} = \frac{2(6 \times 2) \times (2+3) + (20 \times 2)(1)}{2(6 \times 2) + (20 \times 2)} = 2.5 \text{ in}$$



Part	$\bar{I}_{x'}$	$\bar{I}_{y'}$	Area A	d_y	d_x	$\bar{I}_{x'} + Ad_y^2$	$\bar{I}_{y'} + Ad_x^2$
1	$\frac{2 \times 6^3}{12}$ = 36	$\frac{6 \times 2^3}{12}$ = 4	6×2 = 12	2.5	9	111	976
2	$\frac{2 \times 6^3}{12}$ = 36	$\frac{6 \times 2^3}{12}$ = 4	6×2 = 12	2.5	9	111	976
3	$\frac{20 \times 2^3}{12}$ = 13.33	$\frac{2 \times 20^3}{12}$ = 1333.3	20×2 = 40	1.5	0	103.33	1333.3
Σ						325.3	3285.3

$$\bar{I}_x = \sum_{i=1}^3 (\bar{I}_{x'} + Ad_y^2)_i \Rightarrow \boxed{\bar{I}_x = 325 \text{ in}^4} \quad \underline{\underline{\text{Ans}}}$$

$$\bar{I}_y = \sum_{i=1}^3 (\bar{I}_{y'} + Ad_x^2)_i \Rightarrow \boxed{\bar{I}_y = 3285 \text{ in}^4} \quad \underline{\underline{\text{Ans}}}$$

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PROBLEM # 3

Given: The same section as given in problem #2

Required: Moment inertia of the section w.r.t an axis passing through AB, I_{AB} .

Solution:

Part	$\overline{I_{x'}}$	Area A	d_{AB}	$\overline{I_{x'}} + A d_{AB}^2$
1	$\frac{2 \times 6^3}{12}$ = 36	6×2 = 12	5	336
2	$\frac{2 \times 6^3}{12}$ = 36	6×2 = 12	5	336
3	$\frac{20 \times 2^3}{12}$ = 13.33	20×2 = 40	1	53.33
Σ				725.33

$$I_{AB} = \sum_{i=1}^3 (\overline{I_{x'}} + A d_{AB}^2)_i \Rightarrow \boxed{I_{AB} = 725 \text{ in}^4} \text{ Ans}$$

Problem 2 required to find location of centroidal axes of the section, while in this problem the axis was pre-located. \Rightarrow It is easier.

It should also be noted that the moment of inertia is always smallest about centroidal axis as compared to any other parallel axis.

④

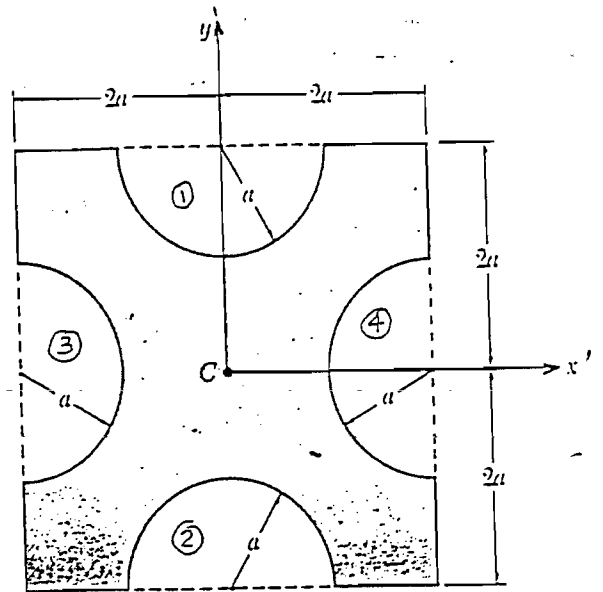
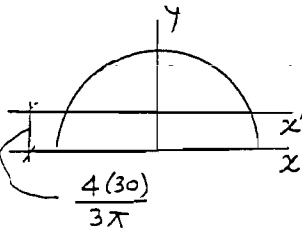
PROBLEM #4Given: $a = 30 \text{ mm}$ Required: I_x and I_y

