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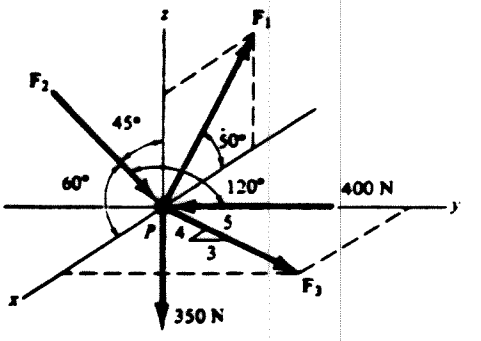
Examples

Equilibrium of a Particle
(3-D Force Systems)

Example 1:



Given:

The forces shown on P; Equilibrium

Req'd.:

The magnitudes of $\vec{F_1}$, $\vec{F_2}$, and $\vec{F_3}$

Soln.:

$\vec{F_1} = -F_1 \cos 50^\circ \vec{i} + F_1 \sin 50^\circ \vec{k}$

$\vec{F_2} = -F_2 \cos 60^\circ \vec{i} + F_2 \underbrace{\cos 60^\circ}_{-\cos 120^\circ} \vec{j} - F_2 \cos 45^\circ \vec{k}$

$\vec{F_3} = 0.8 F_3 \vec{i} + 0.6 F_3 \vec{j}$

Note that one of the angles (60°, 120°, 45°) can be calculated, if not given. Show!

$\Sigma F_x = 0 \Rightarrow -F_1 \cos 50^\circ - F_2 \cos 60^\circ + 0.8 F_3 = 0$ ①

$\Sigma F_y = 0 \Rightarrow F_2 \cos 60^\circ + 0.6 F_3 - 400 = 0$ ②

$\Sigma F_z = 0 \Rightarrow F_1 \sin 50^\circ - F_2 \cos 45^\circ - 350 = 0$ ③

From ①, $F_1 = \dfrac{350 + F_2 \cos 45^\circ}{\sin 50^\circ}$ ④

From ②, $F_3 = \dfrac{400 - F_2 \cos 60^\circ}{0.6}$ ⑤

From ④ and ⑤ into ①,

$-(350 + F_2 \cos 45^\circ) \underbrace{\dfrac{\cos 50^\circ}{\sin 50^\circ}}_{1/\tan 50^\circ} - F_2 \cos 60^\circ + \dfrac{0.8}{0.6}(400 - F_2 \cos 60^\circ) = 0$

$\Rightarrow 1.76 F_2 = 239.6$

$\Rightarrow \boxed{F_2 = 136 \text{ N}}$

$\Rightarrow F_1 = \dfrac{350 + 136 \cos 45^\circ}{\sin 50} \Rightarrow \boxed{F_1 = 583 \text{ N}}$

$F_3 = \dfrac{400 - 136 \cos 60^\circ}{0.6} \Rightarrow \boxed{F_3 = 553 \text{ N}}$

Note: 3 equations & 3 unknowns

Steps:

① Express F_i in Cartesian V.

② Add $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$.

③ Solve system of eqs.

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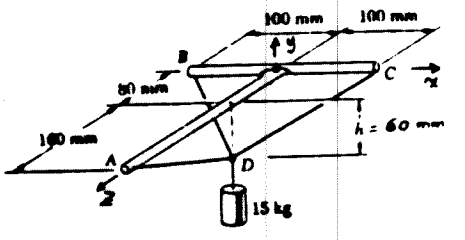
Example 2:



Given:

The figure shown (in equilibrium)

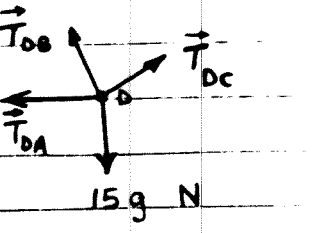
Req'd.:

The tension in the wires

Soln.:

A(0, 0, 240), B(-100, 0, 0)

C(100, 0, 0), D(0, -60, 80)



$\vec{DA} = 0\vec{i} + 60\vec{j} + 160\vec{k} \Rightarrow DA = 170.9$ mm

$\Rightarrow \vec{T}_{DA} = \frac{T_{DA}}{170.9}(0\vec{i} + 60\vec{j} + 160\vec{k})$ ← Note that $\vec{T}_{DA} = T_{DA}\vec{u}_{DA} = T_{DA}\frac{\vec{DA}}{DA}$