Solution:

$$\bar{I}_{x'} = I_x - Ad^2$$

$$\therefore \bar{I}_{x'} = \frac{\pi(30)^4}{8} - \frac{\pi(30)^2}{2} \left(\frac{4(30)}{3\pi} \right)^2$$

$$\Rightarrow \bar{I}_{x'} = 88.90 \times 10^3 \text{ mm}^4$$



Part	$\bar{I}_{x'}$	$\bar{I}_{y'}$	Area A	d_y	d_x	$\bar{I}_{x'} + Ad_y^2$	$\bar{I}_{y'} + Ad_x^2$
Square $4a \times 4a$	$\frac{(120)^4}{12}$ $= 17.28 \times 10^6$	$\frac{(120)^4}{12}$ $= 17.28 \times 10^6$	120×120 $= 14400$	0	0	17.28×10^6	17.28×10^6
1	-88.90×10^3	$-\frac{\pi(30)^4}{8}$ $= -318.1 \times 10^3$	$-\frac{\pi(30)^2}{2}$ $= -1414$	$60 - \frac{4(30)}{3\pi}$ $= 47.27$	0	-3.248×10^6	-318.1×10^3
2	-88.90×10^3	-318.1×10^3	-1414	47.27	0	-3.248×10^6	-318.1×10^3
3	$-\frac{\pi(30)^4}{8}$ $= -318.1 \times 10^3$	-88.90×10^3	-1414	0	47.27	-318.1×10^3	-3.248×10^6
4	$= -318.1 \times 10^3$	-88.90×10^3	-1414	0	47.27	-318.1×10^3	-3.248×10^6
Σ						10.15×10^6	10.15×10^6

$$I_x = \sum_{i=1}^4 (\bar{I}_{x'} + Ad_y^2)_i \Rightarrow \boxed{I_x = 10.1 \times 10^6 \text{ mm}^4} \quad \underline{\underline{\text{Ans}}}$$

$$I_y = \sum_{i=1}^4 (\bar{I}_{y'} + Ad_x^2)_i \Rightarrow \boxed{I_y = 10.1 \times 10^6 \text{ mm}^4} \quad \underline{\underline{\text{Ans}}}$$

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PROBLEM # 5

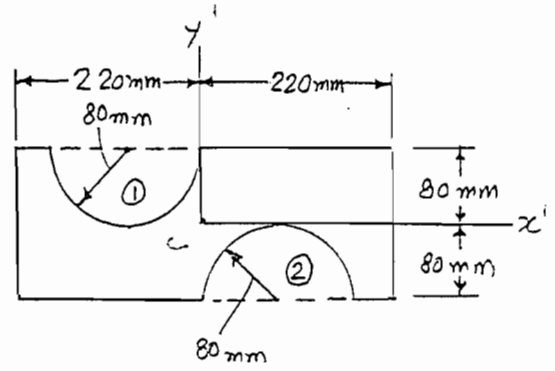
Given: The section shown

Required: \bar{I}_x and \bar{I}_y

Solution:

∵ Section area is symmetrical
w.r.t both horizontal and vertical
axes

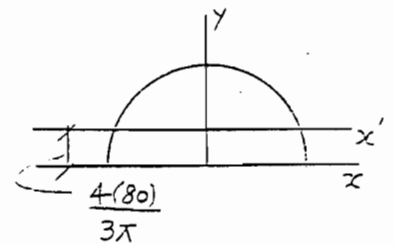
∴ the centroidal x - y axes are located as shown.



Part	$\bar{I}_{x'}$	$\bar{I}_{y'}$	Area A	d_y	d_x	$\bar{I}_{x'} + Ad_y^2$	$\bar{I}_{y'} + Ad_x^2$
Rectangle 440 x 160	$\frac{440(160)^3}{12}$ $= 150.2 \times 10^6$	$\frac{160(440)^3}{12}$ $= 1135.8 \times 10^6$	160×440 $= 70400$	0	0	150.2×10^6	1135.8×10^6
1	-4.496×10^6	$-\frac{\pi(80)^4}{8}$ $= -16.08 \times 10^6$	$-\frac{\pi(80)^2}{2}$ $= -10.05 \times 10^3$	$\frac{80 - 4(80)}{3\pi}$ $= 46.05$	80	-25.81×10^6	-80.4×10^6
2	-4.496×10^6	-16.08×10^6	$-\frac{\pi(80)^2}{2}$ $= -10.05 \times 10^3$	$= 46.05$	80	-25.81×10^6	-80.4×10^6
Σ						98.58×10^6	975×10^6

$$\bar{I}_{x'} = \sum_{i=1}^n (\bar{I}_{x'_i} + Ad_y^2) \Rightarrow \boxed{\bar{I}_{x'} = 98.6 \times 10^6 \text{ mm}^4} \quad \underline{\underline{\text{Ans}}}$$

$$\bar{I}_{y'} = \sum_{i=1}^n (\bar{I}_{y'_i} + Ad_x^2) \Rightarrow \boxed{\bar{I}_{y'} = 975 \times 10^6 \text{ mm}^4} \quad \underline{\underline{\text{Ans}}}$$



$$\bar{I}_{x'} = I_x - Ad^2$$

$$\therefore \bar{I}_{x'} = \frac{\pi(80)^4}{8} - \frac{\pi(80)^2}{2} \left\{ \frac{4(80)}{3\pi} \right\}^2$$

$$\Rightarrow \bar{I}_{x'} = 4.496 \times 10^6 \text{ mm}^4$$