$\vec{DB} = -100\vec{i} + 60\vec{j} - 80\vec{k} \Rightarrow DB = 141.4$ mm

$\Rightarrow \vec{T}_{DB} = \frac{T_{DB}}{141.4}(-100\vec{i} + 60\vec{j} - 80\vec{k})$

$\vec{DC} = 100\vec{i} + 60\vec{j} - 80\vec{k} \Rightarrow DC = 141.4$ mm

$\Rightarrow \vec{T}_{DC} = \frac{T_{DC}}{141.4}(100\vec{i} + 60\vec{j} - 80\vec{k})$

From equilibrium and the FBD shown,

$\Sigma F_x = 0 \Rightarrow$

$$\frac{-100}{141.4}T_{DB} + \frac{100}{141.4}T_{DC} = 0 \quad (1)$$

$\Rightarrow T_{DB} = T_{DC}$

$$\Sigma F_y = 0 \Rightarrow -15(9.81) + \frac{60}{170.9}T_{DA} + \frac{60}{141.4}T_{DB} + \frac{60}{141.4}T_{DC} = 0 \quad (2)$$

$$\Sigma F_z = 0 \Rightarrow \frac{160}{170.9}T_{DA} - \frac{80}{141.4}T_{DB} - \frac{80}{141.4}T_{DC} = 0 \quad (3)$$

Solving eqs. (1), (2), and (3) yields

$T_{DA} = 139.8$ N

$T_{DB} = 115.6$ N

$T_{DC} = 115.6$ N

Steps:

1. Draw FBD
2. Write coordinates
3. Form position vectors
4. Form force vectors
5. Add $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$
6. Solve system of eqs.

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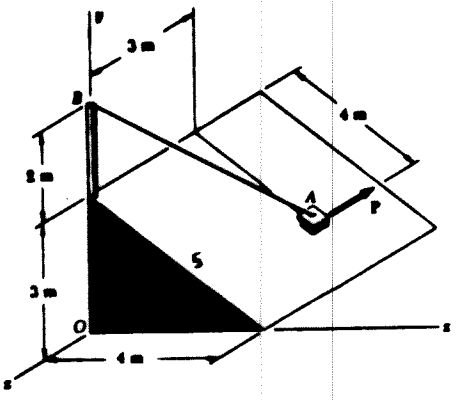
Example 3:

Given & Reqd.:

The 30-kg crate is held on the incline by the wire *AB* and by the horizontal force P which is directed parallel to the *z* axis. Since the crate is mounted on casters, the force exerted by the incline on the crate is perpendicular to the incline. Determine the magnitude of P and the tension in wire *AB*.



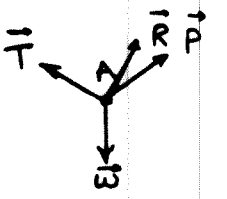
Soln.:

$A\left(4\left(\frac{4}{5}\right), 1\left(\frac{3}{5}\right), -3\right) = A\left(\frac{16}{5}, \frac{3}{5}, -3\right)$

$B(0, 5, 0)$

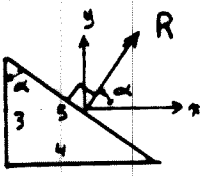
$\vec{AB} = -\frac{16}{5}\vec{i} + \frac{22}{5}\vec{j} + 3\vec{k}$

$= -3.2\vec{i} + 4.4\vec{j} + 3\vec{k} \Rightarrow AB = 6.213 \text{ m}$



$\vec{T} = \frac{T}{6.213}(-3.2\vec{i} + 4.4\vec{j} + 3\vec{k})$

$= T(-0.5151\vec{i} + 0.7082\vec{j} + 0.4829\vec{k})$



R is ⊥ to the plane shown ⇒

R is in the xy plane

$R_x = \frac{3}{5}R$

$R_y = \frac{4}{5}R$

$\Rightarrow \vec{R} = R(0.6\vec{i} + 0.8\vec{j})$

$\vec{P} = -P\vec{k}$

$\Sigma F_x = 0 \Rightarrow 0.6R - 0.5151T = 0$

$\Rightarrow R = 0.8583T$

$\Sigma F_y = 0 \Rightarrow -30(9.81) + 0.7082T + 0.8R = 0$

$\Rightarrow -294.3 + 0.7082T + 0.8(0.8583)T = 0$

$\Rightarrow \boxed{T = 211 \text{ N}}$

$\Sigma F_z = 0 \Rightarrow -P + 0.4829T = 0$

$\Rightarrow P = 0.4829(211)$

$\Rightarrow \boxed{P = 102 \text{ N}}$

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Example 4:

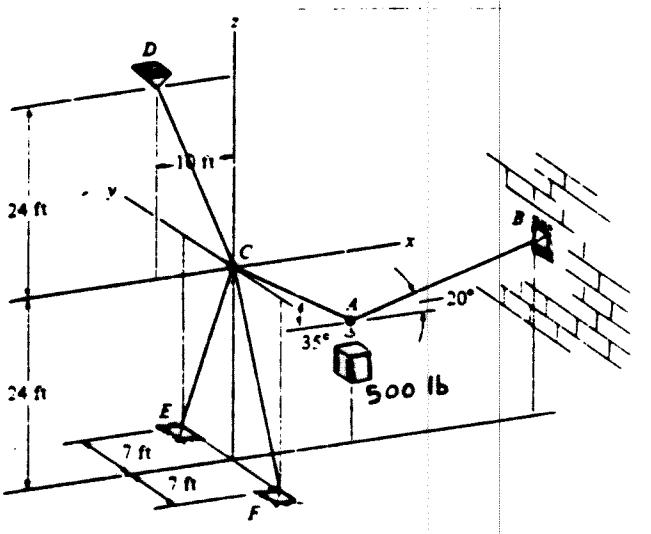
Given:

The system shown

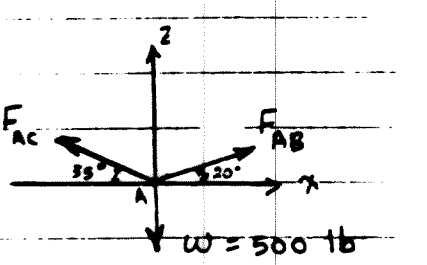
Req'd:

The forces in the cables

Soln.:



Note that the FBD of point A or C can be taken. However, by examining both, it can be concluded that we can not start with point C as there are 4 unknowns.



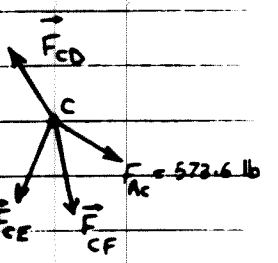
$\Rightarrow$ From FBD ①, $\Sigma F_x = 0 \Rightarrow$

$F_{AB} \cos 20^\circ - F_{AC} \cos 35^\circ = 0$ ①

$\Sigma F_z = 0 \Rightarrow F_{AB} \sin 20^\circ + F_{AC} \sin 35^\circ - 500 = 0$ ②

Solving ① and ② gives $\boxed{F_{AB} = 500 \text{ lb}}$; $\boxed{F_{AC} = 573.6 \text{ lb}}$ FBD ①

Now, FBD ② can be utilized,



C(0,0,0), D(-10,0,24), E(0,7,-24), F(0,-7,-24)

$\vec{CD} = -10\vec{i} + 24\vec{k} \Rightarrow CD = 26$ ft

$\Rightarrow \vec{F}_{CD} = \frac{F_{CD}}{26}(-10\vec{i} + 24\vec{k})$ ④

$\vec{CE} = 7\vec{j} - 24\vec{k} \Rightarrow CE = 25$ ft

$\Rightarrow \vec{F}_{CE} = \frac{F_{CE}}{25}(7\vec{j} - 24\vec{k})$ ⑤ FBD ②

$\vec{CF} = -7\vec{j} - 24\vec{k} \Rightarrow CF = 25$ ft

$\Rightarrow \vec{F}_{CF} = \frac{F_{CF}}{25}(-7\vec{j} - 24\vec{k})$ ⑥

$\vec{F}_{CA} = 573.6 \cos 35^\circ \vec{i} - 573.6 \sin 35^\circ \vec{k} = 469.9\vec{i} - 329\vec{k}$ ⑦

From eqs. ③-⑦

$\Sigma F_x = 0 \Rightarrow \frac{-10}{26} F_{CD} + 469.9 = 0 \Rightarrow \boxed{F_{CD} = 1220 \text{ lb}}$

$\Sigma F_y = 0 \Rightarrow \frac{7}{25} F_{CE} - \frac{7}{25} F_{CF} = 0 \Rightarrow F_{CE} = F_{CF}$

$\Sigma F_z = 0 \Rightarrow \frac{24}{26}(1220) - \frac{24}{25} F_{CE} - \frac{24}{25} \underbrace{F_{CF}}_{= F_{CE}} - 329 = 0 \Rightarrow \boxed{F_{CE} = F_{CF} = 416 \text{ lb}}$